Analysis of coil slumping

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Abstract

Steel strip is usually stored as a coil, which will slump to some degree after the removal of the mandrel. More often than not, the amount of slumping is so minor that it is assumed not to have occurred. Occasionally, the amount, though minor, is sufficient to compromise the integrity of the cylindrical bore which compromises subsequent handling of the coil. In extreme situations, the slumping progresses to a complete collapse of the coil. Such a collapse is rare. It occurs when a coil cannot hold up its own mass and loses its circular cross-section, as illustrated in Figure 2 below. It is thought to be principally associated with the size and weight of the coil, inappropriate coiling tensions and/or poor re-coiler equipment design. Strip properties, especially inter-strip contact characteristics, have been demonstrated experimentally to be crucial determinants of whether or not coil collapse is likely to occur. The particular kind of slumping/collapse of interest to BlueScope Steel, who proposed this Study Group problem, is the minor slumping that compromises cylindrical bore integrity. It is referred to as *coil slump*.

The Study Group was asked to investigate and model the phenomenon of coil slumping, and, if possible, to quantify the effect of critical parameters, especially coil mass, strip thickness and inter-strip friction. In particular, it was suggested that deliberations should aim to characterize the geometry of slumping and to predict the deformation profile at the innermost and outermost wraps.

For BlueScope Steel, the long term objectives are: (1) the formulation of the governing equations for the stresses in a coil under self-weight, (2) the identification of analytical solutions and/or numerical schemes for the final coil shape after slumping, and (3) the formulation of exclusion rules-of-thumb which predict when a particular form of slump (oval or triangular, as illustrated in Figure 2 below) is likely to occur.

The Study Group made some progress with (1), limited progress with (2) and most progress with (3). Though various computer programs were written to explore different force and energy balance scenarios, they only scratched the surface with regards to (2). Success with it is heavily dependent in substantial progress being made with (1). As explained in detail in the sequel, the Study Group's deliberations resulted in an improved understanding of the coil slumping/collapse problem by identifying a number of specific issues that should be of direct assistance to BlueScope Steel's future management of coil slumping/collapse.

In particular, such issues included the need, from a modelling perspective, to draw a clear distinction between minor slumping and major slumping which can subsequently lead to collapse; the formulation of a heuristic hypothesis about the dynamics of coil slumping/collapse which can be compared with historical data and act act as a conceptualization guide for further investigations; the identification of a *"tension-weight ratio"* $\mathcal{R}$ as the relevant dimensionless group which represents an indicative rule-of-thumb which can be applied in practice; and proposed, on the basis of the hypothesis, an efficient procedure for recording collapse events and statistically identifying possible collapse situations.

1 Introduction

It is well known that removing the cardboard 'mandrel' from a toilet roll causes slumping and often subsequent collapse. That is one reason why it is there. The same can happen for steel coils after the mandrel is removed and they are laid on their sides. However, steel coils are much stiffer by comparison so that slumping is usually only minor, and subsequent complete collapse rare. More common is the situation where the minor slumping, which always occurs after the removal of the mandrel, results in a distorted cylindrical bore that compromises subsequent handling.

BlueScope Steel manufactures, stores and transports coils of steel strip. They have asked the Study Group to consider the slumping/collapse of these coils, where the coil cannot hold up its own mass and maintain the integrity of its cylindrical shape when it is stacked on its cylindrical side (as can be seen from the examples in Figure 2). In particular, the inner bore of a slumped coil is distorted from a circular cross-section, and subsequent handling of the coil is impeded.

Coils are wound with carefully chosen tension in the form of a tightly-wound cylindrical spiral. Too much tension causes a different kind of slumping/collapse than too little. Inter-wrap slippage is believed to be associated with slumping, and it is known that changing the surface characteristics of the steel can reduce the likelihood of slumping. For BlueScope Steel, soft slumping/collapse, associated with too little winding tension, is of more interest than tight-bore collapse, associated with excessive winding tension. The observed shape and location of the critical curve, in tension-friction space, is as seen schematically in Figure 3. A coil will be able to resist slump/collapse, if the tension is large and the frictional force is large with a trade-off operating between them.

If coils fail to maintain their axial circular cross-section, they become unusable and must be scrapped at considerable cost. This happens rarely, but even minor slumping is a major concern. Because of the fine circular tolerances involved, the insertion of the mandrels, onto which the coils are loaded at various stages of their processing, becomes difficult, time consuming and problematic, if the inner bore is even mildly distorted (slumped). The obvious solution of retaining the mandrel in place or cradling the coil was considered to be inappropriate.

The major challenge posed to the Study Group was the development of a comprehensive understanding of the initiation and subsequent evolution of the slumping/collapse. In particular, BlueScope Steel was interested in quantifying the effect of coil mass, strip thickness, winding tension and inter-strip friction. They were also interested in the ability to predict the final shape of the coil cross-section, which, from their experience, tends to have either an elliptical-type or a triangular-type of shape.

Though there is a comprehensive literature about coil winding, rewinding, optimal tension and more, it was of only marginal use for the current investigation. The specific types of questions that BlueScope Steel wanted investigated had not been previously studied, at least in the open literature.

The situation confronting the MISG participants was an excellent example of one where the problem description was simple, clear and unambiguous, but actually getting a hook into the problem was quite challenging. A number of approaches were investigated - balance the forces in a localized representative increment; determine and then minimize the total energy contained in a wound coil (gravitational, bending, stretching and frictional energies) to find the related Euler-Lagrange equations; for a simpler system determine the total energy and corresponding Euler-Lagrange equations in order to obtain a better understanding of the underlying mathematics and insight about the nature of the problem; concentrate on an appropriate linearization and solve that first; treat it as a classical continuum mechanics problem and attack it directly; understand the role of the frictional forces on inter-layer slip in maintaining the shape of a wound coil; analyse the interplay between gravitational and bending energies of a simple system to understand how a coil supports its own mass; and the geometric structure of the collapse of a coil.

At various stages of the deliberations, all of these options were examined and discussed in various levels of detail. It was a necessary part of the brainstorming process required to generate useful insight. However, because of the inherent complexity, it took time to see a pattern that identified, at least heuristically and phenomenologically, the nature of the dynamics of coil slumping/collapse and how it could be modelled.

Gradually, in conjunction with the understanding that unfolded as the various deliberations and discussions progressed, it became clear that pictures of the various types of coil collapse could be utilized to hypothesize about the nature of the dynamics of the slumping/collapse process. This in turn was utilized to formulate a practical rule-of-thumb for assessing coil configurations. It was found that the various deliberations represented to a lesser or greater extent validation of this hypothesis, adding support to the potential utility of the rule-of-thumb.

This report has been organized in the following manner. After presenting the dimensional and physical details about the coils in Section 2, a brief literature review is given in Section 3. The hypothesis is formulated in Section 4 along with the corresponding rule-of-thumb. Many of the ideas proposed and discussed during the deliberations about coil slumping/collapse can be viewed as topics for further research. Some suggestions of this nature are discussed in Section 5. Conclusions are discussed in Section 6.

2 Background knowledge and physical properties

The basic coil geometry is illustrated in Figure 1.

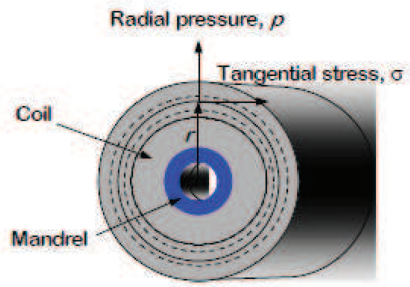


Figure 1: Basic Coil Geometry

The steel strip has typical (geometric) dimensions of thickness $0.2 - 0.5$ mm, width $\sim$ 1000 mm and length $\sim$ 5 kilometres, and is stored in a coil with typical dimensions of inner radius $\sim$ 250 mm, and outer radius $\sim$ 750 mm. They are laid on their curved (cylindrical) sides, as shown in Figure 2. The density of the steel is 7.8×10^3 kg/m^3, elastic modulus 200 GPa, Poisson's ratio 0.3 and inter-wrap coefficient of friction $0.1 - 0.2$.

A coil of outer diameter 1.5 metres and width 1000 mm has a mass of approximately 1.2×10^4 kg (12 tons). Coil masses vary from $5 - 20$ tons. Thus, in its uncollapsed form, the gravitational potential energy (GPE) of the mentioned representative coil is $DMg/2 = 7.5 \times 10^4$ Joules. Since, when they collapse, coils have a height of about two thirds of their initial circular diameter, they lose about $(1/2 - 1/3)DMg = DMg/6$ of their GPE. This is indicative of the amount of total elastic energy and frictional energy (TEE) that a wound coil must have to avoid collapse.

Typical coiling stress for thin gauge coils is 50 MPa, though, during the initial early phase of the winding, the stress can be as high as $100 - 150$ MPa. It varies depending on the particular processing line, and can be a bit lower for paint lines and metal coating lines. The initial tension does not affect coil slumping. A summary of this information is given in Table 1.

The purpose of this section is to give a listing of the various facts, experimental results and beliefs about coil winding and slumping that will be utilized in the sequel.

1. When the mandrel is removed from a wound coil, slumping commences immediately. However, in most situations, it is only slight as the trade-off between GPE and TEE adjusts to the equilibrium determined by the minimum of the total energy $GPE+TEE$. It is for this reason that a clear distinction is and must be drawn between slumping and collapse.

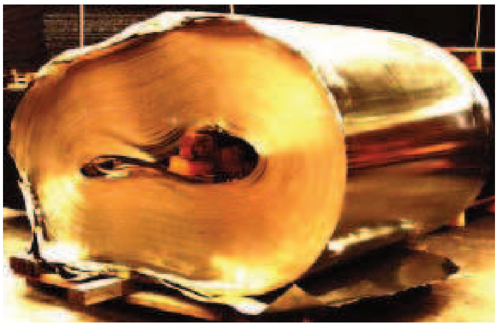
2. Even slight slumping becomes a problem if it results in the integrity of the cylindrical bore of the coil being compromised to the point where the reinsertion of the mandrel becomes either problematic or impossible.

3. Acceptable industry jargon appears to be that "no slumping" occurs if the integrity of the cylindrical bore is not compromised, "slumping" occurs if the integrity of the bore

(a) Colorbond steel coil with racetrack bottom

(b) Metal coil with racetrack bottom

(c) Compromised racetrack with triangular top



(d) Complete collapse

Figure 2: Some examples of coil slumping.

Symbol	Name	Value	Units
E_r	effective radial modulus	2×10^6	Pa
E_t	elastic modulus	2×10^{11}	Pa
h	thickness	0.3	mm
R_b	inner coil radius	250	mm
R_c	outer coil radius	750	mm
S_c	coiling stress	50	MPa
μ	coeff friction	0.1	
ν	Poisson's ratio	0.28	
ρ	density	7.8×10^3	kg/m^3

Table 1: Table of properties of steel and steel coils

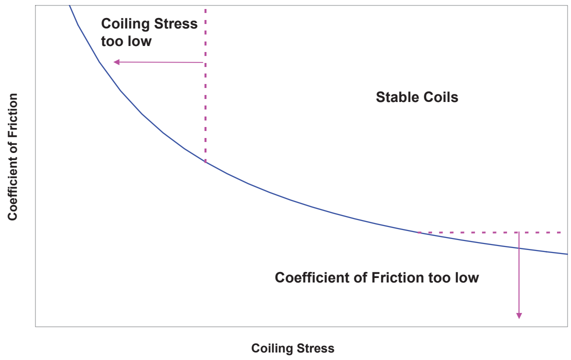


Figure 3: Critical tension-friction curve

is slightly compromised to the point where mandrel insertion becomes problematic, and "collapse" occurs if the bore is visibly distorted. For a coil slumped to the point where subsequent mandrel insertion is compromised, the coil will be visibly distorted.

4. As explained in the literature review below, there is a considerable source of information and data about coil winding, but much less about the geometry of coil slumping and virtually nothing about the dynamics of coil collapse.

5. How tightly a coil is wound is critical. Too tightly, and a different kind of collapse occurs, a kinking due to excessive stresses at the inner wraps of the coil. Too loose, and each wrap of a coil acts independently and slumps gently since it cannot hold up its own weight. In between, and friction links adjacent wraps to strengthen the structure, effectively thickening wraps so that they can hold up their own weight.

6. Though a wide range of collapses have been observed, they can be notionally categorized into the following equivalence classes

   (i) a racetrack bottom with a slightly distorted circular top which has become elliptical in structure in order to accommodate the formation of the racetrack at the bottom (as illustrated in Figure 2(a)),

   (ii) a racetrack bottom with a strongly distorted circular top which has collapsed inwardly to have a curly M-shaped structure in order to accommodate the formation of the racetrack at the bottom (as illustrated in Figure 2(d)),
   and

   (iii) a slightly/strongly distorted racetrack bottom with a slightly distorted upside-down V top (corresponding to the circular top jumping upwards) to produce the triangular feature seen in some examples of slumped coils (as illustrated in Figure 2(c)).

3 Literature review

Even though there are numerous publications on coil winding models, particularly for materials such as paper, not much literature is available on coil slump. One of the first models of coil winding (and perhaps one of the most influential models) was the model for linear orthotropic material developed by [1]. Since then various enhancements and extensions have been made to the linear model. These include non-linear material properties [2-5], large deformations of soft materials [6], relaxation of the winding material [7-10] and three-dimensional effects [11-13]. A comprehensive recent review of coil winding models can be found in [14].

Mathematically, the modelling of coil slump is much more difficult, since the coil can no longer be treated as axisymmetrical, as assumed by the coil winding models, due to the gravitational force of the coil's self weight acting on the coil. Hence, one common method to deal with this loss of symmetry is to use FEA (Finite Element Analysis) to determine the stresses in the coil and the final deformation of the coil. One such model was developed by [15]. The authors first calculated the stresses in the coil after winding from standard coil winding models (assuming both linear and non-linear material properties). The effect of gravitational loading of the coil was then determined from a multi-layered FEA model, with the material properties used in the model dependent on the calculated stress distribution. Each layer was modelled as a plane strain solid element with the radial stiffness dependent on the average radial pressure for the layer. Layer to layer interaction was modelled using contact surfaces, with the shear modulus dependent on both the average pressure and the coefficient of friction. [15] used this model to study the effect of factors such as coiling tension, radial stiffness, lubrication, and creep behaviour on coil deformation. Their calculations, interestingly, showed an increase in coil deformation with increasing number of layers used in the FEA model for the same initial stress distribution and material properties.

[16] used a FEA model with continuum linearly elastic material properties to study the effect of the self weight of steel coils with residual stresses. Interlayer slippage and opening was allowed for by using a jointed material model. (A jointed material model has joints or cracks that can open or close depending on stresses and strains). The initial stresses in the coil were calculated from a coil winding model for a linear elastic, isotropic material, hence the model neglected the compressibility of the inter-wrap gaps. The FEA calculations showed that the coiling stress has a significant effect on coil deformation. Interestingly, the results presented showed an increase in coil deformation with increasing coiling stress, whereas the calculations by [15] predicted a decrease in coil deformation with increasing coiling stress.

Such contradictory conclusions resulting from FEA calculations are not surprising. The approximate solutions thereby generated correspond, from a backwards error analysis perspective, to the exact minimization of an approximation of the exact Lagrangian. Depending on the form that the approximate Lagrangian takes, different scenarios are identified. In the above situation, it appears that [15] have modelled the situation where the GPE is larger than the TEE, whereas [16] have modelled the situation where TEE is larger than the GPE. Clarification of this point is given in Section 4.

Based on observations for thin gauge coils produced at BlueScope Steel, increasing the coiling stress decreases the likelihood of coil slump and can reduce coil deformation.

[17] developed a FEA model to calculate stresses in coils due to external forces. They used an orthotropic, elastoplastic jointed material model, accounting for interlayer slippage and separation of joints. Rather than determining the effect of gravitational forces due to the coil's weight, the FEA model was used to calculate the stresses induced in a paper coil by the clamping forces of devices used for lifting the roll (in the vertical position), as well as the interlayer slippage in a paper roll in rolling contact against a winding drum.

There appears to be no references on modelling coil deformation due to self-weight, which do not include FEA calculations. This report, in Section 4, examines phenomenonologically the modelling of coil deformation as a function of self-weight as an interplay between GPE and TEE, without specifically addressing interlayer friction.

4 Modelling coil slumping and collapse

A number of different modelling scenarios are considered. Hamilton's principle identifies the framework for a rigorous theoretical analysis, and allows, as a function of the degree of slumping, the interplay between the gravitational potential energy (GPE) and the total elastic/frictional energy (TEE) to be characterized for various coil configurations. However, the formulation and minimization of the corresponding Lagrangian requires a level of detail about the interlayer frictional and tension energy balance which was outside the scope of the study groups deliberations. Nevertheless, this framework is useful in conceptualizing the essence of the problem from an energy perspective.

We will present these ideas and this framework in the form of a set of two hypotheses enumerated below. In an attempt to further understand and quantify slumping/collapse, we will introduce further hypotheses in Section 4.2. The ideas presented here were arrived at after investigating a number of exactly solvable simple models related to the coil problem, the details of which are provided in Appendix 1.

From a pragmatic perspective, what is required is a phenomenological analysis which yields a practical rule-of-thumb which can be utilized by BlueScope Steel. This is the purpose of the discussion of Section 4.2. It is based on a hypothesis about the dynamics of coil slumping, inferred on the basis of energy considerations and the observed shapes of slumped coils as illustrated in Figure 2 and categorized as equivalence classes in 6(i)-(iii) of Section 2.2.

The starting point for a more detailed analysis of a complex situation is the choice of an appropriate modelling framework. Here, the obvious choice is the "slumping/buckling" of a cylindrical spiral spring, lying on its side under gravity. How it slumps/buckles depends on the equilibrium balance between gravitational and tension forces. Hamilton's principle asserts that, for a static conservative mechanical system, the state chosen by nature is the one that renders stationary the Lagrangian (in this case the total potential energy) $\mathcal{V} = \mathcal{E} + \mathcal{G}$, where $\mathcal{E}$ and $\mathcal{G}$ denote, respectively, the stored elastic and gravitational potential energy of the system.

For example, consider the simple mass-spring system, where the spring, sitting upright on a table, has a mass placed on top. The mass will settle, compressing the spring (decreasing $\mathcal{G}$ and increasing $\mathcal{E}$), until static equilibrium is realized with $\mathcal{E} + \mathcal{G}$ a minimum. This trade-off between $\mathcal{E}$ and $\mathcal{G}$ is central to our understanding of slumping. Of course, as with the coil, vibrations of the spring can occur if appropriate care is not exercised when performing the experiment.

A more revealing model is a heavy spring slumping under its own weight described in Appendix 1. In this case, the effect of the spring's self-weight results in a variable slump. In both these simple examples, the system is conservative. In the coil situation, energy is dissipated in the form of frictional losses, the effect of these losses on the equilibrium state is examined in a sliding mass spring system (Figure 7) in Appendix 2. With these simple examples in mind, we are now in the position to frame an hypotheses for examining the coil situation.

The formulation of the hypothesis is based on the following observational assessment of the geometry of slumped/collapsed coils and assumptions about the dynamics of the slumping/collapse:

1. The geometry of the slumped coils, as seen from the Figure 2 of Section 2, can be classified into the three distinct equivalence classes discussed in 6(i)-(iii) of Section 2.2.

2. The initiation of the slumping and the resulting shape that it takes corresponds to a failure to have a balance between the gravitational potential energy ($GPE(z)$) (associated with the increasing weight of the coil as a function of the distance z from the top of the coil) and the total elastic energy (TEE) stored in various forms in the final configuration of the coil (associated with the elastic potential energy of its spiral structure, the tension with which the coil has been wound and the friction between the inter-strip layers). As shown in Figure 5, $GPE(z)$ is a strictly increasing function of z, whereas, for a given coil configuration, the TEE is essentially constant.

4.1 The Hypothesis – initiation of slumping/collapse

H1. Essentially no, or very little, slumping occurs if, after winding, $GPE(2R_c) \sim TEE$, where R_c denotes the radius of the coil. Because coil slumping is not a common occurrence, it is natural to assume that the bulk of coils have been wound so that the difference between TEE and $GPE(2R_c)$ is marginal. Consequently, after winding, the configuration of the coil changes slightly to bring the system into equilibrium with $TEE = GPE(2R_c)$. This will involve slight simultaneous changes in both $GPE(z)$ and TEE in terms of slight changes in the shape of the coil.

H2. Immediately after winding, coil slumping will occur either quite quickly, when there is a significant mis-match between TEE and $GPE(2R_c)$, or slowly, when the mis-match is minor. It is the former situation that often results in a significant distortion of the inner bore, usually without a major collapse of the type illustrated in Figure 2. The latter situation will always occur as, initially, there will always be a minor mismatch. At worst, it may lead to a noticeable, but not necessarily significant, distortion of the inner bore.

4.2 A global model

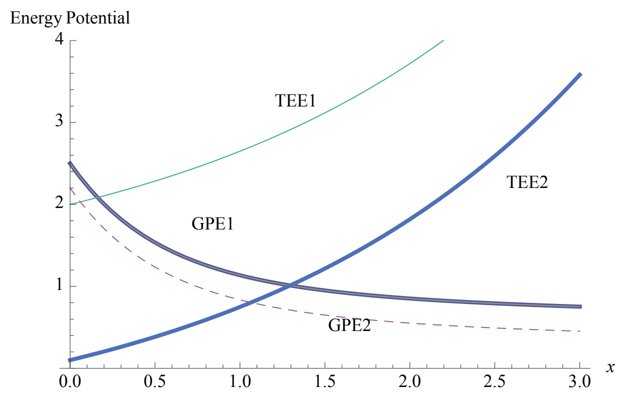


Figure 4: Gravitational and Elastic Potential Curves

We now quantify the situation. If T is the tension used to wind up the steel then the work done to wind it up into a coil is $TEE_0 \sim TL$ where L is the length of the sheet. Some of the work performed will be lost in various forms (friction, work hardening, etc), but one would expect most of the work to be stored up in elastic energy, primarily in the form of bending energy associated with the cylindrical shape, but also in the form of the 'frictional energy' bound up in the contact regions. Tighter coils (smaller mandrel radius) of course require a larger winding tension and, in addition, there will be greater contact energy associated with larger T because the contact area increases monotonically as a function of T. This provides us with a measure for the initial elastic energy stored in the coil, which needs to be 'in balance' with the initial gravitational energy $GPE_0 = MgD/2$ where $D/2$ is the initial height of the centre of mass above the floor, taken as the datum. The ratio of these two terms, to be referred to as the *"tension-weight ratio"*,

$$\mathcal{R} = \frac{2TL}{DMg}$$

is the relevant dimensionless group, and this quantifies the balance referred to earlier.

The possible shape changes that can occur, after the mandrel is removed, can be parameterized in terms of the distance x which measures the height loss due to the slumping. The GPE and the TEE now become functions $GPE(x)$ and $TEE(x)$ of x. Then, at least theoretically, the gravitational energy and elastic energy associated with such changed circumstances can be quantified and the state with minimum Lagrangian (Min-L) identified. In general terms, the shapes of $GPE(x)$ and $TEE(x)$, as a function of x, are displayed in Figure 4[1]. In particular, as x increases, the decreasing GPE curve will asymptote to a value that reflects the shape flattening (with an approximate race-track structure), while the TEE curve will asymptote towards (but not reaching) infinity because of the rapidly increasing stretching and bending needed to realize the more squashed shape. The Min-L state will be close to the intersection of these two curves, as it determines a lower bound for the value of the minimum of the Lagrangian. Evidently, given the shape of these curves there will be a Min-L state. If the coil is 'correctly' wound, then $\mathcal{R} \approx 1$, so that the curves will cross close to $x = 0$, with the Min-L state corresponding to a marginal change in state with virtually no slumping. In Figure 4, the TEE1 and GPW1 curves represent such a scenario. If, however, the tension used is too small as in TEE2, then the intersection of GPE1 and TEE2 corresponds to a larger x value corresponding to equilibrium, $\mathcal{R} << 1$, and slumping is inevitable. If, alternatively, the applied tension is too large (so that $\mathcal{R} >> 1$), then the coil will 'unwind' thus reducing its elastic energy in favor of gravitational energy. This is unlikely to be the case for BlueScope Steel coils.

Now once movement occurs (as in the sliding supported spring situation) frictional forces come into play and the overall energy status of the coil will be altered so that the GPE curve will drop and the associated equilibrium x will be altered. In Figure 4, the GPE1 curve slips to the dotted GPE2 curve. The drop will be dependent on coefficient of friction μ (as in the spring case) and there is no obvious way of estimating the location of this second curve without solving the complete problem. It represents a major undertaking.

Whilst solving the complete problem represented an unrealistic objective for the study group, the above work does suggest a useful experimental procedure for identifying critical (slumping) situations. As indicated above, $\mathcal{R}$ is the appropriate dimensionless group for assessing the $\mathcal{E}$ vs $\mathcal{G}$ balance and the only other (obvious) dimensionless group in the problem is the frictional constant μ. Dimensionality arguments thus indicate that critical situations must be defined by a relationship of the form $\mathcal{R} = \mathcal{R}(\mu)$ in the ($\mathcal{R}$, μ)-plane. Such a plot

[1] There will be different curves for the two situations identified earlier as H1 and H2.

would provide a MUCH improved characterization of collapse than that given in Figure 3, in that the essential dynamical features of the problem are condensed into the simplest form, enabling sensible extrapolation.

The proposed procedure, which requires further investigation, reduces to: Plot out points in the ($\mathcal{R}$, μ)-plane corresponding to various recorded collapses. This will identify a critical patch in the ($\mathcal{R}$, μ)-plane. A statistical analysis can then be performed to determine the mean and standard deviation associated with this critical patch, so that a probability of collapse can be determined.

Aside 1. Of course, a strap placed around the coil could prevent slumping. The Lagrangian now needs to include the stretching energy associated with the strap as it accommodates any coil shape change. If the mandrel is retained in position, then elastic energy is stored in the mandrel preventing slumping. Similarly, if cradled the system described must include cradling elastic energy.

Aside 2. A strap placed around the coil only constrains the outer circumference, so slumping, that involves slipping towards the inner hole of the coil, can still occur. Note also that such slipping, for a coil, means more wraps in total, but this may be achieved with no change in total volume, since the shape is distorting away from the optimal (for volume enclosed) circle shape. The assumption that volume does not change then provides a way to calculate total slippage.

4.3 A more detailed model

Whilst the above work provides a general framework for understanding collapse it lacks specific detail. As pointed out the processes involved are indeed complex so that any complete model could not be attempted at the MISG. However based on the observations presented earlier about the shape of the collapsed coil we extend the hypotheses in a way that we hope will fill the gap. To do this we apply Hamiltonian ideas to sections of the coil. The following additional hypotheses are made:

The formulation of the hypothesis is based on the following observational assessment of the geometry of slumped/collapsed coils and assumptions about the dynamics of the slumping/collapse:

1. The geometry of the slumped coils, as seen from the Figure 2 of Section 2, can be classified into the three distinct equivalence classes discussed in 6(i)-(iii) of Section 2.2.

2. The initiation of the slumping and the resulting shape that it takes corresponds to a failure to have a balance between the gravitational potential energy ($GPE(z)$) (associated with the increasing weight of the coil as a function of the distance z from the top of the coil) and the total elastic energy (TEE) stored in various forms in the final configuration of the coil (associated with the elastic potential energy of its spiral structure, the tension with which the coil has been wound and the friction between the inter-strip layers). As shown in Figure 5, $GPE(z)$ is a strictly increasing function of z , whereas, for a given coil configuration, the TEE is essentially constant. A graphical representation of the failure situation is given in Figure 5.

The Hypothesis – The Dynamics of Coil Slumping/Collapse after Initiation. The dynamics for the three equivalence classes of coil slump/collapse list above are:

H3. A 6(i) slump occurs when $GPE(z) > TEE$ with $R_c < z < R_c + R_b$.

H4. A 6(ii) slump occurs when $GPE(z) > TEE$ with $z < R_c$.

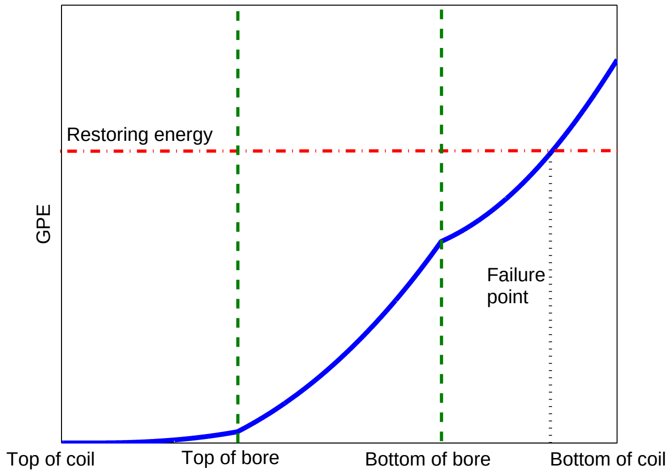


Figure 5: Gravitational potential energy of the self-weight of a coil

H5. For the transition zone where $GPE(z) \sim TEE$, with $z \sim R_c$, the configuration assumed by the slump will be 6(i), or 6(ii) (of Section 2.2) or a combination of these two possibilities. What happens will depend on the circumstances.

H6. When $GPE(z) > TEE$ only in the bottom region of the coil, where $2R_c > z > R_c + R_b$, it is assumed that no collapse occurs. Clearly, in a more definitive model, the occurrence of a 6(i) slump when $GPE(z) > TEE$ for z slightly less than $R_c + R_b$ would need to be considered.

H7. A 6(iii) slump occurs when the value of TEE is significantly larger than $GPE(2R_c)$. In a way, this corresponds to a type of bifurcation buckling phenomenon with the jump upwards increasing the $GPE(2R_c)$ to balance off the higher value of the TEE.

4.4 Validation of hypothesis

There are different ways in which the above hypothesis can be validated and utilized. They include:

- As outlined above, the circumstantial evidence suggests that there is an interplay between the tightness with which the coil is wound and the mass that a coil has before it collapses.
- From a visual inspection of the examples in Figure 2, the lower (compressed) part of a collapsed coil has a racetrack shape.
- A cylindrical racetrack shell (lying on its flat side) has a lower GPE than any circular/elliptical cylindrical shell from which it has been formed.
- The upper half of a collapsed coil has either a cylindrical arch shape or a buckled M or inverted V shape.

- The cylindrical arch shape corresponds to situations where the "tension-weight ratio" $\mathcal{R} << 1$, which is indicative of a situation where the weight of the top half or so of the coil exceeds the TEE.
- The buckled shapes correspond to situations where the "tension-weight ratio" $\mathcal{R} >> 1$, which is indicative of a situation where the TEE of the wound coil is greater than its total GPE. In such situations, a buckling upwards of the wound coil is a possibility so that the GPE comes into balance with the TEE.

5 Other deliberations and future research

For a challenging MISG problem, for which the Coil Slumping problem is a good example, much of the deliberations involved looking at the problem from different points of view, in a wide-ranging brainstorming manner. The role of this section is to give a list of the various ideas discussed which are not covered in the body of the Report. The following represent possibilities for future research.

- Comparisons of the $GPE(z)$ calculations for different coil and shell shapes.
- The energy balance for the equilibrium of a spiral spring, sitting vertically, under its own and added weight.
- Energy balance between GPE and bending energy.
- Rigorous calculation of bending energy.
- Some consideration was given to the detailed force balance along the lines of [18], when slippage between wraps is allowed to occur. The following comments summarise this:

  The frictional force F is zero if the coil is in (radial) tension and $F = \mu\sigma_{rr}$ otherwise, F is non-zero. It remains to do this properly for large deformations in the sense that the radial direction may not remain the normal direction. On taking moments, it follows that $F = \sigma_r\theta - \sigma\theta_r$, so that when there is no friction the stresses return to being symmetric. In particular, for a material in compression, equilibrium plus taking moments gives three relationships between the σ's. Since four are required, some additional constitutive constraint must be invoked to close the problem.
- Frictional contact. Firstly note that the wound coil problem (determining the tension in the wound coil before placing it on the floor) is statically indeterminate in the sense that the different tension distributions $T(s)$ along the wound coil can correspond to a wound coil anchored at the mandrel and with prescribed tension at the end $T(L) = T_0$. Thus, certain patches of a winding may be unstretched (ie.$T(s) = 0$), with neighbouring patches on the point of slipping with $T(s) = \mu_c N(s)$ locally. Other patches will be barely 'stuck' (with $T(s^{\pm}) < \mu_c N(s)$ so that no surface slip will occur with the actual value of $T(s)$ determined by local equilibrium). Furthermore the longitudinal stress acting on a winding may vary across the observed winding so that the sheets above and below can slide in opposite directions or in the same direction. A suitable coil winding/contact surface model together with an appropriate prescribed applied tension model eg $T(L) = T_0 + \epsilon T'$ can be used to determine the initial tension state in the coil.

(Some details about the issues involved can be found in [5]). The important point here is that it is likely that the initial tension distribution probably determines which coils in a batch slump and which don't, so tension control during windup is important. Now both during and after windup localized slipping will occur, perhaps commencing at one location and setting in place a cascading series of slippages which will eventually cease when all surfaces are 'stuck', but with some portions of the windings 'about to slip' and other portions 'under stressed' or even 'stress free'.

6 Conclusions

Because of the inherent complexity of this problem in terms of the different factors (e.g. geometry (of collapse), weight, tension, friction, etc) involved, it was not possible to formulate a defnitive model for the dynamics of the slumping/collapse of a steel coil. However, the deliberations have lead to an effective way of thinking about collapse based on Hamiltonian principles which has been summarized in the form of a set of hypotheses. Based on these hypotheses, the tension-weight ratio has been identified as the key parameter for quantifying collapse, so that the determination of the critical value of this parameter as a function of the coefficient of frictional between sheets is seen as being the essence of the problem. In the absence of an adequate dynamical model, one may make use of historic data to identify this function as described in Section 5. We believe this to be an effective way for BlueScope Steel to immediately proceed. In more detail, our improved understanding has resulted in:

(i) the need to draw a clear distinction between minor slumping and major slumping which can subsequently lead to collapse,

(ii) the formulation of a heuristic hypothesis about the dynamics of coil slumping/collapse which can be compared with historical data and act act as a conceptualization guide for further investigations,

(iii) the most appropriate framework for a more detailed modelling endeavour is variational using Hamiltonian/Lagrangian energy principles,

(iv) the identificaiton of the *"tension-weight ratio"* $\mathcal{R}$ as the relevant dimensionless group which represents an indicative rule-of-thumb which can be applied in practice,

(v) the identification of the model problems of spring collapse under its own weight and interlayer slip modelling, and

(vi) some possibilities for future research projects.

For BlueScope Steel, it is hoped that this enhanced understanding will become the basis for their continuing investigation of coil slumping/collapse. From the Coil Slumping team, who enjoyed the challenges posed by such a daunting problem, thanks go to BlueScope Steel for bring it to MISG 2009.

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The appendices

Within the context of the questions that are being investigated, the starting point for the formulation of a simple representation of a problem is the identification of features which appear to play a primary role. Though, in the coil slumping/collapse problem, interlayer slip is clearly involved the moment a coil starts to slump after the removal of the mandrel, it is only a first order effect if the slumping is not minor. Even though interlayer slip continues to occur as the coil collapses, it is abundantly clear from the collapses illustrated in Figure 2, and the discussion of Section 2.2, that the interlayer slip is second order because a collapse is either a self-weight buckling (because the tension is too small) or a tension buckling (because the tension is too high).

The minor slumping situation was not examined. It is not necessary for the situations where it is so slight that the integrity of the cylindrical bore is not compromised. It is necessary for situations where the slumping compromises the integrity of the cylindrical bore. This did not become a major focus for the study group. An exemplification of the frictional issues involved is discussed below under the heading "Interlayer Slip Modelling" in Appendix 2.

Appendix 1: Simple spring modelling

Thus, for a simple spring mass system, there is a trade-off between $\mathcal{E}$ and $\mathcal{G}$ with $\mathcal{E}$ increasing and $\mathcal{G}$ decreasing. As is clear from Figure 4, this guarantees that the total energy $\mathcal{V}$ has a well-defined minimum.

As a simpler model problem, such a framework could be used to analyse the slumping of a vertical spring under its own weight. This is a more complex situation than the compression (extension) of a weightless spring when a point mass M is attached or placed on the top, as illustrated in Figure 6. In that situation, the Lagrangian takes the form

$$\mathcal{V}(h) = \mathcal{E}(h) + \mathcal{G}(h) = \frac{1}{2}k(h_0 - h)^2 + Mgh$$

where k denotes the elastic modulus of the spring, h and h_0 the height of the point mass on the stretched and unstretched spring above some reference point, and g the gravitational acceleration. The minimization of this Lagrangian yields the classical result $k(h - h_0) = Mg$.

However, in the latter, it is assumed, in terms of how the situation is modelled, that the coils of the spring remain evenly distributed as the spring extends, whereas in the former, this is not and cannot be assumed as the increasing weight of the spring, as one moves down from the top, generates an increasing compression of the spring. It is this difference which exemplifies why the modelling of the slumping of a steel coil is much more involved than it might appear at first sight.

For a simple model for self-weight buckling, an appropriate choice is the extension (compression) of a vertical spring under its own weight or under its own weight with an additional mass placed on top, as illustrated in Figure 6. It highlights how including the weight of an object can change the nature of modelling. There is a limited literature on this problem (cf.

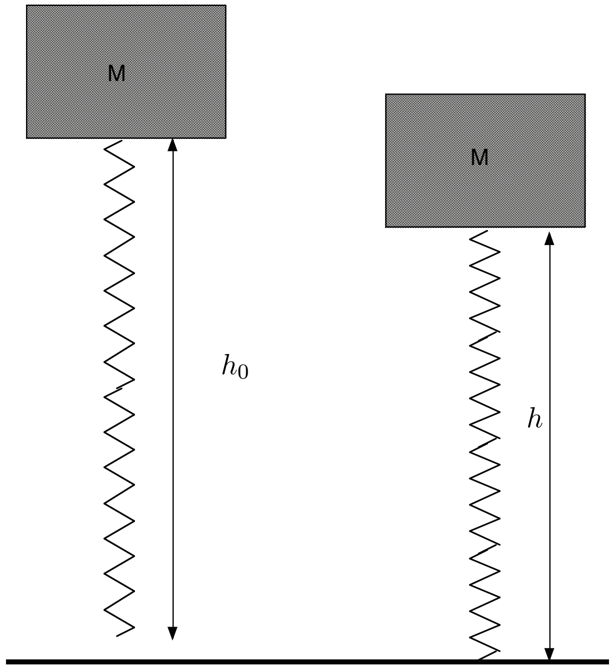

Figure 6: A mass spring system: the figure shows the system just before the the spring makes contact with the floor and after the system has relaxed.

Champion and Champion [19, 20] and the references cited there). It is often discussed in papers that model the oscillation of a spring with an attached mass, where the motivation is to model the vibration of a spring so that theoretical estimates of the period of oscillation match the experimentally measured ones. In such situations, the assumption that the spring is weightless is too simplifying an assumption. The analysis for a Hookian spring can be found in [19], while that for a non-Hookian spring is discussed in [20].

Appendix 2: Interlayer slip modelling

A weight is placed on a rough table angled at θ to the horizontal and height h_0 above a datum and an initially uncompressed spring is in a position to support the weight but also the frictional contact can help support the weight as it slides downwards (Slider in Figure 7). We have in this case $N = Mg\cos\theta$, and if $\mu_c = \tan\alpha_c$ is the critical coefficient of friction associated with the contact, then:

1. If $Mg\sin\theta < F = \mu_c Mg\cos\theta$ (so $\theta < \alpha_c$), then the frictional support up the plane supplied by the contact is adequate by itself to support the weight, and it will remain in position. There will be elastic energy bound up in the contact region, which can be thought of as being a 'sheared' elastic zone. Importantly (in the coil case) the amount of stored energy will depend on the orientation θ. In the sliding mass case this stored energy is usually ignored.

2. On the other hand if $Mg\sin\theta > F$, then friction cannot by itself support the mass, and

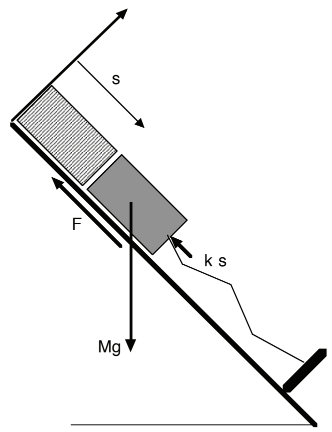

Figure 7: A Sliding Weight

it will slide down the table, losing energy as it goes until the combined 'elastic' supports bring it to rest. If s is the distance it moves down the plane (so that $s \sin\theta = x$ is the total vertical drop of the weight) then the gravitational energy Mgx loss is either frictionally dissipated ($\mathcal{F} = Fs = \mu_c Nx / \sin\theta$) or elastically stored in the spring. We have

$$\mu_c Mg \cos\theta - ks = Mg \sin\theta, \quad \text{so } \frac{k}{Mg} s = \tan\theta - \tan\alpha_c$$

determines the displacement s down the plane.

In the coil case, one can think of M as referring to a section of a wrap, however the normal force N acting on the wrap element (and thus the frictional force) varies greatly with location and orientation, with elements close to the floor supporting the total coil weight and those at the top of the coil supporting little of the coil weight; of course the tension in the wrap will also vary greatly and influence N. Sections close to the floor will thus have a tendency to stick whereas those higher up will have a tendency to move thereby losing energy (frictional dissipation) and reapportioning the available gravitational energy in the structure as a whole (the spring) either in the form of elastic energy (bending, stretching) or in the form of surface contact energy. Of course the geometry of the coil will change as a result so that there will be a changed balance between the bending and stretching elastic components. Of course the sections in a single wrap are connected, so that compatibility requirements constrain the displacements in that wrap and indeed the coil as a whole. This may mean that sections will lift away from the lower layer $N \to 0$. One might expect wraps to be pinned immediately under the centre of the coil, so that in plane movement may ordinarily be not transmitted from one wrap to the next. It seems likely however that 'slump' refers to situations in which 'the pin' fails; models examining this are being developed. It would appear from the above description that any useful predictive model must include localized frictional losses and contact energies, local winding orientation, globalized compatibility constraints and force balance. One might hope however that the above details can be avoided so that a simple (global) description can be developed.

Appendix 3: The buckling of rings and arches

A simple model for tension buckling is the equations derived by [21] for the elastic stability (buckling) of weightless rings and arches. By ignoring gravitational effects, the mathematics simplifies to the point where explicit solutions are derived.

The critical buckling load Q_{crit} is determined as a function of the arch properties (Young's modulus (E), second moment of inertia (I), the arch thickness to ratio (S)), and support conditions (in our case pin jointing being most appropriate);

$$Q_{crit} = \frac{4EI}{R^2}\bar{Q}(S, \lambda)$$

where λ is the first eigenvalue of the associated homogeneous problem. Here $\bar{Q}$ is the eigen solution of a boundary valued problem that needs to be determined numerically for values of S of interest and for boundary conditions appropriate for each of the two modes identified earlier. Details can be found in [21].

References

[1] Altmann, H.C. (1968) Formulas for computing the stresses in center-wound rolls, *Tappi*, **4**, 176-179.

[2] Hakiel, Z. (1987) Non linear model for wound roll stresses, *Tappi*, **70** (5), 113-117.

[3] Willett, M.S. & Poesch, W.L. (1988) Determining the stress distributions in wound reels of magnetic tape using a nonlinear finite difference approach, *Trans. ASME, Ser. E, J. Appl. Mech.*, **55**, 365-371.

[4] Zabaras, N., Liu, S., Koppuzha, J. & Donaldson, E. (1994) A hypoelastic model for computing the stresses in center-wound rolls of magnetic tape, *Trans. ASME, Ser. E., J. Appl. Mech.*, **61**, 290-295.

[5] Cozijnsen, M. & Yuen, W.Y.D. (1996) Stress distributions in wound coils, *Proc. 2nd Biennial Australian Engineering Mathematics Conference*, July 15-17, Sydney, 117-124.

[6] Benson, R.C. (1995) A nonlinear wound roll model allowing for large deformation, *Trans. ASME, Ser. E, J. Appl. Mech.*, **62**, 853-859.

[7] Qualls, W.R. & Good, J.K. (1997) Orthotropic viscoelastic winding model including a nonlinear radial stiffness, *ASME J. Appl. Mech.*, **64**, 201-208.

[8] Tramposch, H. (1965) Relaxation of internal forces in a wound reel of magnetic tape, *ASME J. Appl. Mech.*, **32**, 865-873.

[9] Tramposch, H. (1967) Anisotropic relaxation of internal forces in a wound reel of magnetic tape, *ASME J. Appl. Mech.*, **34**, 888-894.

[10] Lin, J.Y. & Westmann, R.A. (1989) Visco-elastic winding mechanics, *J. Appl. Mech.*, **56**, 821-827

[11] Cole, K.A. & Hakiel, Z. (1992) A nonlinear wound roll stress model accounting for widthwise web thickness nonuniformities, *Proc. Web Handling Symposium, ASME Applied Mech. Div., AMD-Vol 149*, 13-24.

[12] Hakiel, Z. (1992) On the effect of width direction thickness variations in wound rolls, *Proc. 2nd Int. Conf. Web Handling*, Oklahoma State University, May, 79-98.

[13] Lee, Y.M. & Wickert, J.A. (2002) Stress field in finite width axisymmetric wound rolls, *ASME J. Appl. Mech.*, **69** (2), 130-138.

[14] Good, J.K. (2005) The abilities and inabilities of wound roll models to predict winding defects, *Proc. 8th Int. Conf. Web Handling*, Oklahoma State University, June 5-8, 1-71

[15] Li, S. & Cao, J. (2004) A hybrid approach for quantifying the winding process and material effects on sheet coil deformation, *Transactions of the ASME*, **126**, 304-313.

[16] Smolinski, P., Miller, C.S., Marangoni,R.D. & Onipede, D. (2002) Modeling the collapse of coiled material, *Finite Elements in Analysis and Design*, **38**, 521-535.

[17] Arola, K. & von Hertzen, R. (2006) An elastoplastic continuum model for a wound roll with interlayer slippage, *Finite Elements in Analysis and Design*, **42**, 503-517.

[18] Timoshenko, S.P. & Goodier, J.N. (1970) *Theory of Elasticity*, McGraw-Hill, New York.

[19] Champion, R. & Champion, W.L. (2003) The oscillations of a loaded spring, *Math. Scientist*, **28**, 67-78.

[20] Champion, R. & Champion, W.L. (2007) The extension and oscillation of a non-Hooke's law spring, *Euro. J. Mech. A/Solids*, **26**, 286-297.

[21] Timoshenko, S.P. & Gere, J.M. (1961) *Theory of Elastic Stability*, McGraw-Hill, New York.