Akonni Biosystems: Wicking in Microchannels on Biochips

Report prepared by:
Yousef Daneshbod, University of La Verne
Ali Nadim, Claremont Graduate University

Participants:
Jason Kreykes, Appalachian State University
Jakub Kominiarczuk, University of California, Berkeley

Problem Description (by Dr. Chris Cooney)

A microfluidic biochip device has been designed and proven to withstand temperatures up to approximately 90°C. Although effective for some of our applications, there is a desire to increase this temperature an additional 5 to 7 degrees. The chamber design (schematic is shown in Figure 1) consists of an inlet, a reaction chamber, a waste chamber, a channel connecting the reaction chamber to the waste chamber, and a vent hole.

The reaction chamber volumes are low enough that surface tension dominates. The current microfluidic design does not require hydrophobic stops for the liquid containment, and for manufacturing reasons, it is preferred not to have this feature added. The inlet is sealed prior to the increase in temperature elevation, but the vent remains open. Our goal is to design a connecting channel which prevents the liquid in the reaction chamber from entering the waste chamber during a 5 minute temperature elevation step.

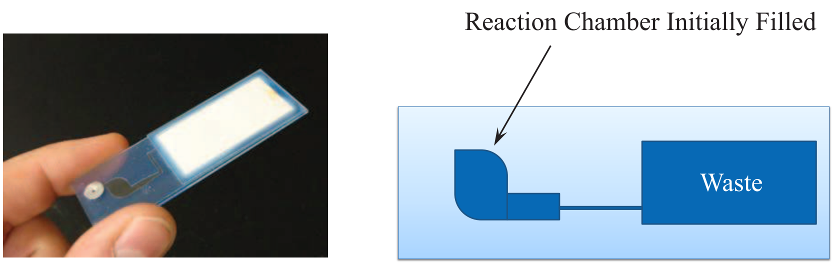

Figure 1: The device along with a schematic diagram describing the chamber design.

Introduction

Microfluidics is the science of designing and manufacturing devices and processes for manipulation of extremely small volumes of fluid, typically micro to nanoliters [11].The most mature application of microfluidics technology is ink-jet printing, which uses orifices less than 100 μm in diameter to generate drops of ink. Today, ink-jet printing is being adapted in biotechnology for delivering reagents to microscopic reactors and depositing DNA into arrays on the surface of biochips [10]. The complex devices now being developed for biological applications involving the analysis of DNA (in genetics and genomics) and proteins (in proteomics) and bio-defense typically involve aqueous solutions and channels 30 to 300 μm in diameter. Unlike microelectronics, in which the current emphasis is on reducing the size of transistors, microfluidics is focusing on making more complex systems of channels with more sophisticated fluid-handling capabilities, rather than reducing the size of the channels [9]. Although micro- and macro-fluidic systems require similar components including pumps, valves, mixers, filters, and separators, the small size of microchannels causes their flow to behave differently. At micron scales, fluid motions are primarily dominated by surface tension and viscous forces.

In the problem under consideration, the issue is one of wicking or leaking of the sample from the reaction reservoir to the waste region at elevated temperatures. A mechanism responsible for this phenomenon was thought to be the "wedge effect," described below, which refers to the tendency of liquids to move along a sharp corner by capillary effects if the conditions are right. The analysis performed during the workshop also mainly focused on this effect.

The Wedge Effect

If two glass plates are brought into contact at one of their edges to form a small wedge and the system is immersed in water, water rises between them as it does in a capillary tube. Also, it rises higher, the nearer the plates approach each other. Hence the upper surface of the water column forms a hyperbolic shape with asymptotes at the bottom and at the closed edge of the plates. The eighteenth century mathematician Brook Taylor was the first to analyze this phenomenon mathematically [12]. The wedge effect has been extensively studied by Finn and Concus [5, 6] and they have observed that the actual behavior of the capillary surface varies quite dramatically depending on the contact angle of the fluid, γ, and the opening angle of the wedge, 2α, see Figure 2. The same phenomenon happens to be the primary mechanism for the transport of water to the upper leaves of tall tress. Computations based on the height of rise of a circular meniscus indicate that the xylem conduits of trees are much too large to raise water to the necessary levels by capillary action alone. Hollow filaments of polygonal cross-section will carry liquids to arbitrarily large heights provided that at least one of the interior angles is such that $\alpha + \gamma < \pi/2$. This appears to be the case of tracheids of conifers, as well as conduits occurring in woody angiosperms [6]. Darcy Thompson in his celebrated work *On Growth and Form*[8] refers to the phenomenon in which one cell-wall always tends to align itself at right angles to another wall as the *Sach's Rule* in botany. Related to this is *Hofmeisters Rule* which states that if an organ grows in different directions, cell division planes are perpendicular to the direction of the fastest growth. The corner flow problem was also studied by Dong and Chatzis [4] who employed 0.3-0.5 mm diameter tubes of square cross-section in the analysis and experiment on imbibition relating to flow in porous media.

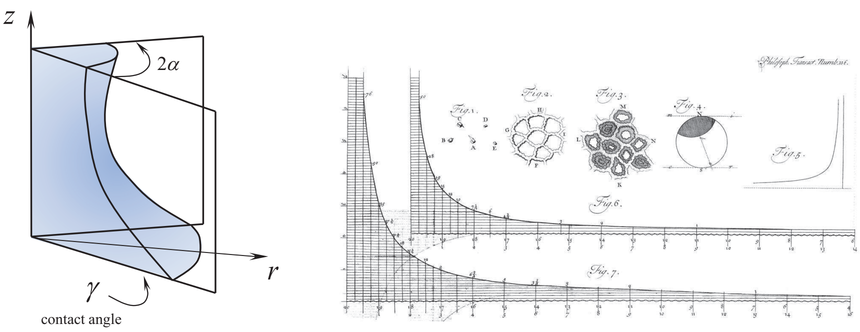

Figure 2: Liquid trapped inside a glass wedge and diagrams from Brook Taylor's capillary experiment.

In the present problem, we believe that the geometry of the interior corners in the channel is the primary mechanism responsible for the flow of liquid. This can be illustrated by the following example given by Weislogel and Lichter [3]. Figure 4 shows a partially liquid filled container with a square cross-section in the presence of gravity. It is commonly observed that the interface is everywhere flat except at the corner regions where the interface curves in order to satisfy the contact angle wetting condition along the perimeter of the interface. The magnified view of the corner region shows that the local radius of curvature of the meniscus decreases as we get closer to the corner. Since the pressure drop across the meniscus is inversely proportional to local radius of curvature, a pressure gradient along the corners will be established. In the absence of gravity, the balance of this gradient with the hydrostatic forces is disrupted and, as a result, a slender column of liquid is pumped up the corners via the capillary forces.

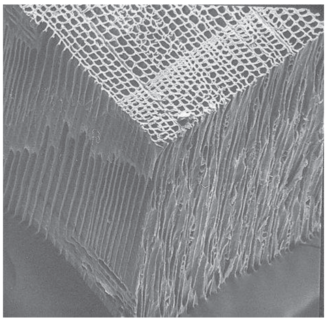

Figure 3: Image of tracheids of conifers.

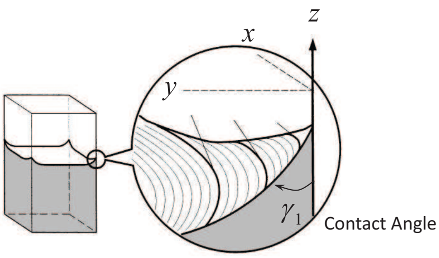

Figure 4: Contact angle at corner (courtesy of Weislogel and Lichter *JFM* 1998).

Interface Statics

The Young-Laplace equation has long been the focus of attention for physicists, fluid mechanicians, and applied mathematicians. Originated by Young and Laplace's pioneering work in 1805 and 1806, this equation determines the free surfaces of a static fluid under gravity. A detailed history on the development of this equation is provided in Finn's classical book on capillary surfaces [7].

Using polar coordinates (r, θ) (where $x = r\cos\theta$, $y = r\sin\theta$) based upon the apex of the wedge geometry being at $r = 0$ and the wetted vertical contact boundaries at $\theta = \pm\alpha$ as shown in Figure 5, the governing non-dimensional equation for the shape of the fluid surface is expressed as [1]

$$\begin{aligned} \nabla \cdot \left(\frac{\nabla h}{\sqrt{1 + (\nabla h)^2}} \right) &= \frac{\partial}{r\partial r} \left(\frac{r h_r}{\sqrt{1 + h_r^2 + h_\theta^2/r^2}} \right) + \frac{\partial}{r^2 \partial \theta} \left(\frac{h_\theta}{\sqrt{1 + h_r^2 + h_\theta^2/r^2}} \right) \\ &= h, \quad -\alpha < \theta < \alpha \end{aligned} \tag{1}$$

along with the boundary conditions

$$\begin{aligned} h_\theta &= 0 \quad \text{at } \theta = 0 \\ h_n &= \frac{1}{r} h_\theta = \left(\sqrt{1 + h_r^2 + h_\theta^2/r^2} \right) \cos\gamma \quad \text{at } \theta = \alpha, \end{aligned} \tag{2}$$

where $h = h^*\sqrt{\kappa}$, $r = r^*\sqrt{\kappa}$, κ is the capillary constant and h^*, r^* are the dimensional variables. Here the gravitational constant has been absorbed within κ so that r measures the distance from the corner in units of capillary length.

We consider similarity solutions of the form [2]

$$h(r, \theta) = \frac{1}{r} f(\theta). \tag{3}$$

Substituting this solution into equation (1), yields the following ordinary differential equation.

$$-\frac{f}{\sqrt{f^2+(f')^2}}+\frac{d}{d\theta}\left(\frac{f'}{\sqrt{f^2+(f')^2}}\right)=f \tag{4}$$

where $f' = df/d\theta$. The solution $f(\theta)$ in the presence of gravity has the form

$$f(\theta)=\frac{\cos\theta-\sqrt{k^2-\sin^2\theta}}{k} \tag{5}$$

where $k = \sin\alpha/\cos\gamma$. The solution $f(\theta)$ in the absence of gravity has the form

$$f(\theta)=\csc(\theta+\alpha+\gamma_1), \tag{6}$$

However, the latter similarity solution only holds when the contact angles on the two walls and the wedge angle satisfy the special relation: $\gamma_1+\gamma_2+2\alpha=\pi$.

Wicking Dynamics

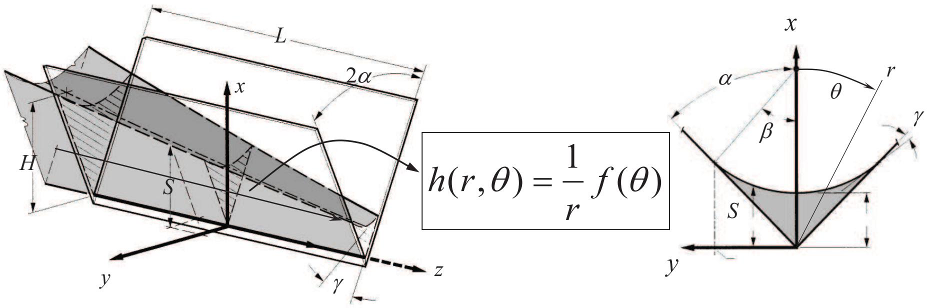

Figure 5: The fluid geometry at an isolated corner (courtesy of Weislogel and Lichter *JFM* 1998).

Aside from the statics of the interface, it is instructive to consider the rate of wicking when an interface begins to advance along a corner. This problem has also been studied in the literature and some of the key results are summarized here. Figure 5 describes the geometry of a fluid column at an isolated corner of angle 2α, with a contact angle of γ. The characteristic height and length of the column are H and L respectively. The following formula [3] can be shown to describe the wicking length $L(t)$ as a function time t

$$L(t)=1.702\sqrt{GHt}. \tag{7}$$

The variable G is defined as

$$G=\frac{\sigma F_i\sin^2\alpha}{\mu f}, \tag{8}$$

where σ is the surface tension, μ the viscosity, $F_i\sim\mathcal{O}(.1)$ the flow resistance and f is a geometric function describing the radius of curvature of the meniscus in the x-y plane given by

$$f=\left(\frac{\sin\gamma}{\sin\alpha}-1\right)^{-1}. \tag{9}$$

Note that as $\gamma\to\pi/2-\alpha$, there will be no wicking. For the case of $\gamma=0$ and $\alpha=\pi/4$ we have

$$L(t)\approx 0.3\sqrt{\sigma Ht/\mu} \tag{10}$$

In our case $L\approx 2$cm and $H\approx 100\mu$m, resulting in $t\approx 0.3$ sec. As such, under conditions when wicking occurs, it may do so rather fast, resulting in the sample escaping toward the waste reservoir.

In conclusion, while a definitive solution to this challenging problem posed in the workshop was not identified, it was felt that using a manufacturing process that can affect the corner angles in the channels may hold the most promise, allowing the wicking mechanism to be controlled without surface treatments that insert hydrophobic stops in the channel. For instance by "rounding" the side walls to increase the corner angles from 90 toward 180 degrees, the leaking of the sample away from the reaction chamber might be delayed.

References

[1] J. Norbury, G. C. Sander and C. F. Scott, *The Laplace-Young Equation Near a Corner*, Q. J. I. Mech. Appl. Math, Vol. 60(1), 1-16, 2004.

[2] J. R. King, J. R. Ockendon and H. Ockendon, *Corner Solutions of the Laplace-Young Equation*, Q. J. I. Mech. Appl. Math, Vol. 52(1), 73-97, 1999.

[3] M. M. Weislogel and S. Lichter, *Capillary Flow in an Interior Corner*, J. Fluid Mech, Vol. 373, 349-378, 1998.

[4] M. Dong and I. Chatzis, *The Imbibition and Flow of a Wetting Liquid along the Corners of a Square Capillary Tube*, J. Coll. Int. Sci, Vol. 172, 278-288, 1995.

[5] R. Finn, *On the Behavior of a Capillary Surface in a Wedge* , PNS, Vol. 63(2), 292-299, 1969.

[6] P. Concus and R. Finn *Capillary Surface Interfaces*, Notices of the AMS, Vol. 46(7), 770-781, 1999.

[7] R. Finn, *Equilibrium Capillary Surfaces*, Springer Verlag., Berlin, 1986.

[8] D. W. Thompson, *On Growth and Form*, Dover Publications Inc., New York, 1992.

[9] T. M. Squires and S. R. Quake *Microfluidics: Fluid Physics on the Nanoliter Scale*, Reviews in Modern Physics, Vol. 77, 977, 2005.

[10] G. M. Whitesides and A. D. Stroock *Flexible Methods for Microfluidics*, Physics Today, Vol. 54, 42, 2001.

[11] H. A. Stone and A. D. Stroock and A. Ajdari, *Engineering Flows in Small Devices: Microfluidics Toward a Lab-on-a-chip*, Annual Review of Fluid Mechanics, Vol. 36, 381, 2004.

[12] B. Taylor, *Concerning the Ascent of Water Between Two Glass Planes* , Annual Review of Fluid Mechanics, Vol. 538, 27, 1710.