Math in Industry Siemens Health Care Slugging Along Tube CGU, 2009

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OutLine:

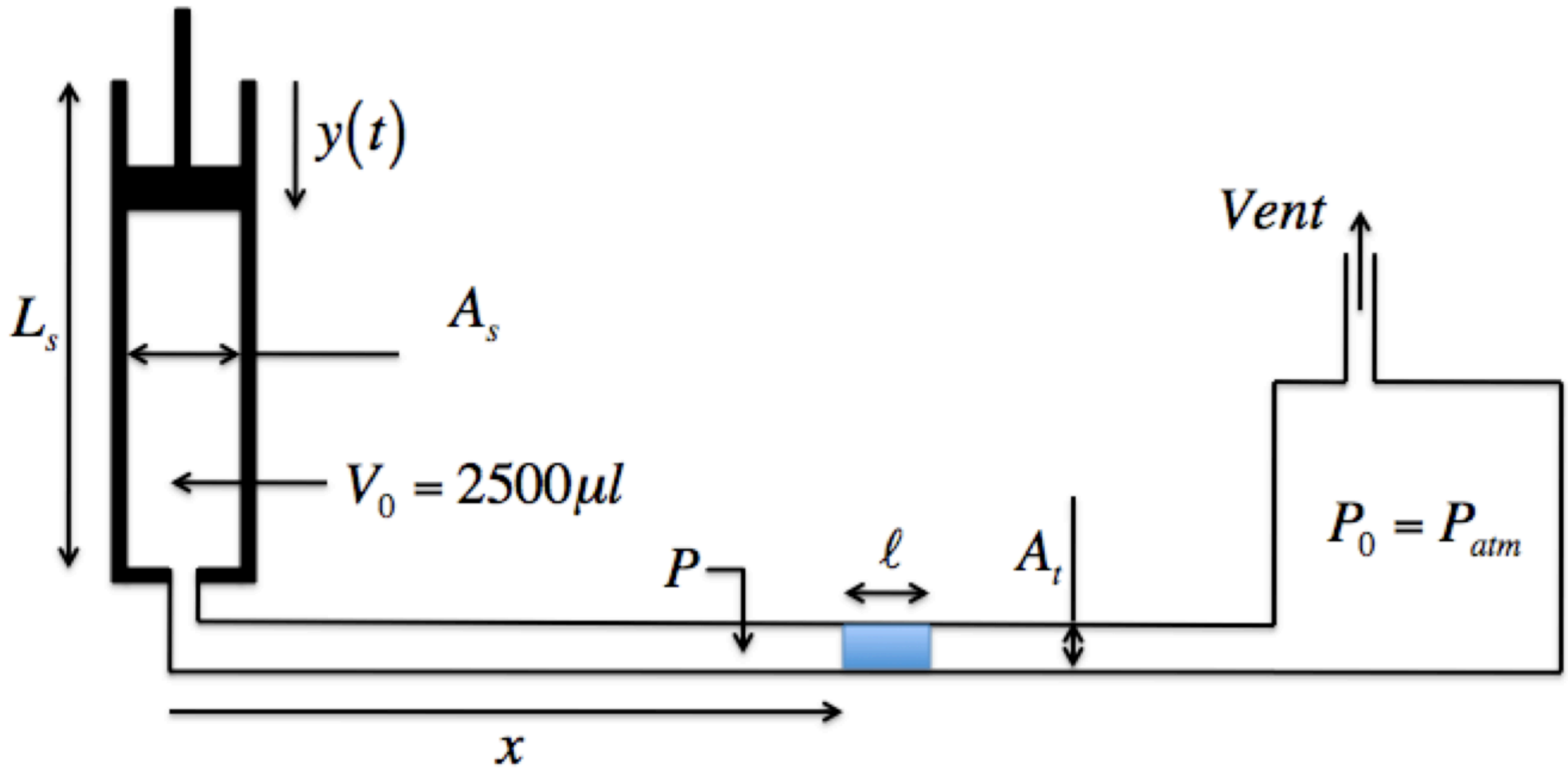
- Problem Definition
- Incompressible Flow
- Compressible Flow
    - Dry Chamber:
        - One Slug
        - Two Slugs
    - Wet Chamber:
        - Bubble Formation and One Slug
- Recommendations

Problem Definition:

- System Description:
  - A liquid slug of length l is driven into a cylindrical reaction chamber by a flux of gas, which then vents to atmosphere
- Inquiries:
  - What is the pumping protocol for the delivery of a slug down a prescribed distance x into the reaction chamber?
  - What is the effect of having more than one slug in the tube at one time?
  - What is the cause of foam formation within the reaction chamber, and how may this be avoided?

Part I: Incompressible Flow

Schematic View (One Slug):



$$A_s = 3.14\times10^{-6} m^2; \quad \ell = 0.5 \sim 5cm; \quad A_t = 1.96\times10^{-7} m^2$$

Incompressible Case:

- In incompressible case, everything is rather easy
- The relation between the plunger and the slug is:

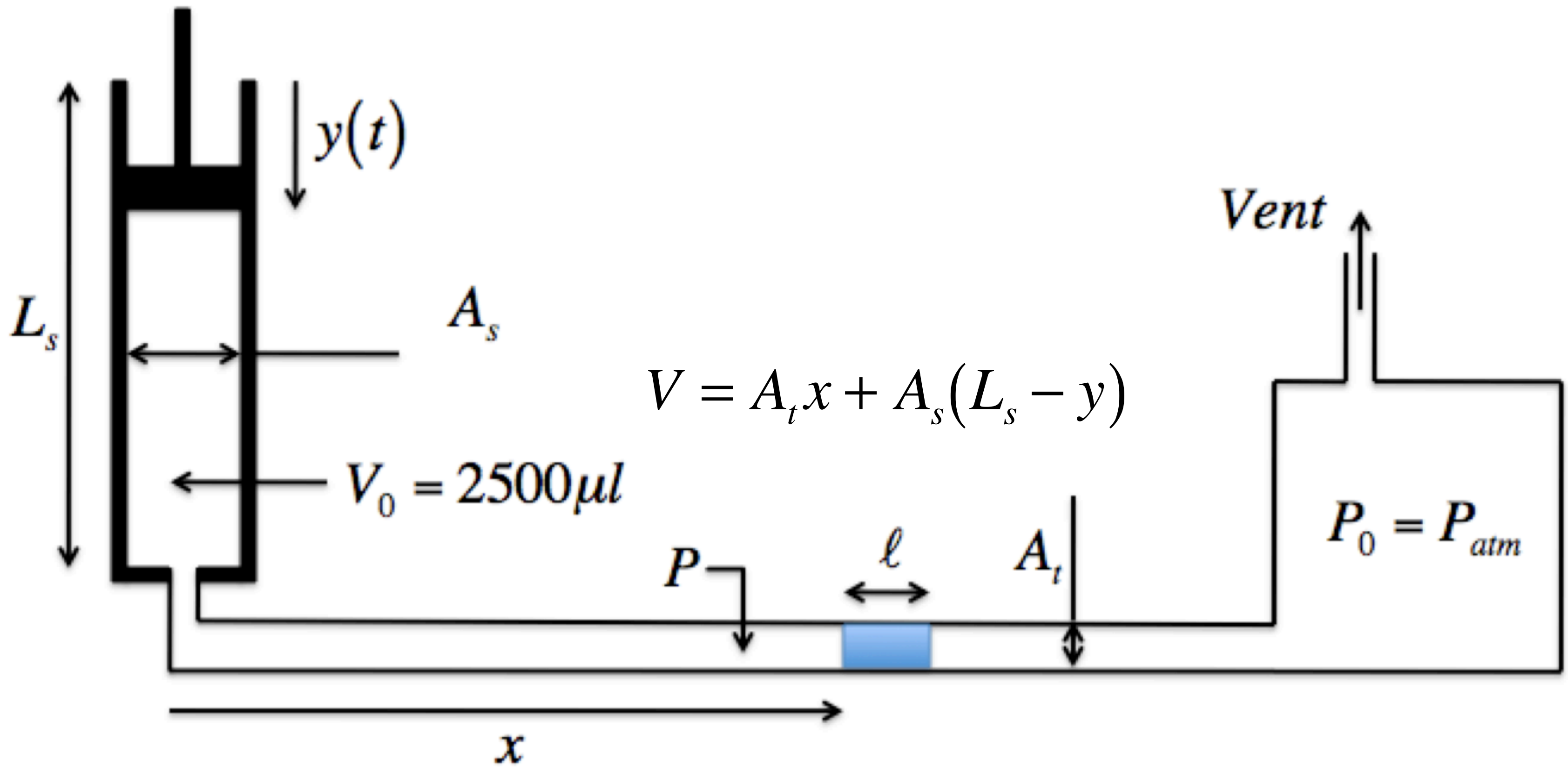
$$x = \frac{A_s}{A_t} y$$

- So, If we assume $A_s = A_t$, we need a 6 m long syringe to push the slug 6 m down the tube.

Part II: Compressible Flow

Dry Chamber - One Slug

Schematic View (One Slug):



$$A_s = 3.14\times10^{-6}m^2;\quad \ell = 0.5\sim 5cm;\quad A_t = 1.96\times10^{-7}m^2$$

Model for single slug:

- We have:

$$(P - P_0)A_t = m\ddot{x} + c\dot{x}$$

- Ideal Gas Law at constant temperature (Boyle's Law):

$$PV = P_0 V_0$$

$$V_0 = 2500\mu l$$

- Remember:

$$V = A_t x + A_s(L_s - y)$$

- Finally:

$$P_0 A_t \left(\frac{V_0}{A_t x + A_s(L_s - y)} - 1 \right) = \underbrace{\rho_w A_t \ell}_{m} \ddot{x} + \underbrace{8\pi\mu\ell}_{c} \dot{x}$$

Finding ‘c’:

- We have:

$$\Delta P = P_b - P_c - P_0 + P_c$$
$$\ell_p = \ell - 2r_w$$

- From Pipe Flow:

$$\bar{u} = \frac{\Delta P}{8\mu\ell_p} r_w^2 = \dot{x}$$

- Using force balance:

$$c\dot{x} = A_t (P_b - P_0) = A_t \Delta P$$
$$c \frac{\Delta P}{8\mu\ell_p} r_w^2 = \Delta P \pi r_w^2$$
$$c = 8\pi\mu\ell_p$$

Non-Dimensional Form:

- Length Scales:

$$x \Rightarrow L_t x$$

$$y \Rightarrow \frac{A_t L_t}{A_s} y$$

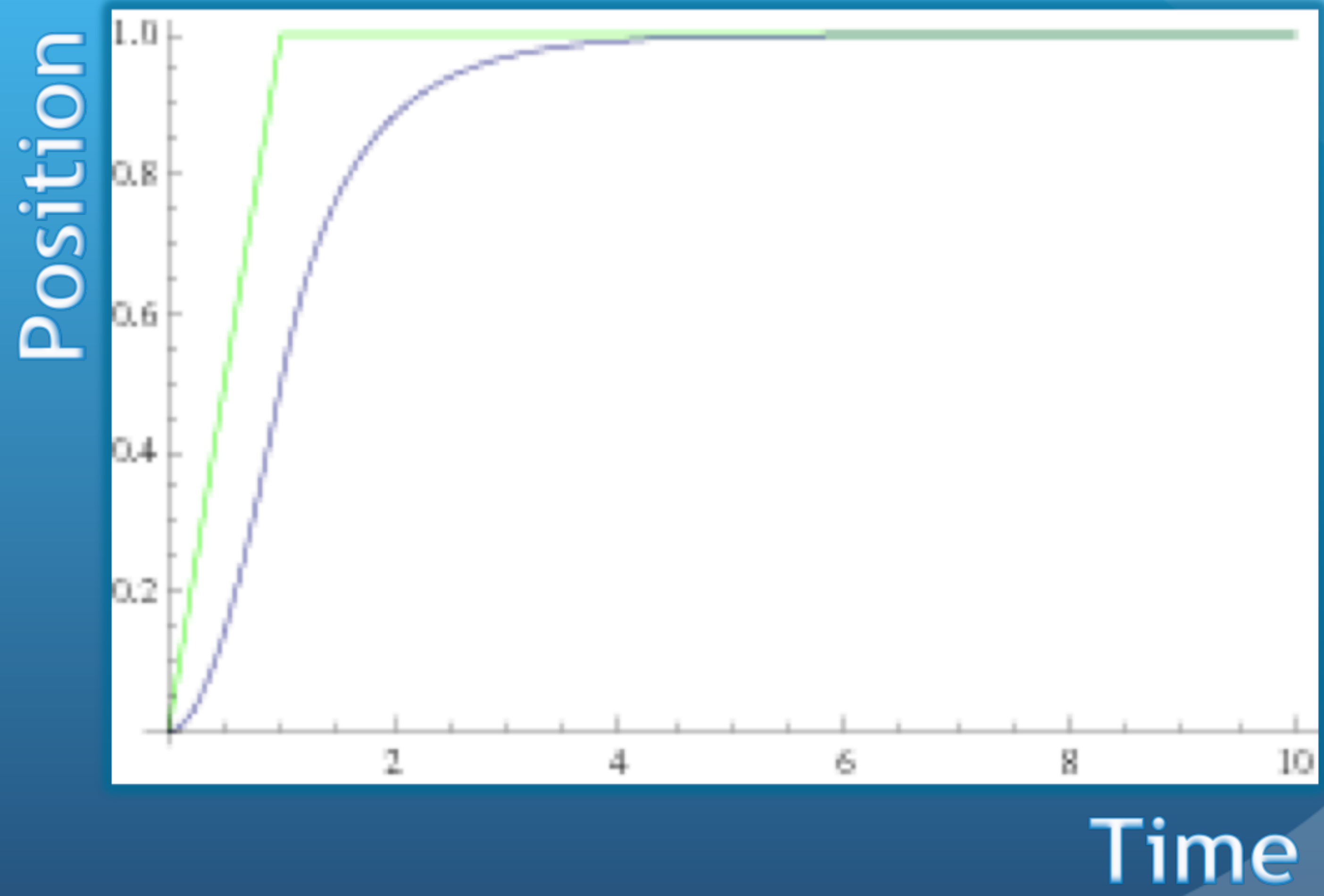
- The non-dimensional form would be:

$$\left(\frac{1}{1+\delta(x-y)} - 1 \right) = \alpha \ddot{x} + \beta \dot{x}$$

- Where:

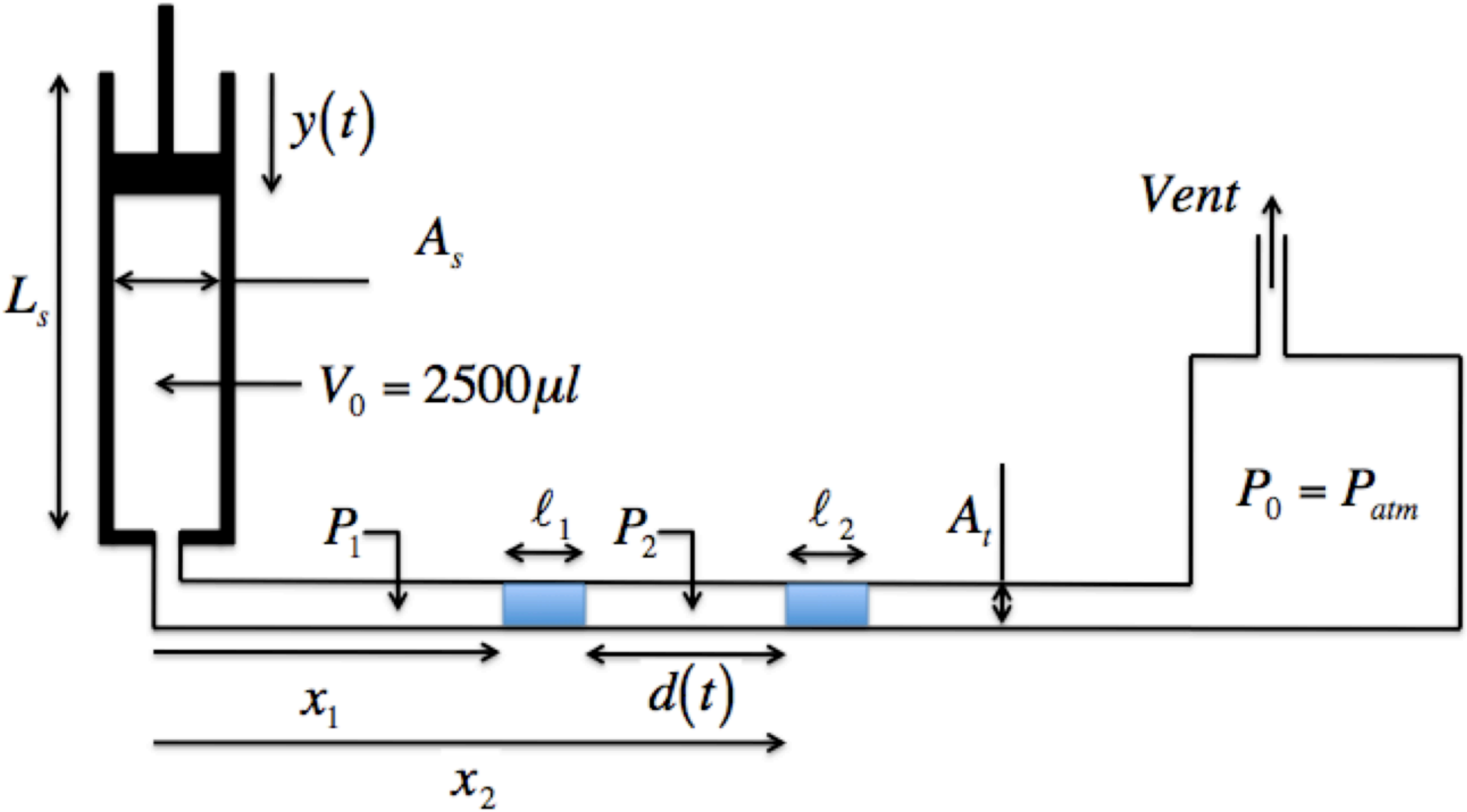
$$\alpha = \frac{\rho_w \ell L_t}{P_0 T^2}; \quad \beta = \frac{8\pi\mu\ell L_t}{A_t P_0 T}; \quad \delta = \frac{A_t L_t}{A_s L_s}$$

Results:



Dry Chamber - Two Slugs

Schematic View of Two Slug:



$A_s = 3.14 \times 10^{-6} m^2; \quad \ell = 0.5 \sim 5cm; \quad A_t = 1.96 \times 10^{-7} m^2$

Model for Two Slugs:

- We have:

$$\begin{cases} A_t(P_1 - P_2) = m_1\ddot{x} + c_1\dot{x} \\ A_t(P_2 - P_0) = m_2\ddot{x} + c_2\dot{x} \end{cases}$$

- Choosing the Scale:

$$x \Rightarrow L_t x; \quad y \Rightarrow \frac{A_t L_t}{A_s} y$$

- We have:

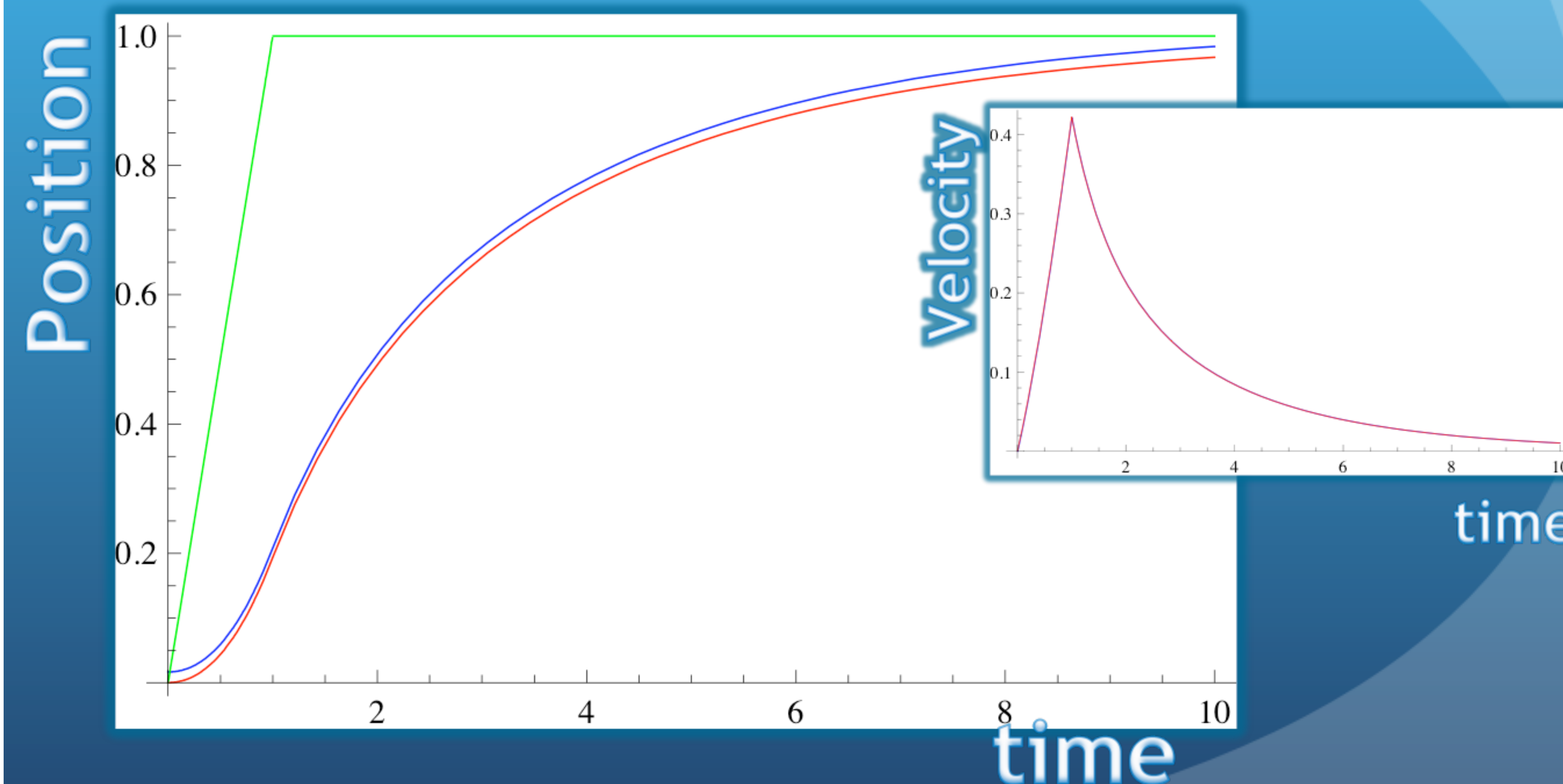
$$\begin{cases} \left[\dfrac{1}{\delta_1(x_1 - y) + 1} - \dfrac{\delta_3}{(x_2 - x_1) - \delta_2}\right] = \alpha_1\ddot{x}_1 + \beta_1\dot{x}_1 \\ \left[\dfrac{\delta_3}{(x_2 - x_1) - \delta_2} - 1\right] = \alpha_2\ddot{x}_2 + \beta_2\dot{x}_2 \end{cases}$$

- Where:

$$\alpha_i = \frac{\rho_w \ell_i L_t}{P_0 T^2}; \quad \beta_i = \frac{8\pi\mu\ell_i L_t}{A_t P_0 T}; \quad \delta_1 = \frac{A_t L_t}{A_s L_s}; \quad \delta_2 = \frac{\ell_1}{L_t}; \quad \delta_3 = \frac{d_1}{L_t}$$

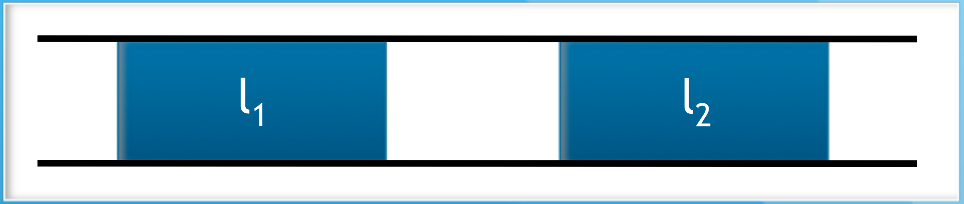
Case I: (Sub 1→Red; Sub 2→Blue)

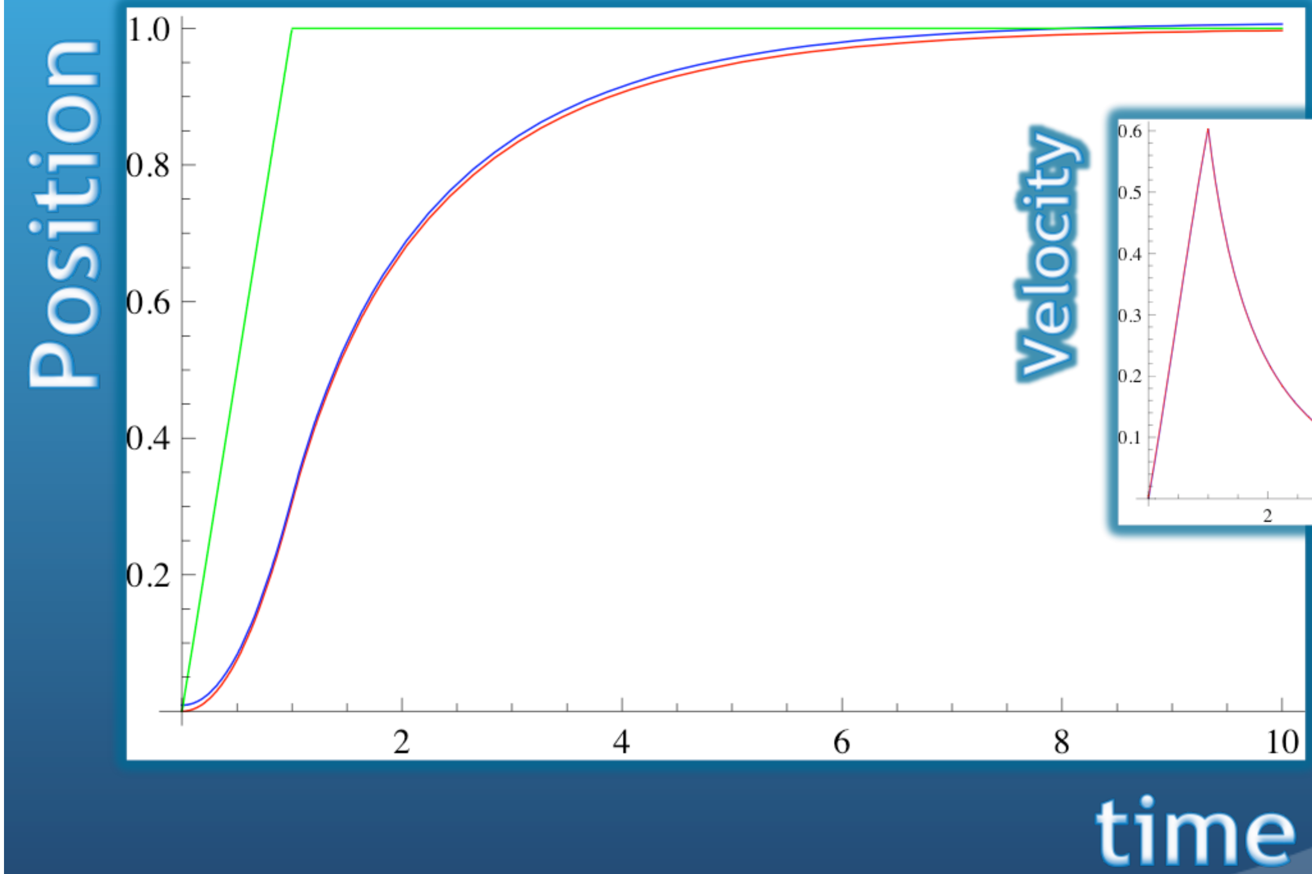
- We have: $\ell_1 > \ell_2$

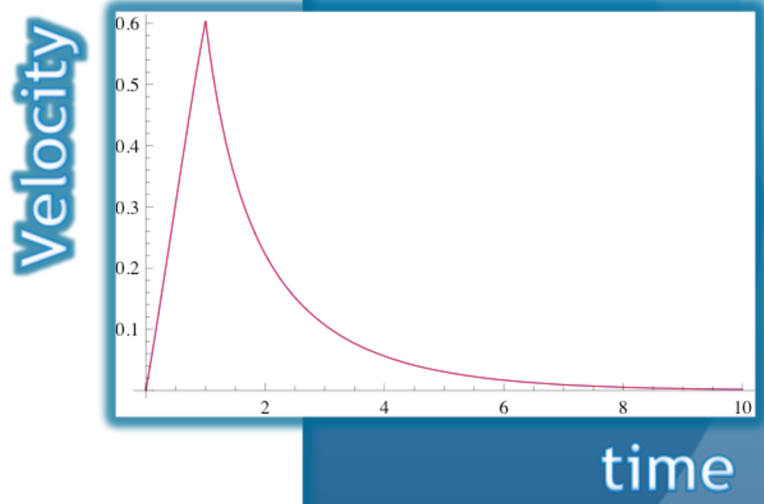


Case II: (Sub 1→Red; Sub 2→Blue)

- We have: $\ell_1 = \ell_2$

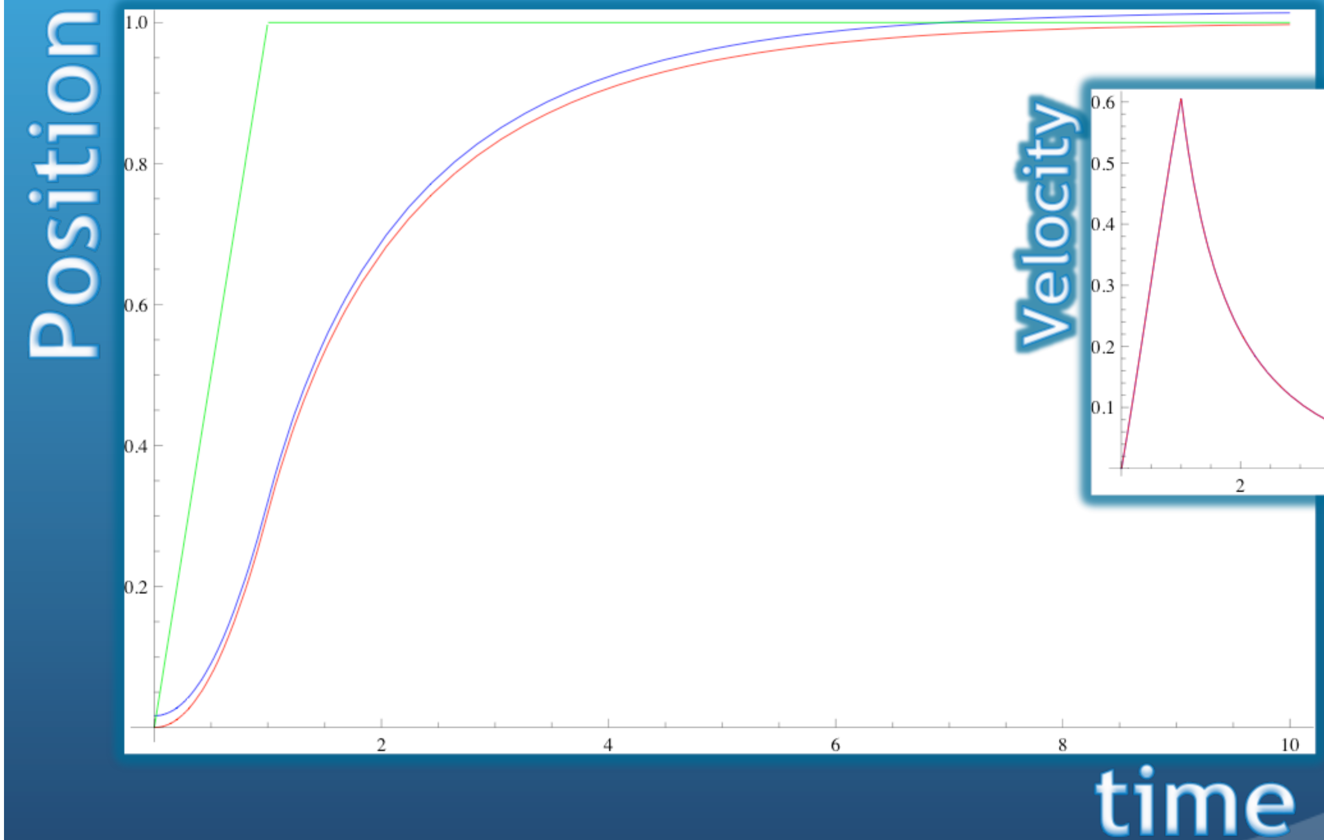


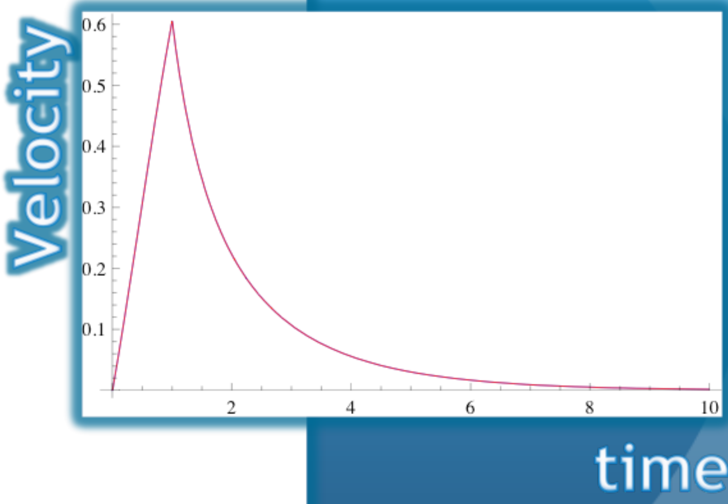




Case III: (Sub 1→Red; Sub 2→Blue)

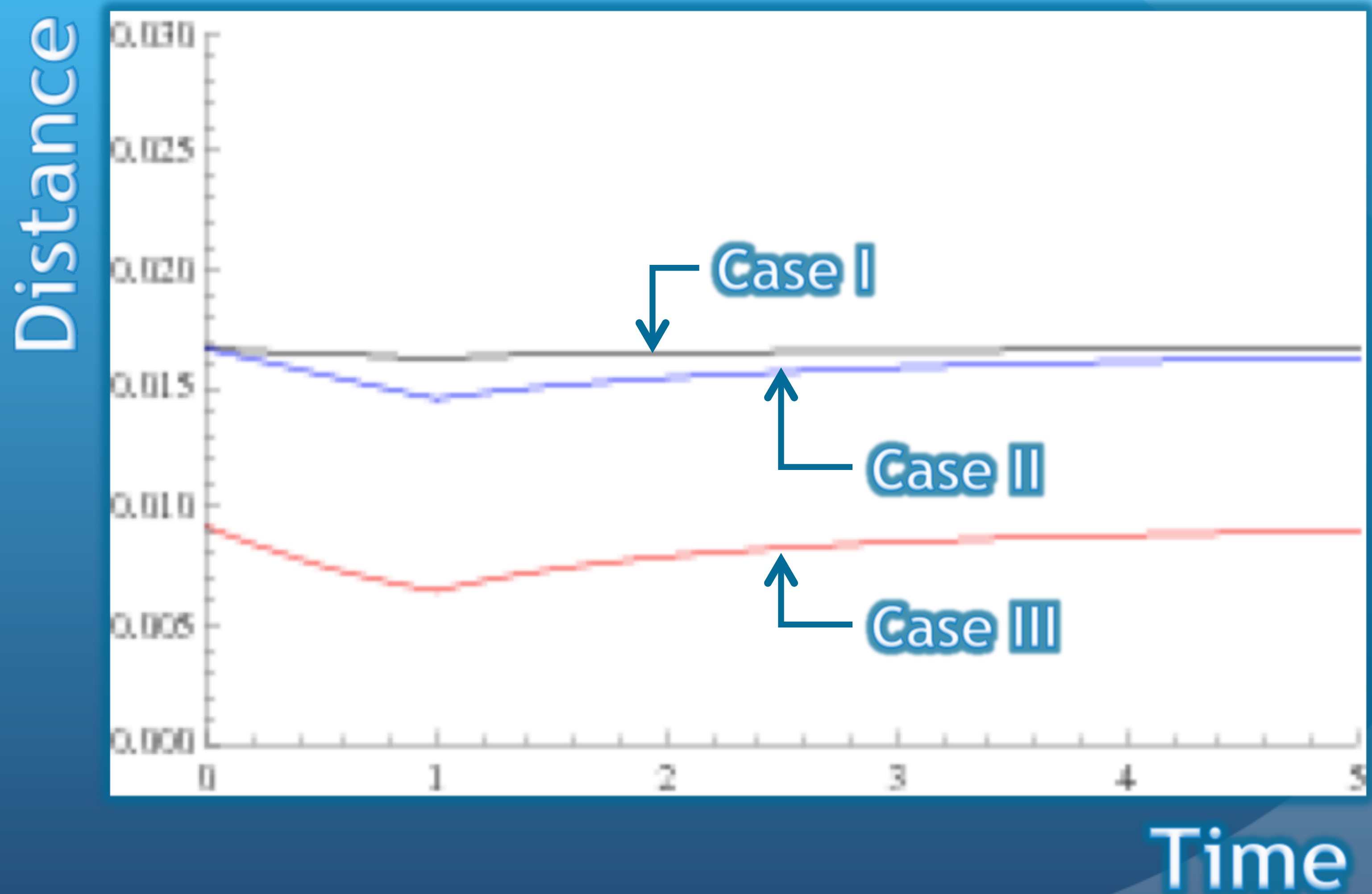
- We have: $\ell_1 < \ell_2$





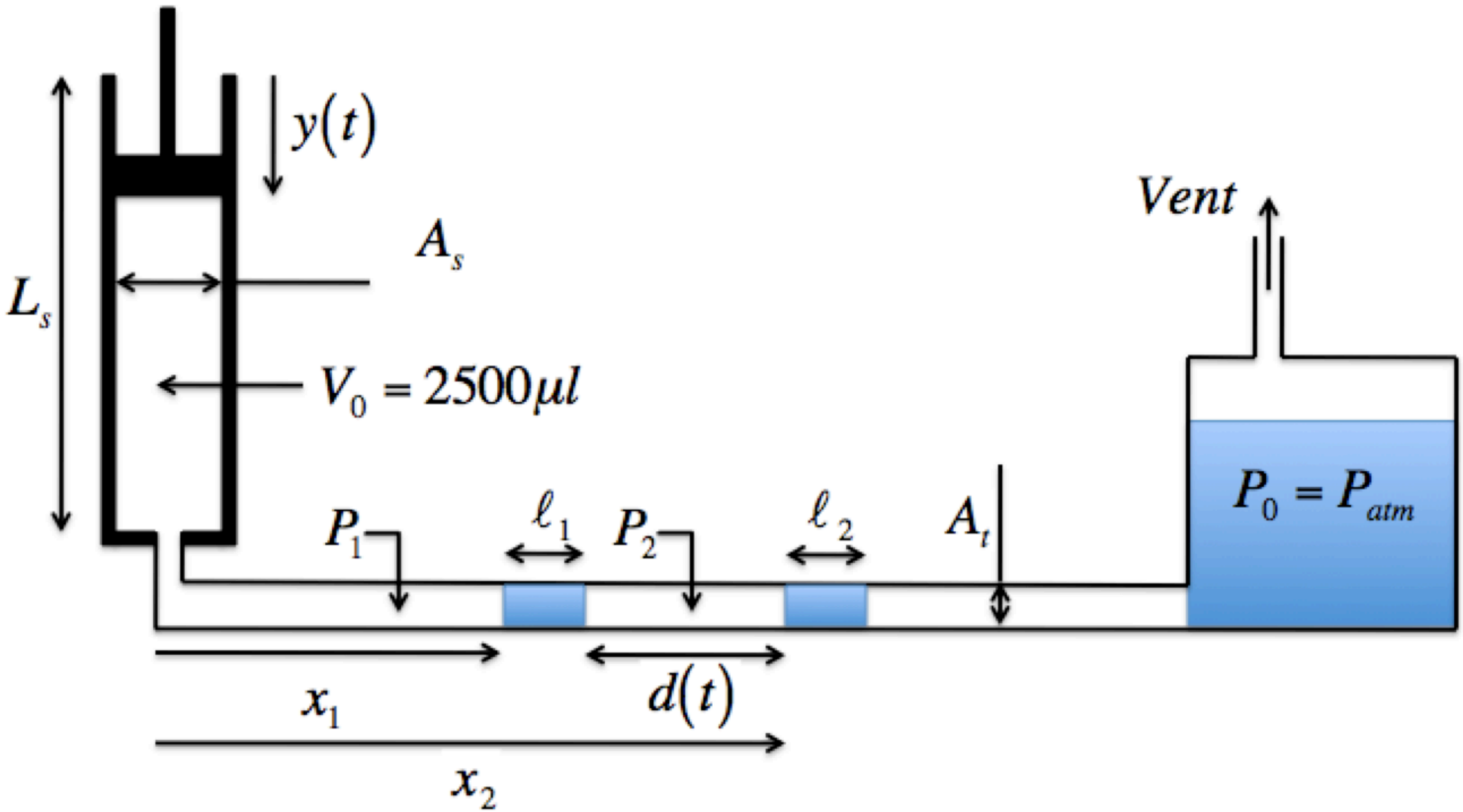
The Gap:

- Changes in the distance between the two slugs are:



Wet Chamber: Bubble Formation & One Slug Model

Schematic View of Wet Chamber:



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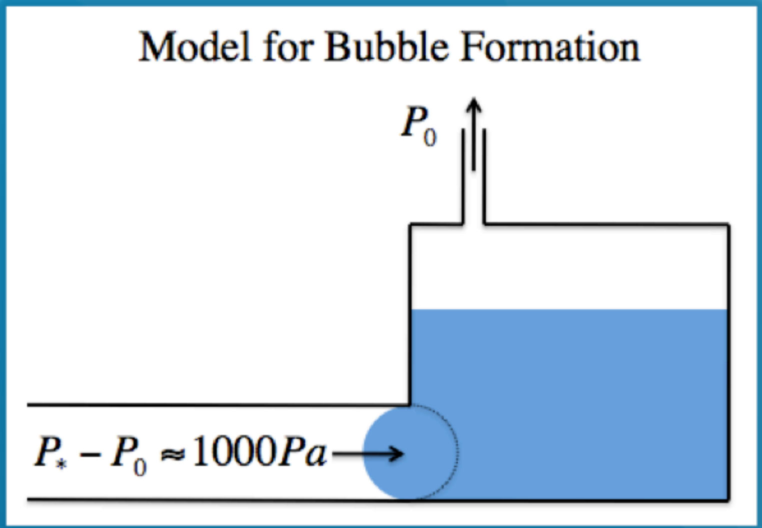
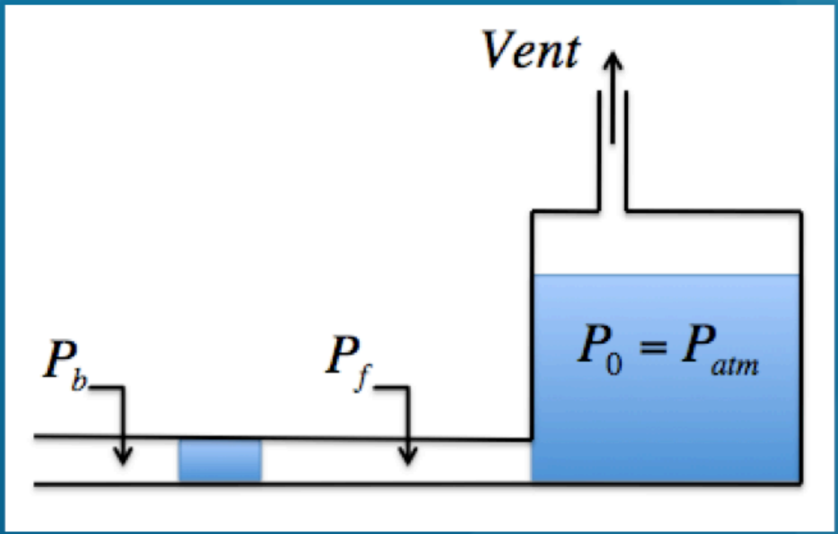
Bubble Formation

- This is the modified equation for one Slug with a wet chamber.

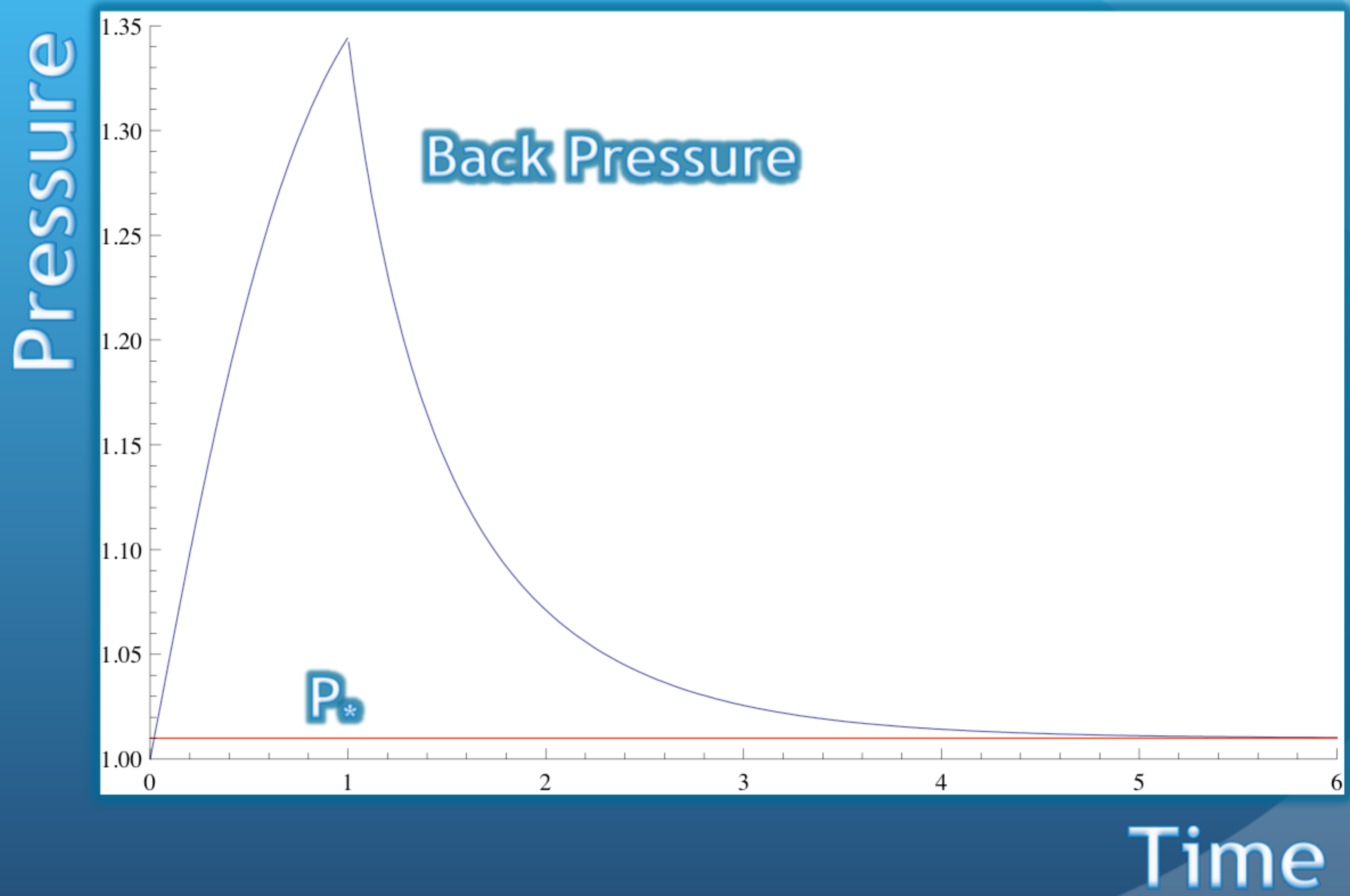
$$\left(\frac{P_b}{P_0} - \frac{P_f}{P_0}\right) = \alpha\ddot{x} + \beta\dot{x}$$

$$\frac{P_f}{P_0} = \min\left(\frac{V_f^0}{A_p(L_p - (x_n + l_n))}, \frac{P_*}{P_0}\right)$$





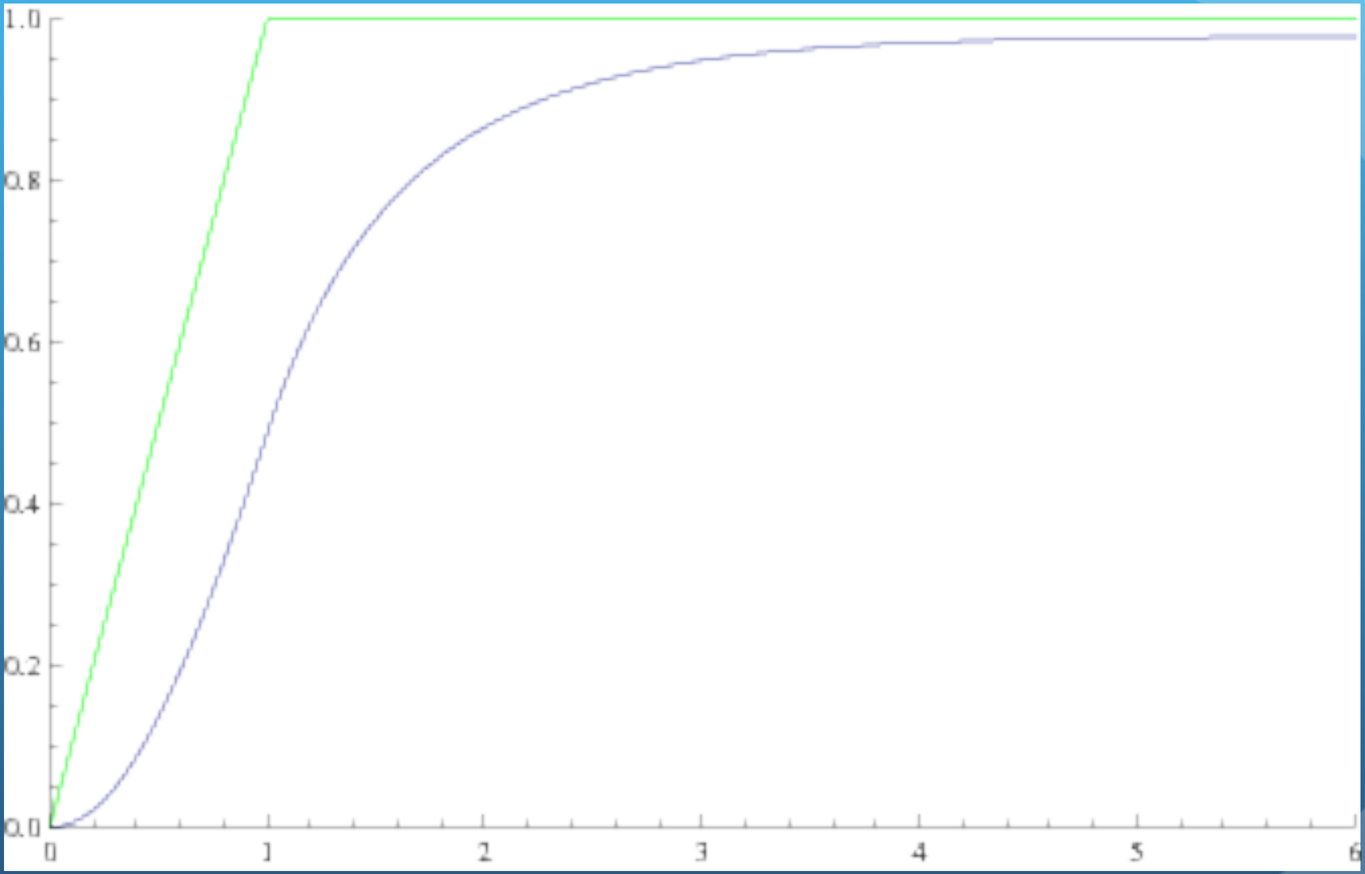
Results:



Pressure
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
Back Pressure
P*
0
1
2
3
4
5
6
Time

Results: (cont.)

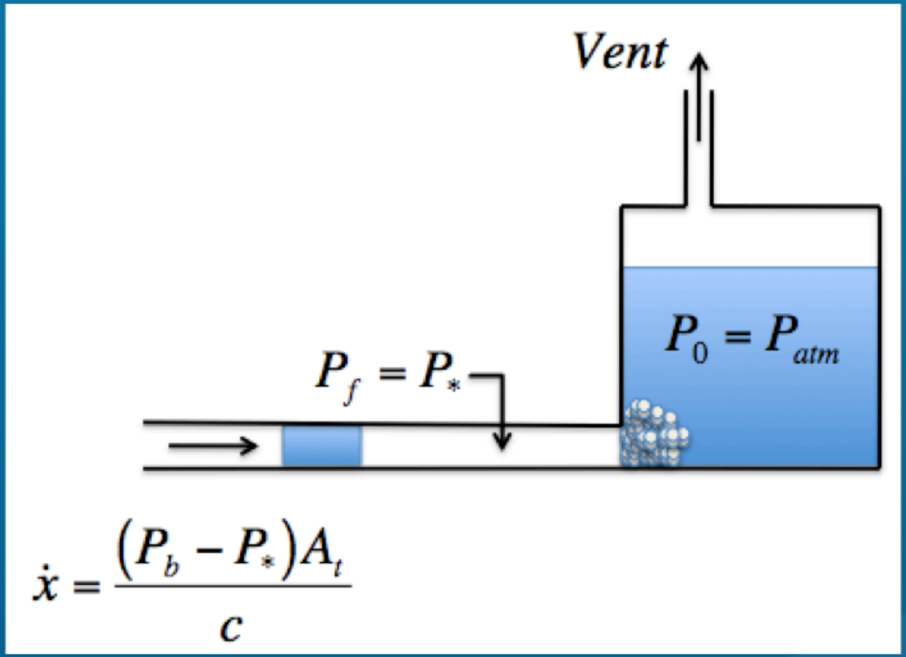
Position



Time

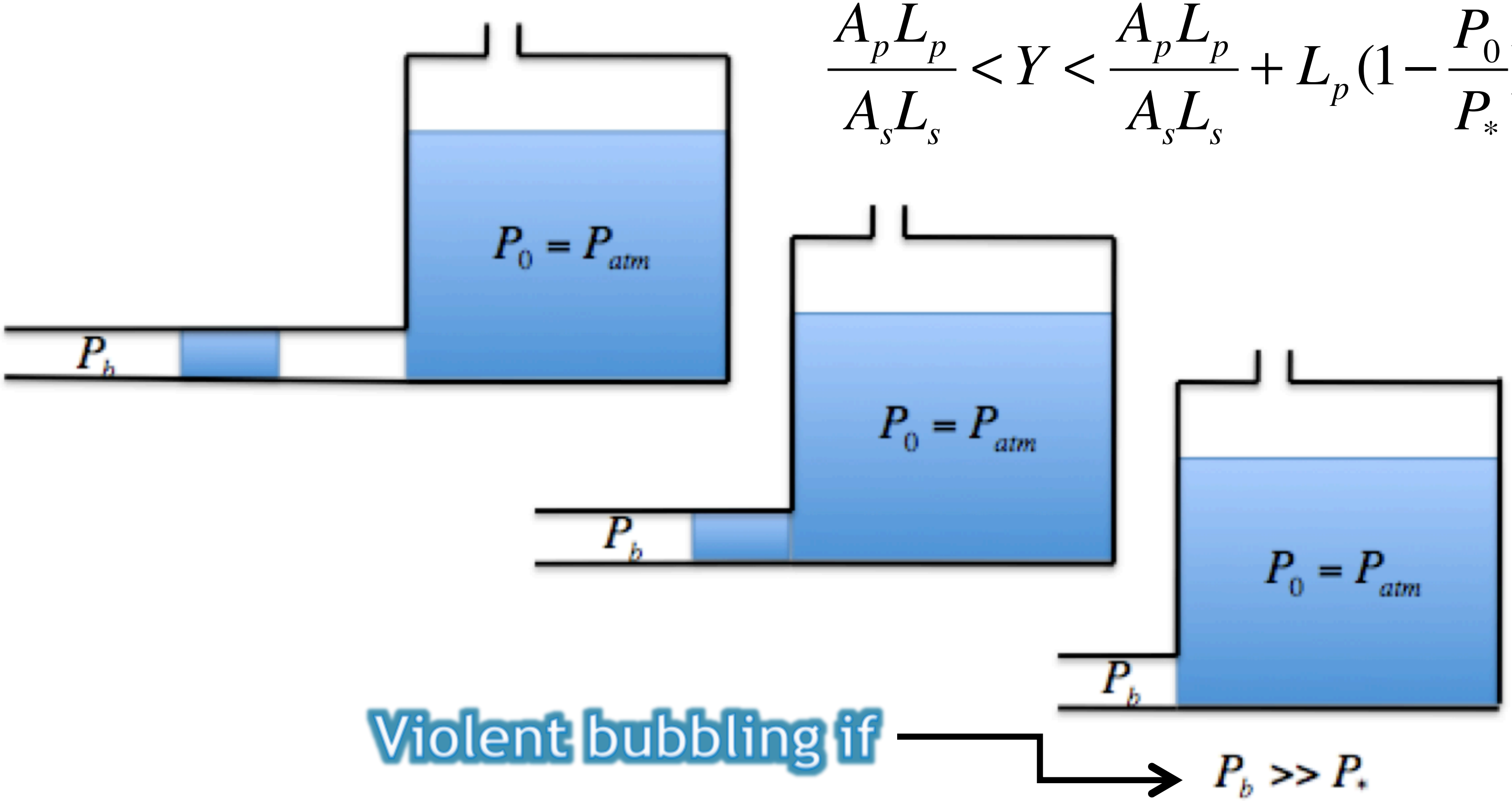
Control Problem:

- To have an idea of how to control the rate of bubbling, we need to consider:
  - Bubbling due to the air in front of the slug,
  - Bubbling due to the air pressure built up in the back.



Back Pressure Problem:

$$\frac{A_p L_p}{A_s L_s} < Y < \frac{A_p L_p}{A_s L_s} + L_p(1 - \frac{P_0}{P_*})$$



Part IV: Recommendations

Recommendations:

- Install air permeable membrane near reaction chamber to allow air to escape (but not liquid).
- Don’t push too hard
- Scheduling of the arrival time of the reagents.

Thank You