Chapter 1

Optimal Strategy for Imperial Oil's Cold Lake Facilities

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1.1 Introduction

Imperial Oil is Canada's largest producer of crude oil and a major producer of natural gas. It is also the largest refiner and marketer of petroleum products – sold primarily under the Esso brand – with a coast-to-coast supply network and a major supplier of petrochemicals. The Resources Division manages Imperial's natural-resource operations and is located in Calgary, Alberta. It is a major developer of Canada's vast reserves of oil sands through its operation at Cold Lake, Alberta, and its participation in Syncrude Canada.

At Cold Lake, Alberta, Imperial Oil uses a cyclic steam stimulation process to produce heavy oil from oil sands formations. High pressure steam is generated at central plant facilities. The steam is distributed through a pipeline system and injected into the reservoir at wells located

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at a distance from the central plant facilities. Steam injection continues until the oil viscosity is such that the oil can be pumped to surface.

Oil, water and gas are produced during the production part of the cycle and are returned to the central plant facilities. Produced water is mixed with make-up water and is treated and reused as feed water for the steam generators. The produced gas supplements the purchased natural gas used as fuel for the steam generators. The produced oil is processed to remove sand and water. It is then diluted with a lighter hydrocarbon ("diluent") to meet pipeline viscosity specifications. The diluted bitumen is sold to refiners.

There are three central plant facilities at Cold Lake. A total of 3500 wells have been drilled to date of which 3200 are still active. Each well is associated with a single plant. The older wells have completed up to 10 cycles of steaming and production. New wells are periodically added. The duration of the steaming and production phases depends on the age of the well. The steaming phase lasts from 28 to more than 150 days. The production cycle lasts from 100 to 1600 days.

An important issue related to the operation of the Cold Lake system is whether the performance of the overall system can be optimized. Given the vast scale of the operation and the complicated interdependencies of the system, finding an optimal strategy for the entire operation seems to be an impossible task at the first glance. It was decided by the group during the workshop that solving a scaled-down version of the problem would be more productive. It was argued that a problem with many fewer wells could help the group to understand the full problem better and to identify suitable mathematical models.

The first part of this report summarizes the discussions and the models proposed for a four-well problem. With the help of our industrial participants from Imperial Oil, we have made suitable assumptions for the operation conditions and constraints and formulated the problem using several approaches. Results were obtained for some of the models. Because the models are nonlinear, one of the major difficulties associated with these models is that it becomes very expensive computationally to find the solution when the size of the problem grows. In other words, it becomes impractical when the number of wells reaches the level of the Cold Lake system.

The second part of the report discusses a new approach which was developed after the workshop by a smaller group of people. It turns out that the problem can be formulated as a linear programming (or linear optimization) problem in which the size of the problem is not related to the number of wells in the summary as in the non-linear approach, at least under special circumstances. Both continuous time formulation and the discrete time linear programming (LP) approach are discussed. The LP formulation can be viewed as a special discretization (maximizing sequence) of the continuous problem. Results using the LP approach are presented for a test case with 3200 wells at the beginning and another 6000 new wells added during the operation of the system, which is comparable to the size of the Cold Lake system. A number of simplifications are made for the test case. For example, we assumed that all the wells are the same and the performance of the wells is known. We have also assumed that the price of the oil and interest rate are given.

The organization of the rest of this report is as follows. In Section 2 a detailed description of the Cold Lake system is given, which is followed in Section 3 by the nonlinear models studied during the workshop. In Section 4, we introduce the linear models from the follow-up study after the workshop. A discussion of possible directions for future study is given in Section 5.

1.2 Problem Description

The Cold Lake facilities can be viewed as a complex system. One of the challenges for the group is to identify the most important features of the system and to formulate the problem mathematically under reasonable assumptions.

After lengthy discussions among the group including the industrial partners, we made the following observations/assumptions.

- The oil production and steam consumption of each well is not known in general. Accurate prediction requires detailed modelling using fluid dynamics and statistical tools. However, on the scale of the Cold Lake operation, it is not unreasonable to model the production and steam consumption by a simple mathematical formula, based on the data from field operations. Typically, the production rate is high in the early stages and decreases as the wells become old.

- The actual cost of developing and maintaining wells during production cycles often varies, depending on many factors. For simplicity, we will use an empirical formula fitted to the field operation data.

- We recognize that wells are in general different. We assume that they can be categorized into different classes: from good wells with high production level and low cost to bad ones with low production.

- The steam is distributed from three plants to wells at various locations through the pipelines. A main physical constraint is the daily available steam. In reality, the steam available to wells also differs from one location to another. However, for simplicity, we will assume that the location factor can be ignored and the only constraint is the total daily available steam.

- The oil recovered from the wells (with water, sand and gas) is 'cleaned up', diluted, transported, and sold. The water is treated and recycled back to the steam plants as part of the water supply needed for generating the steam. There is a limit on the amount of water that can be used at a given time. However, this constraint is completely ignored. The price of the crude oil is obviously volatile. However, we will ignore the stochastic nature of the oil price and treat it as a constant.

- Another physical constraint is the daily oil production for the entire system (an upper bound) due to the ability in transportation and treatments. The constraint on each individual well is location dependent. However, we will ignore this and only impose a constraint on the total production level.

- Economic consideration provides a lower bound on the production level, which likely becomes important only towards the end of the entire operation. Again, a constraint will only be placed on the total production level.

- Finally, the unit lifting cost (ULC), defined as cost divided by the oil production is of practical concern, which appears as a constraint. We believe that if the right objective function is chosen, this constraint is likely satisfied automatically.

In light of the observations from field operation data and assumptions made above, we now introduce the daily production function for a typical well

$$\mathcal{P}(t) = 18e^{-0.0003t},$$

where $t \geq 0$ is the time (measured in days since the beginning of the operation of the well). Production is measured in m^3/day. Note that the production is averaged over the production and steam cycles of the well. In reality, a well only produces oil when it is in the production cycle. The averaged daily steam rate can be expressed as a function of time as

$$\mathcal{S}(t) = 115(t+1)^{-0.18} \tag{1.2}$$

m^3/day. Again, in reality, the steam is only required during the steam cycles. The averaged cost of maintaining normal production of a well is given as

$$\mathcal{C}(t) = 601(t+1)^{-0.105} \tag{1.3}$$

in dollars. All three are monotonically decreasing functions. Finally the cost of developing a new well is assumed to be a constant D_0. In addition, we also assume that the discount rate is given as a constant. With these assumptions we now formulate a four-well problem.

1.3 The Four-Well Problem

We consider a small oil field with only four wells, two existing wells and two new wells to be drilled. The question is when we should developed the remaining two wells so that some objective function can be maximized, subject to the constraints on the production level, unit lifting cost, and the steam consumption rate.

Three approaches (two of them related) which were discussed and formulated during the workshop will be presented here. We will discuss the semi-continuous approach first, followed by the discrete non-linear programming and the depth-first (dynamic programming) approaches.

1.3.1 Semi-continuous approach

The participants working on this approach are *Yonqiang Cao, Glynis Carling, Huaxiong Huang, Dong Liang, Margaret Liang, Jim Muldowney, Marc Paulhus and Shane Stark.*

We first assuming that all four wells are of the same class and a well can be terminated only once. Cases with different classes of wells can be treated similarly. The problem can be formulated as an optimization problem with the profit functional defined as

$$J(\vec{T}, \vec{E}) = \int_0^{T_\infty} \sum_{j=1}^{4} \left(p_0 \mathcal{P}(t - T_j) - \mathcal{C}(t - T_j) - D_0 \delta(t - T_j)\right) H(E_j - t) e^{-Rt} dt$$

where p_0 is the crude oil price. T_j and E_j are the starting and terminal time of well j, for $j = 1, 2, 3, 4$, or in vector form we have $\vec{T} = [T_1, ..., T_4]$ and $\vec{E} = [E_1, ..., E_4]$. $H(x)$ is the Heaviside function, i.e., $H(x) = 1$ when $x \geq 0$ and $H(x) = 0$ when $x < 0$. $\delta(x)$ is the Dirac delta function. It has the following properties: $\delta(x) = 0$ when $x \neq 0$ and $\int_{-\infty}^{\infty} \delta(x)dx = 1$. The objective is to find $\vec{T}$ and $\vec{E}$ such that the profit J is maximized subject to the following constraints.

1. The first constraint which needs to be satisfied is on the steam consumption,

$$\sum_{j=1}^{4} \mathcal{S}(t - T_j) \leq S^H \tag{1.5}$$

   for $0 \leq t \leq T_\infty$. S^H is the limit on the total daily steam consumption.

2. The constraints on the production level can be written similarly as

$$P^L \leq \sum_{j=1}^{4} \mathcal{P}(t - T_j) \leq P^H \tag{1.6}$$

   for $0 \leq t \leq T_\infty$. Here P^L and P^H are the lower and upper bounds for the total daily production.

3. Finally the constraint on the unit lifting cost is

$$\sum_{j=1}^{4} \mathcal{C}(t - T_j) \leq ULC^H \sum_{j=1}^{4} \mathcal{P}(t - T_j) \tag{1.7}$$

   for $0 \leq t \leq T_\infty$. Here ULC^H is a given upper bound.

Since the functions $\mathcal{P}$, $\mathcal{S}$ and $\mathcal{C}$ are monotonic, it is sufficient to enforce constraints before and after each T_i as

$$\begin{aligned}
\sum_{j<i} \mathcal{S}(T_i - T_j) &\leq S^H, \\
\sum_{j<i} \mathcal{P}(T_i - T_j) &\leq P^H, \\
\sum_{j\leq i} \mathcal{P}(T_i - T_j) &\geq P^L, \\
\sum_{j<i} \left(\mathcal{C}(T_i - T_j) - ULC^H \mathcal{P}(T_i - T_j)\right) &\leq 0, \quad \text{for} \quad i = 1, ..., 4.
\end{aligned} \tag{1.8}$$

The problem with two existing wells and two new wells was attempted using the Matlab Optimization toolbox and Monte-Carlo simulation. Matlab uses some standard methods for solving nonlinear optimization problems with constraints. The Monte-Carlo simulation is done by generating the starting and ending time of each new well randomly during a given time interval using a uniform distribution random number generator. The profit associated with each realization is computed accordingly until one of the constraints is violated. Surprisingly (or perhaps not surprisingly), Matlab failed to generate any results. The Monte-Carlo simulation with 10,000 trials, on the other hand, takes less than a minute on a Dell laptop based on a 366 MHz Intel II CPU with 128M RAM.

1.3.2 Non-linear programming approach

The participant working on this approach is *Kyle Biswanger*.

We attempted to represent this problem as a linear program. In this formulation the problem is represented as a set of linear constraints, which form an n-dimensional polyhedron, and an objective function, which is optimized over the feasible region defined by the polyhedron. We can define the objective function in many ways, in this case we choose to maximize net present value. The feasible region is defined by two classes of constraints

1. those that are related to set limitations of the plant such as steam availability and daily operating cost; and
2. those that ensure the model behaves appropriately, for example ensuring that a well must start producing oil before it can be terminated.

Formally there is no distinction made between the constraints, however the distinction can help us understand the formulation. In defining the behavioral constraints, we encounter non-linearities. We have chosen to keep the non-linearities, and represent the problem as a non-linear program instead of a linear one. Non-linear programs are in general more difficult to solve. The formulation of the nonlinear program is achieve the objective

$$\max \sum_j \sum_i \frac{(Px_{ij} - c_{ij} - \psi_j)}{(1+r)^j} \tag{1.9}$$

subject to the following plant constraints

$$\sum_i x_{ij} \le Q_H^x, \quad \sum_i x_{ij} \ge Q_L^x, \quad \sum_i s_{ij} \le Q_H^s, \quad \sum_i \frac{c_{ij}}{x_{ij}} \le Q_H^c, \tag{1.10}$$

and the behavioral constraints

$$\begin{gathered} \sum_j T_{ij} = 1, \quad \sum_j E_{ij} = 1, \quad \sum_{j=1}^{q} T_{ij} \ge \sum_{j=1}^{q+1} E_{ij}; \\ q \in [1,m], \quad T_{ij} \ge 0, \quad E_{ij} \ge 0, \quad T_{ij}, E_{ij} \in [0,1], \end{gathered} \tag{1.11}$$

where

$$\begin{aligned} x_{ij} &= \gamma_{ij} \sum_{k=1}^{m} A_k^x T_{i(j+1-k)}, \quad s_{ij} = \gamma_{ij} \sum_{k=1}^{m} A_k^s T_{i(j+1-k)}, \\ c_{ij} &= \gamma_{ij} \sum_{k=1}^{m} A_k^c T_{i(j+1-k)}, \quad \gamma_{ij} = 1 - \sum_{k=1}^{j} E_{ik}. \end{aligned}$$

In this formulation, the start times (T_{ij}) and end times (E_{ij}) are the decision variables. The definitions give relations, and are actually equality constraints on the problem. However, they relate the decision variables to variables that appear in the objective function, and in this sense are really nothing more than definitions. It is worth noting that when time is discretised, this approach is essentially the same as the continuous time model discussed earlier.

1.3.3 Depth-first approach

The participants working on this approach are

Brian Alspach, Calin Anton, Amos Ben-Zvi, Mufeed Mahmoud, Reza Naserasr, Cristina Popescu, Bruce Rout, and Tzvetalin Vassilev.

Again, we will explore the case of four wells, two of them working (one is in cycle nine and other in cycle nineteen of their producing life) and two to be drilled.

First we will make some definitions. We will define each well w_i as an ordered pair $w_i(c, y)$, where $c \in \mathcal{Z}$ is the class of the well (1-good, 2-medium, 3-poor) and $y \in \mathcal{Z}$ is the cycle of production in which the well is currently, $y = 0, 1, ..., 25$ (0 meaning that the well has not been developed yet and 25 meaning that the well has been abandoned, any other odd cycle number is a steaming cycle and any other even cycle number is a production cycle).

Let W be the set of all possible combinations (c_i, y_j), $i = 1, 2, 3$, and $j = 0, 1, ..., 25$, which is a 3×26 set. Further, we will define $\Lambda = \{w_1, ..., w_r \mid w_i \in W, i = 1, ..., r\}$. This must be regarded as the set of all possible combinations over the given set of wells. Then, the allowable set of wells can be defined by $\Lambda^* \subset \Lambda$ such that $\Lambda^* = \{w_1 ..., w_r \mid f_k \geq 0\}$. Here $f_k \geq 0$ represents our constraints, and as above $w_i \in W, i = 1, ..., r$. Let $a, b \subset \Lambda^*$, then we will define binary operation "$*$" on Λ^*: $* : \Lambda^* \times \Lambda^* \to \mathcal{Z}$,

$$a * b = t, \text{where } t = \begin{cases} 0, & \text{if } a = b \\ 1, & \text{if the transition } a \to b \text{ has one coupling line} \\ 2, & \text{if } a \to b \text{ has two coupling lines} \\ \vdots & \\ k, & \text{if } a \to b \text{ has k coupling lines} \end{cases}$$

Having this we can define another binary operation called "$s$uccession". We will say that b succeeds a if there is transition with length one, i.e. if they are immediate neighbours in the manifold. Formally:

$$B_a = \{b \mid b * a = 1\} \quad a, b \in \Lambda^*.$$

The main idea of the approach is to build a manifold of the states and transitions between them using their properties and constraints. Then we will find the best possible path between starting (initial) and target (final) position with respect to given objective function. Final states will be all these states for which we have no transition to an allowable state. For these states it also can be noted that they have no succeeding states, or for a final state z, $B_z = \emptyset$ holds. Building the manifold we will take advantage of the properties of the wells and constraints imposed in a way that will allow us to cut down infeasible transitions and consider only a relatively small number of choices at each step.

Now our problem is formulated as follows: we wish to travel from some state a (start state) to some state p (final point) while optimizing an objective function. At each step we will build the manifold and finally go through all possible paths.

For each well we are given: $\mathcal{C}_i-$ cost of production [\$/day], ULC_i^H- unit lifting cost [\$/m^3], $\mathcal{Q}_i^s-$ steam needed [m^3/day], and $\mathcal{Q}_i^x-$ oil produced [m^3/day]. We are also given the cost of drilling a new well, D_0. Initially we have $N = 4$ wells with two of them producing and two to be drilled.

The following constraints are imposed on our model:

1. Steam capacity: steam used (needed) at each moment must be at most as much as our producing capacity: $\mathcal{Q}^s = \sum_i \mathcal{Q}_i^s \leq S^H$ (an upper bound).

2. Production rate: quantity of oil produced at each moment can be within certain limits: $P^L \leq \mathcal{Q}^x \leq P^H$. Here $\mathcal{Q}^x = \sum\limits_i \mathcal{Q}_i^x$ is our total production of oil and P^L and P^H are the bounds.

3. Unit Lifting Cost (ULC) must be less than a certain value, which is the value that gives us a profitable production. It is calculated by summing all production costs and dividing them by the total quantity of oil produced, i.e. $ULC = \sum\limits_i (ULC_i \mathcal{Q}_i^x / \mathcal{Q}^x)$ and $ULC \leq ULC^H$.

An issue that simplifies the model is classification of wells. Wells are separated into three different classes according to their producing characteristics, for ease of modelling. Approximate differences between oil productions are: wells of class 1 produce roughly about 20% more oil than these of class 2 per working cycle, and these of class 3 produce about 20% less than class 2. We will assume that no more than two wells can be started at the same time.

An additional constraint imposed here is that the maximum number of wells which can be drilled at the same time is five.

With this scheme, we can now build the manifold of all states and find the optimal path on it, using a certain objective function. At each decision moment we need to generate the sequence of actions, which well to be started, developed or abandoned. On the time scale calculated we will consider certain points referred to as "decision moments". Each one of these moments is the point at which we evaluate the objective function and possible ways to act further. These moments can be the ends of steaming or producing cycles of wells that are in operation. We denote them by t_j.

We can choose different objective functions to evaluate and optimize. Comparison between the results obtained can add valuable insight to the solution of the problem. For example, we can choose the unit lifting cost with depreciation as the objective function. If we have an annual interest rate of r, and assume that ULC is the same during the steaming and production periods (costs spread over whole cycle), the following objective function can express our maximal benefit over the period criterion:

$$F = \sum_j \frac{365 C_j}{r} \left(e^{\frac{-r t_{j+1}}{365}} - e^{\frac{-r t_j}{365}}\right)$$

Method	Total Profit (Million $)	$\vec{T}$ (days)	$\vec{E}$ (days)
MC	9.84	[-5370, -1110, 3164, 13082]	[498, 4347, 10227, 18179]

Table 1.1: Summary of Results for the Four-well Problem.

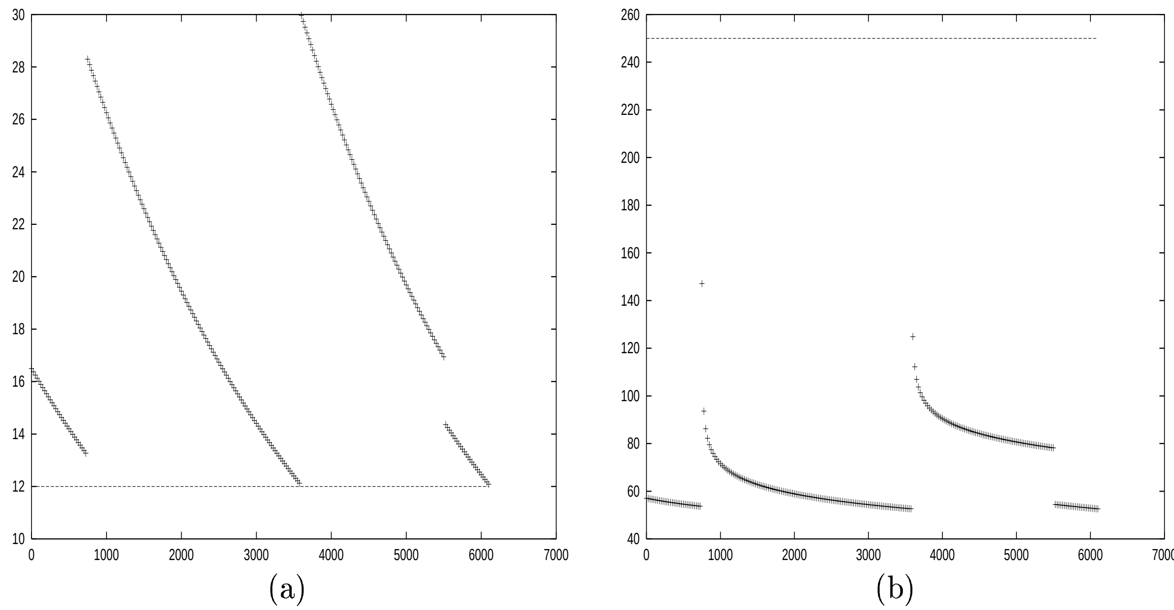

Figure 1.1: Time history of (a) the production level and (b) the steam consumption for the four-well problem.

1.3.4 Results

Attempts were made to solve the problems formulated above by each sub-group. However, due to the extremely limited time available during the workshop, only partial results were obtained.

Table 1.1 summarizes the results using the Monte Carlo simulation with a time step size of 25 days and one million realizations. The upper limit of steam consumption is $S^H = 250m^3$/day, while the daily production level bounds are 12 and 30 m^3. The discount rate is taken as $R = 0.05/365$ annually. The price of the commodity is $p_0 = 200$ $\$/m^3$. The cost of developing a new well is $D_0 = \$500,000$ and the unit lifting cost bound is $ULC^H = 50\$/m^3$. The simulation takes about 30 minutes on a Dell 366 MHz laptop.

Figure 1.1 shows the time history of production level and steam consumption. It can be seen that the operation is regulated mostly by the upper bound of the production level for this particular example. We have assumed that the two existing well were drilled 5370 and 1110 days when the decision to drill two new wells are being made. For a copy of the code, please contact Huaxiong Huang [13].

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1.4 An Alternative Approach for the Cold Lake System

This part of the report is based on the work of *Huaxiong Huang, Marc Paulhus, Miro Powojowski, and Nikhil Shah.*

The main difficulty of extending some of the approaches studied during the workshop to a large problem is that it becomes very expensive to compute the solution as the number of wells increases. However, it has been observed that unless each well is absolutely unique, it is not necessary to distinguish well i from well j.

It turns out that this difficulty can be avoided if we formulate the problem differently when the wells are not very different from each other, or at least that they can be put into a small number of groups. In the second part of this report, we will discuss this new approach, first by presenting a discrete linear programming (LP) formula. It is followed by a continuous formulation with arbitrarily large number of wells. The discrete LP approach is re-derived as a special discretization of the continuous formula. The discrete problem is solved using an LP package and results are given for a test problem. For simplicity, we assume that all the wells are the same. A relaxation of this assumption will be discussed at the end of this section.

1.4.1 An LP approach

We will formulate the problem in such a way that the number of wells is not relevant to the size of the problem. Rather than solving for the starting and ending times of each individual well we will instead determine the number of wells in an age category any particular time in the future. This will allow us to set the problem up as straight-forward (although very large) linear programming problem that we can solve using a mixed-integer programming package written by Michel Berkelaar [2]. This package is freely available for non-commercial use.

Problem Definition

We will start by describing the problem we intend to solve. It has the following input file which we will describe in detail below.

```
Number of undeveloped wells:    6000
Time step (days):    100
1000
Abandon times (days):    300 500 800 1100 1600 2300 3200 4300 5600 7000
Starting well ages (days):    100 200 300 400 500 600 700 800 1000 2000
3200 4000 5600
Number of wells starting with those ages:    250 250 250 250 250 250 250
250 250 250 250 250 250
Max steam use (day):    120000
Max production rate (day):    42000
Min production rate (day):    9000
Discount rate (annual):    0.05
Price of commodity:    200
Well development cost:    500000
```

First is the number of potential wells that could be developed in this field, $N = 6000$. The "time step" is the decision interval for this project, in this case we will make decisions on the management of this project once every $T = 100$ days. The "total time length" is 11000 days (about 30 years). The times that we can abandon a well (presumably just before we steam it) are given as the vector of "abandon times". The abandon times are roughly the same as the steaming times given to us by Imperial Oil rounded to the nearest 100 days.

The next two vectors define the starting state for the field. The first vector defines the ages of the currently producing wells and the second vector defines the number of current wells of each of those ages. In this case we have 3250 active wells varying in age between 100 and 5600 days.

The "max steam use" $S^H = 12000$ is the maximum amount of steam that can be used on any single day. The "max production rate" $P^H = 42000$ and "min production rate" $P^L = 9000$ are bounds of the daily production rate of the entire project. The "discount rate" $R = 0.05/365$ is used to discount future cash flows. The "price of the commodity" $p_0 = 200$ is the market value of one cubic meter of bitumen and the "well development cost" $D_0 = 500000$ is the cost of drilling a new well.

In our model we are only considering a single class of wells, a so-called *class B* well as described during the workshop. If it has been x days since a well was originally developed, its production level $\mathcal{P}(x)$, steam consumption rate $\mathcal{S}(x)$ and cost function $\mathcal{C}(x)$ are given by (1.2)-(1.3). The effect of including more than one well class will be discussed later.

Method

Note that the time step defines a natural classification for the ages of the wells. If a well is of stage s then it is between $(s-1)T+1$ and sT days old. Define

$$a_s = (s-1)T + 1 \tag{1.12}$$

the minimum age of stage s well. In our example the oldest possible well is of age 7000 days (that is the highest abandon time) thus we have $C = 70$ different stages of well ages.

The time step also defines a discretization of time. Since the "total time length" is 11000 days we have $Y = 1099$ decision intervals over the life of the project. Define

$$d_t = (t-1)T + 1 \tag{1.13}$$

the starting time of decision interval t.

We are interested in solving for the number of wells of stage s in the decision interval t for all (t, s) which maximizes the profit function Equation (1.14). Thus we have 76930 decision variables. The variable labelled $z(t, s)$ is the number of wells of stage s in the t^{th} decision interval. We define $z(0, s)$ to be the starting profile of the field as defined by the input file.

The profit function can now be described as

$$\sum_{t=1}^{Y} \sum_{s=1}^{C} v(t, s) z(t, s) \tag{1.14}$$

where

$$v(t, s) = \sum_{k=0}^{T-1} (\mathcal{P}(a_s + k) p_0 - \mathcal{C}(a_s + k)) e^{-R(d_t + k)} - \mathcal{D}(s) e^{-R d_t} \tag{1.15}$$

which is simply the sum of the gross income (production times price) that a well of stage s will generate over a T day interval minus the operating cost of a well of stage s, all discounted.

$$\mathcal{D}(s) = \begin{cases} D_0, & \text{if } s = 1; \\ 0, & \text{otherwise;} \end{cases} \tag{1.16}$$

is the cost of developing a well, which only needs to be paid when a well is first developed. Note that the function $v(t, s)$ is non-linear but for any fixed pair (t, s) it is simply a constant. Thus Equation (1.14) is linear.

There are steam constraints that must be satisfied over every decision interval. That is for all t

$$\sum_{s=1}^{C} \mathcal{S}(a_s) z(t, s) \leq S^H. \tag{1.17}$$

Note that $\mathcal{S}$ is decreasing and thus the constraint only needs to be checked at a_s. Moreover $\mathcal{S}(a_s)$ is a constant for a fixed s and thus Equation (1.17) defines 1099 linear constraints. Similarly we must include 1099 linear constraints for the upper bound on the production rate,

$$\sum_{s=1}^{C} \mathcal{P}(a_s) z(t, s) \leq P^H \tag{1.18}$$

and 1099 linear constraints for the lower bound on the production rate,

$$\sum_{s=1}^{C} \mathcal{P}(a_s) z(t, s) \geq P^L. \tag{1.19}$$

There are also transition constraints. If a_{s+1} is an abandon time then for all t

$$z(t+1, s+1) \leq z(t, s) \tag{1.20}$$

since all wells of stage s must either be abandoned or become wells of stage $s+1$ the next decision interval. If a_{s+1} is not an abandon time then Equation (1.20) will be a strict equality. Also, to insure we do not develop more wells than are available, we must have

$$\sum_{t=1}^{T} z(t, 1) \leq N. \tag{1.21}$$

The solution of this problem is given after the discussion of the continuous formulation. The solution procedure is illustrated via a simpler example problem in the Appendix.

Note that for large problems we drop the integer declaration and solve it as an LP problem for real numbers, rounding the answers down to the nearest integer. If the total number of wells is large enough then this solution should be extremely close to the optimal integer solution. Indeed, this procedure relies on the fact that we are dealing with a large number of wells and will not work for a small number. We also note that the decision making time interval T can be altered as well. An interesting question is whether the value of maximum realized objective function will increase as T is reduced and approach an upper bound in the limit of continuous time. Another interesting question is whether allowing wells to be added/removed more freely instead of at specific time will affect the overall solution. These questions could be investigated using the LP model discussed here. However, to re-formulate the problem from the continuous point of view may also provide some useful insights. This is the rationale behind the continuous formulation to be discussed in the following.

1.4.2 A continuous time formulation

A linear optimization problem

We now re-formulate the problem as a linear continuous optimization problem with linear constraints. Define $n(t, a)$ as the number of wells at time t with age a. $n_0(t) = n(t, 0)$ is used to describe the number of new wells being developed at time t. Let $n_b(a) = n(0, a)$ denote the number of wells of age a at the beginning of the operation. We assume that the total number of wells $N = \int_0^{T-a} n_b(a)da + \int_0^{T_\infty} n_0(t)dt$ is given. Here T is the natural life span of wells (assumed to be the same for all wells) and T_∞ is the time of the entire operation of the oil-field.

Note now the production, steam consumption rate and cost are functions of age, $\mathcal{P}(a)$, $\mathcal{S}(a)$ and $\mathcal{C}(a)$, which are simply (1.2) - (1.3). The development cost of a well is given as $\mathcal{D}(a) = D_0\delta(a+)$. The total profit is a functional $J(n)$ defined as

$$J(n) = \int_0^{T_\infty} \int_0^{T} n(t, a)\mathcal{F}(a) \exp(-Rt) da dt \tag{1.22}$$

where

$$\mathcal{F}(a) = p_0\mathcal{P}(a) - \mathcal{C}(a) - \mathcal{D}(a)$$

is the profit of operating one age a well before discount and p_0 is the crude oil price. The objective is to find a function $n(t,a)$ such that $J(n)$ is maximized subject to the following constraints

$$\int_0^T n(t,a)\mathcal{S}(a)da \leq S^H, \tag{1.23}$$

$$\int_0^T n(t,a)\mathcal{P}(a)da \leq P^H, \tag{1.24}$$

$$\int_0^T n(t,a)\mathcal{P}(a)da \geq P^L, \tag{1.25}$$

$$\int_0^T n(t,a)\mathcal{C}(a)da \leq ULC^H \int_0^{T_a} n(t,a)\mathcal{P}(a)da. \tag{1.26}$$

Again S^H is the upper steam rate limit, P^H and P^L are the upper and lower production level, and ULC^H is the unit lifting cost.

Dynamic constraint on $n(t,a)$

We now discuss the dynamic constraint satisfied by $n(t,a)$. We note that the total number of wells is a conserved quantity except when they are shut down or removed from operation. Define $f(t,a) \geq 0$ as the number of wells of age a removed during a unit time interval at time t. Then we must have

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a} = -f(t,a). \tag{1.27}$$

This is a first order hyperbolic equation and it can be easily solved using the method of characteristics

$$n(t,a) = n(0,a-t) - \int_{a-t}^{a+t} f((\eta - a + t)/2, (\eta + a - t)/2)d\eta$$

for $a > t$ and

$$n(t,a) = n(t-a,0) - \int_{t-a}^{a+t} f((\eta - a + t)/2, (\eta + a - t)/2)d\eta$$

for $t \geq a$. Note that $n(t-a,0) = n_0(t-a)$ is unknown and $n(0,a-t) = n_b(a-t)$ is given. It can be observed as well that $n(t,a)$ is non-increasing function and the inequalities

$$n(t,a) < n_0(t-a) \;\text{ or }\; n_b(a-t)$$

hold when the wells are being removed along the characteristics, $\xi = a-t =$ constant. Otherwise, $n(t,a)$ is simply $n_0(t-a)$ or $n_b(a-t)$.

Discretization and numerical approximations

One approach for finding the extremum of a functional is to use direct methods. For example, one can use the Ritz method to search for an optimization sequence [1]. We now discuss a special case of the Ritz method where the continuous problem is approximated by a numerical quadrature. We will show that it can be related with the LP formulation proposed earlier when a special discretization is used.

Suppose that a grid in (t_r, a_s) is set up to cover the solution domain $[0, T_\infty]\times[0, T]$, with $t_r = t_{r-1} + T$, $a_s = a_{s-1} + T$, for $r = 1, 2, ...r_{\max}$ and $s = 1, 2, ..., s_{\max}$. The functional $J(n)$ can be re-written

$$\begin{aligned} J(n) &= \sum_r \sum_s \int_{t_{r-1}}^{t_r} \int_{a_{s-1}}^{a_s} n(t,a)\mathcal{F}(a)\exp(-Rt)dadt \\ &\approx \sum_r \sum_s T^2 \int_{t_{r-1}}^{t_r} n(t_{r-1}, a_{s-1}) \int_{a_{s-1}}^{a_s} \mathcal{F}(a)\exp(-Rt)dadt \\ &= \sum_r \sum_s \int_{t_{r-1}}^{t_r} T^2 n_{r-1,s-1} v_{r-1,s-1} \end{aligned}$$

where the grid function $n_{r,s}$ is the approximation of $n(t, a)$. This is equivalent to (1.14) in the LP formulation if $z(r, s) = T^2 n_{r,s}$ and $v_{r,s}$ is evaluated numerically using the Riemann sum with $\delta t = 1$ day. The constraints can be discretized similarly as

$$\sum_s n_{r,s}\mathcal{S}_s \le S^H, \quad \sum_s n_{r,s}\mathcal{P}_s \le P^H, \quad \sum_s n_{r,s}\mathcal{P}_s \ge P^L, \quad \sum_s n_{r,s}\mathcal{P}_s \le ULC^H \sum_s n_{r,s}\mathcal{C}_s.$$

Finally the dynamic equation can be discretized and replaced by the inequality $n_{r,s} \le n_{r-1,s-1}$ which is equivalent to $z(r+1, s+1) \le z(r, s)$.

Obviously other types of discretization can be used. For example, instead of using a grid function, we can use piecewise polynomials on each subinterval. From a numerical point of view, using grid functions fits into a finite difference framework while using piecewise polynomial approximation is a finite element method. From LP point of view, using grid functions can be interpreted as restrict adding/removing wells at specific times and using piecewise polynomials means that we will drill/abandon wells continuously with some restrictions.

Other related issues are the convergence of the discrete solution to the solution of the continuous problem (if it exists) in general and the speed of convergence associated with a particular discretization. However, addressing these issues properly is beyond the scope of this report. Instead, we now present the results obtained using this special discretization, which is equivalent to the LP formulation discussed earlier.

1.4.3 Solution

To solve the discrete LP problem, we wrote a program that takes an input file which defines a problem and writes it as an lp-file, which was submitted to lp-solver. The whole process takes about 30 minutes on a Pentium 400 machine with 256MB RAM. A copy of the code and the complete solution can be obtained from Marc Paulhus[14]. To summarize, we now have a linear objective function with 76930 decision variables, 1099 linear constraints for the steam requirements, 1099 linear constraints for each of the production bounds, and 76931 linear transition constraints. Remarkably we can solve this system. The optimal solution is an assignment to

[14] `marc@ndtechnologies.com`

the decision variables which is too much information to present here, but we will attempt to provide a brief glimpse into the optimal strategy for managing this project.

To this end we have provided 12 snapshots of what the field will look like at 1000 day intervals in Figures 1.2 and 1.3. These plots show the profile of the oil field at particular times in the future. The title of the plot indicates the future time displayed in that plot. The x-axis is the age of the wells in the project and the y-axis is the number of wells of each age.

- The *day 100* plot is almost identical to the starting profile we specified in the input file.
- The *day 1000* through *day 4000* plots still show the effect of the starting profile. Most of the wells that were active at the start are still active in 1000 days but nearly all of the starting wells have been abandoned by day 4000.
- The *day 5000* and *day 6000* plots show the middle evolution of the project. All wells in this window are abandoned at age 4300 days.
- The *day 7000* through *day 10900* plots show the winding down of the project. In this phase we allow wells to mature much more than in the earlier phases of the project. After 8300 days we have exhausted our supply of undeveloped wells and thus no new wells can be drilled.

Next we look at the constraints. Figure 1.4 shows the daily production rate over the life of the project and the daily steam requirements over the life of the project. We can see that with this particular input file the steam constraints are much more restricting than the production constraints.

Plotted in Figure 1.5 are the costs and profits over the life of the project. These plots show the daily profits and costs and do not include the one-time costs associated with drilling a new well. The magnitude of those development costs can be determined by looking at Figure 1.6a which shows the number of new wells started at future times.

Also plotted in Figure 1.6 are the number of wells abandoned when they reach the age of 4300 days. In total 4746 wells are abandoned before their life's end. Most of those (4496 to be precise) are abandoned when they reach the age of 4300 days. Finally we show the unit lifting cost as a function of time in Figure 1.7. Note how closely related this graph is to the profit graph (Figure 1.5b) which is to be expected.

A summary of some interesting statistics is presented in Table 1.2.

Before we finish this section, we note that adding more constraints should not be a problem as long as they are linear. If we expand our problem to include different classes of wells then the complexity of the problem increases dramatically. If we have m categories of wells there will be m times as many decision variables and m times as many transition constraints. Nevertheless, given enough time, computing power, and the proper software, this problem should be solvable for a realistic model of the Cold Lake project as long as $m \ll N$ (the total number of wells).[15] There are some very powerful commercial software packages available although we should stress how impressed we are with the power of Berkelaar's solver. See [3] and [4] for examples of good commercial LP solvers or see [5] for a more complete list.

[15] An interesting comparison on the linear and non-linear approaches can be made when $m = N$.

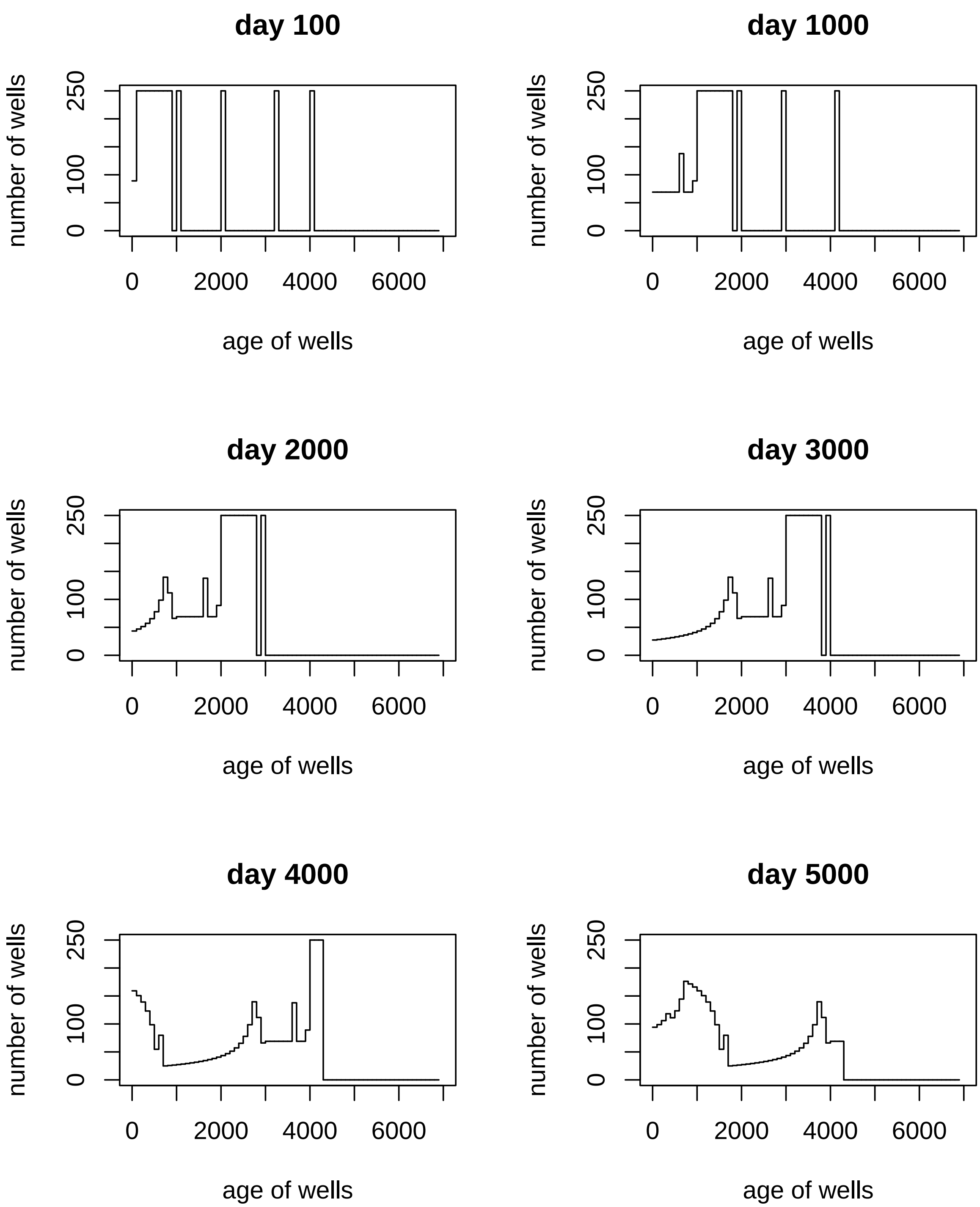


Figure 1.2: Snapshots of the field profile on days 100 through 5000

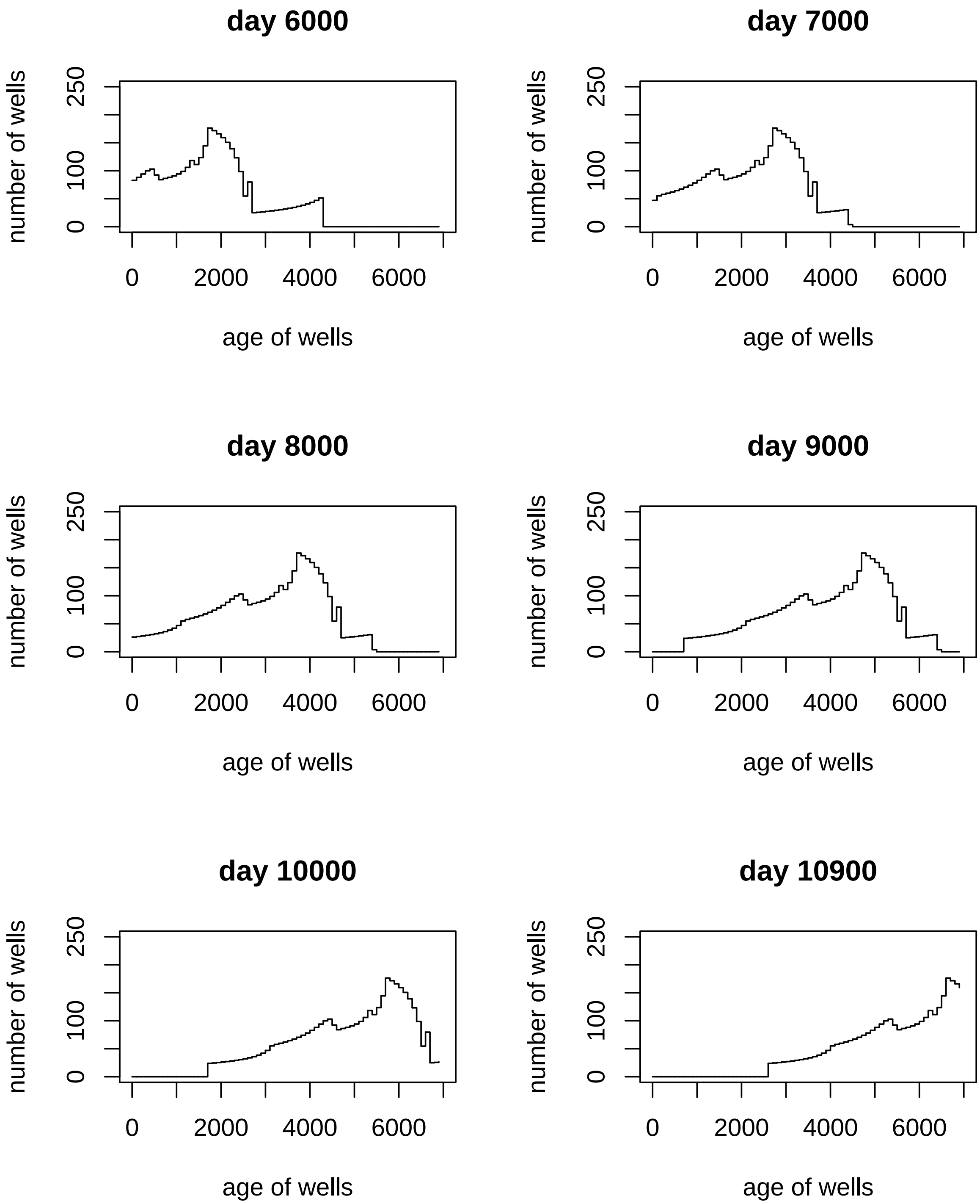
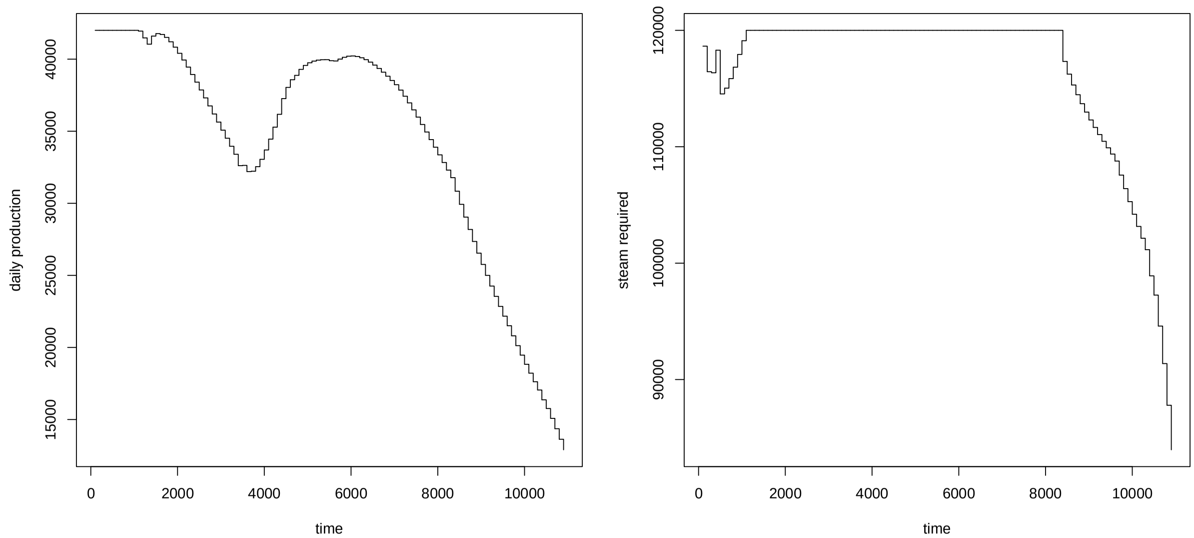
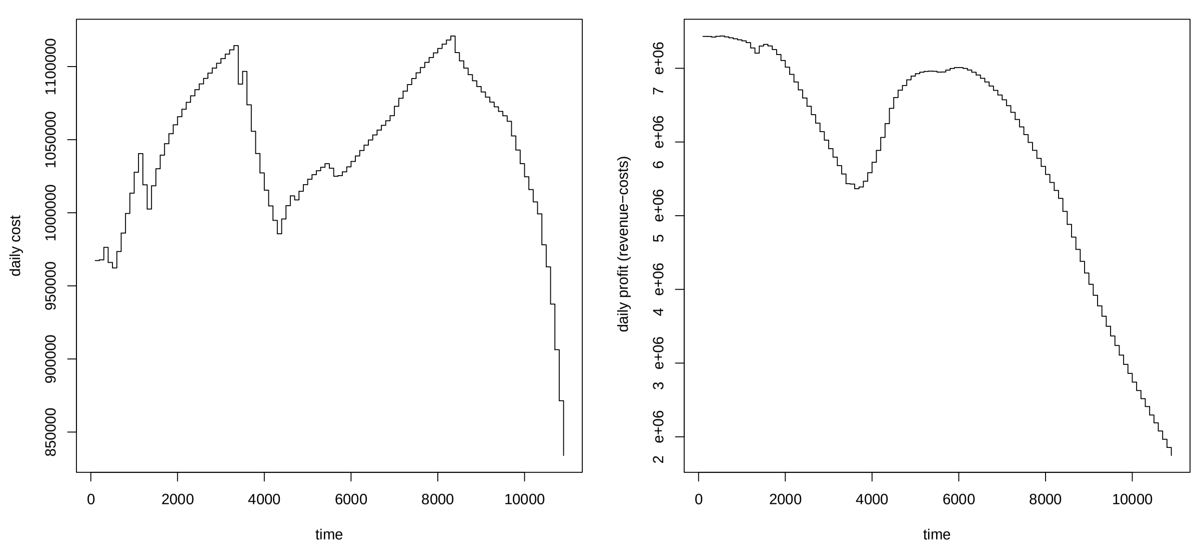


Figure 1.3: Snapshots of the field profile on days 6000 through 10900



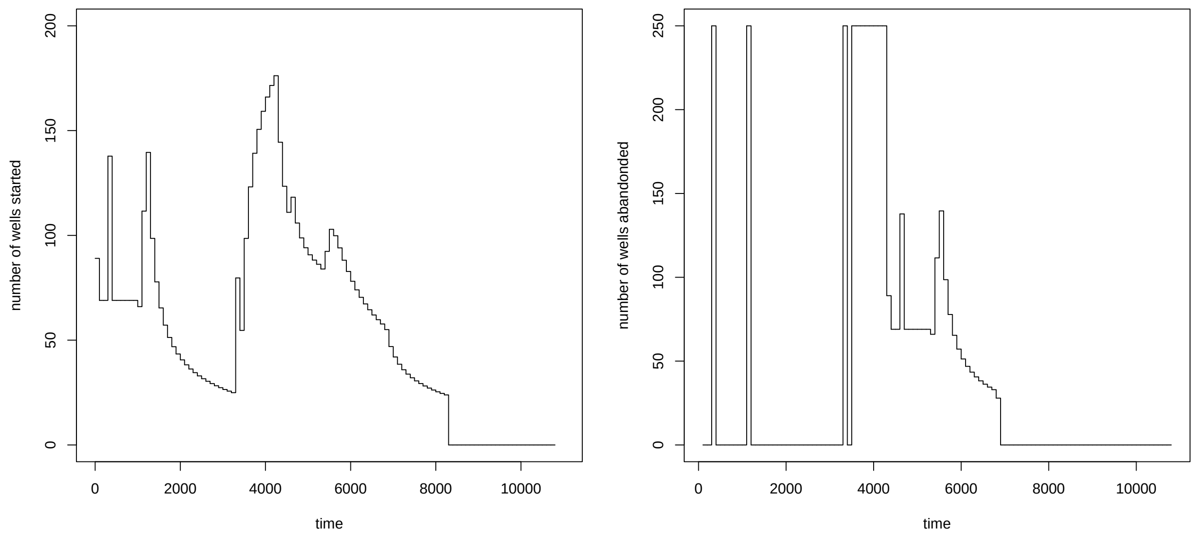
(a) (b)

Figure 1.4: The daily production versus time (a), and daily required steam versus time (b).



(a) (b)

Figure 1.5: The daily costs versus time (a), and daily profits versus time (b).



(a) (b)

Figure 1.6: The number of new wells started versus time (a), and the number of wells abandoned at age 4300 days versus time (b).

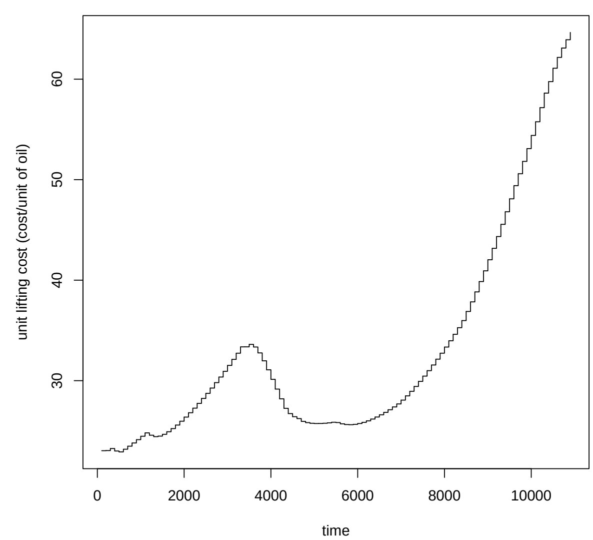


Figure 1.7: The unit lifting cost versus time

Table 1.2: The average, minimum, and maximum values of various characteristics over the life of the project.

	Average	Minimum	Maximum
Active Wells	3794	3089	4253
Daily Production	34145	12989	42000
Daily Steam Use	116297	83961	12000
Daily Costs	1044308	833941	1120901
Daily Profits	5784723	1745679	7437729
Unit Lifting Cost	43.78	22.91	64.66

1.5 Conclusion

The problem that Imperial Oil brought to the PIMS 4th Industrial Problem Workshop was to determine if there is a method to help manage the huge Cold Lake project. After exploring several approaches during and after the workshop, we feel that the answer is yes, as long as the it can be formulated as a linear programming problem in a manner similar to what we have described in the second part of this report.

The major advantage of the linear programming/linear optimization formulation over the non-linear approaches is that the complexity of the solution does not depend on the number of wells that are being modelled if the wells can be put into a small number of groups/classes. We believe that the linear approach is more suitable for a large project such as the Cold Lake system.

As we mentioned earlier, in this report we have assumed a constant value for the price of the commodity produced. An interesting and relevant complication to the problem would be to investigate the optimal strategy when that price is stochastic. A possible approach to this problem will be to use the techniques of *stochastic programming* but such an investigation is beyond the scope of this report.

Finally we note that another relevant issue faced by the managers of the Cold Lake facilities is how to deliver and distribute the steam from the three central plants to each individual well and the product from wells to the plants for treatment. This scheduling problem, which is not included in this report, will further complicate our mathematical models.

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1.6 Appendix - A Sample Problem for the LP Approach

To solidify the idea of using the LP approach, we look at a simpler problem's input and output files. In what follows $t_i s_j = z(i, j)$ in the notation of Section 4.

Input file:

```
Number of undeveloped wells:  3000
Time step (days):  100
Total time length (days):  400
Abandon times (days):  100 300 400
Starting well ages (days):  100 200 300
Number of wells starting with those ages:  400 400 400
Max steam use (day):  60000
Max production rate (day):  20000
Min production rate (day):  9000
Discount rate (annual):  0.05
Price of commodity:  200
Well development cost:  0
```

LP file:

`/* Objective Function */`
`max:` $311216\, t_1s_1 + 306425\, t_1s_2 + 298215\, t_1s_3 + 289585\, t_1s_4$
$+306981\, t_2s_1 + 302256\, t_2s_2 + 294157\, t_2s_3 + 285645\, t_2s_4$
$+302805\, t_3s_1 + 298144\, t_3s_2 + 290155\, t_3s_3 + 281758\, t_3s_4;$

`/* Steam Constraints */`
$115\, t_1s_1 + 50.11\, t_1s_2 + 44.27\, t_1s_3 + 41.17\, t_1s_4 \leq 60000;$
$115\, t_2s_1 + 50.11\, t_2s_2 + 44.27\, t_2s_3 + 41.17\, t_2s_4 \leq 60000;$
$115\, t_3s_1 + 50.11\, t_3s_2 + 44.27\, t_3s_3 + 41.17\, t_3s_4 \leq 60000;$

`/* Lower production bound constraints */`
$17.99\, t_1s_1 + 17.46\, t_1s_2 + 16.95\, t_1s_3 + 16.45\, t_1s_4 \geq 9000;$
$17.99\, t_2s_1 + 17.46\, t_2s_2 + 16.95\, t_2s_3 + 16.45\, t_2s_4 \geq 9000;$
$17.99\, t_3s_1 + 17.46\, t_3s_2 + 16.95\, t_3s_3 + 16.45\, t_3s_4 \geq 9000;$

`/* Upper production bound constraints */`
$17.99\, t_1s_1 + 17.46\, t_1s_2 + 16.95\, t_1s_3 + 16.45\, t_1s_4 \leq 20000;$
$17.99\, t_2s_1 + 17.46\, t_2s_2 + 16.95\, t_2s_3 + 16.45\, t_2s_4 \leq 20000;$
$17.99\, t_3s_1 + 17.46\, t_3s_2 + 16.95\, t_3s_3 + 16.45\, t_3s_4 \leq 20000;$

`/* Transition Constraints */`
$t_1s_1 + t_2s_1 + t_3s_1 \leq 3000;$
$t_1s_2 \leq 400;\ t_1s_3 = 400;\ t_1s_4 \leq 400;$
$t_2s_2 \leq t_1s_1;\ t_2s_3 = t_1s_2;\ t_2s_4 \leq t_1s_3;$
$t_3s_2\ et_2s_1;\ t_3s_3 = t_2s_2;\ t_3s_4 \leq t_2s_3;$

`/* Declarations */`
`int` $t_1s_1,\ t_1s_2,\ t_1s_3,\ t_1s_4,\ t_2s_1,\ t_2s_2,\ t_2s_3,\ t_2s_4,$
$t_3s_1,\ t_3s_2,\ t_3s_3,\ t_3s_4;$

Solution File:

```
Value of objective function:  942033225
```

$t_1 s_1 = 95,\ t_1 s_2 = 400,\ t_1 s_3 = 400,\ t_1 s_4 = 275,$
$t_2 s_1 = 183,\ t_2 s_2 = 95,\ t_2 s_3 = 400,\ t_2 s_4 = 400,$
$t_3 s_1 = 262,\ t_3 s_2 = 183,\ t_3 s_3 = 95,\ t_3 s_4 = 400$

Bibliography

[1] R. Courant and D. Hilbert, Methods of Mathematical Physics, Interscience Publishers Inc., New York, 1953, p. 175.

[2] *www.cs.sunysb.edu/~algorith/implement/lpsolve/implement.shtml*

[3] *www.lindo.com*

[4] *www.cplex.com*

[5] *www-fp.mcs.anl.gov/otc/Guide/SoftwareGuide/index.html*