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AEA Technology - Compact Heat Exchangers

Surface tension and interface shear effects in plate-fin passages

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Introduction

AEA Technology reported an interest in quantifying more completely the heat flow characteristics of a vertical liquid condensate flow in a plate-fin condenser, particularly under conditions of low flow rate. An illustration of the geometry of a typical plate-fin heat exchanger is given as Figure 1. This shows the existance of a complex geometry for each of the flow channels as generated by the corrugated fins. Experimental measurements of heat transfer coefficients made under conditions of low flow rates showed enhanced values relative to the more predictable coefficients measured at higher flow rates. Typical experimental results were presented at the meeting and are reproduced as Figure 2. A postulated mechanism active in the two-phase drainage flow is that of surface tension effects which would inherently be expected to contribute more significantly at lower flow rates. This report describes progress made on a theory to incorporate geometrical considerations into the plate-fin configuration and to investigate the effects of surface tension and interface shear effects generated by the core flow on the heat transfer coefficients.

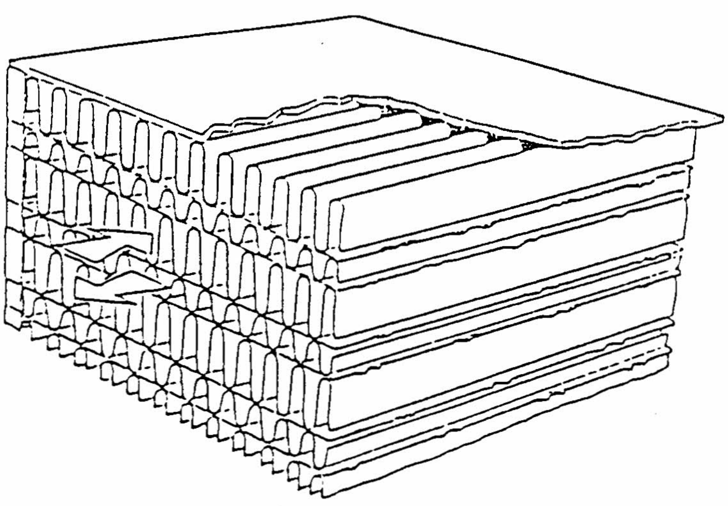

Figure 1. Plate-fin heat exchanger cross-section

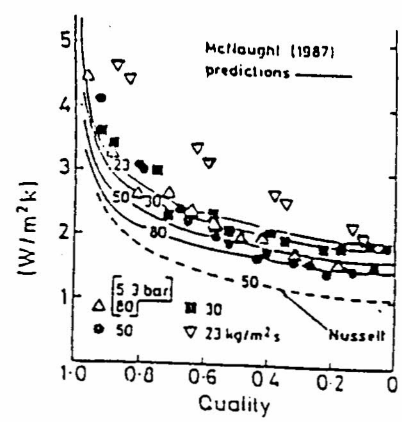

Figure 2. Experimental measurements of heat transfer coefficients

Background Theory

The underlying approach to the theoretical analysis cited by AEA Technology (Clarke(1990)) is that of the Nusselt theory and is described in full within numerous texts including Bennett and Myers (1962) and Collier(1981). In a simplest attempt to analyse the film-wise condensation of a liquid vapour onto a vertical plate the physical situation is modelled as depicted in Figure 3.

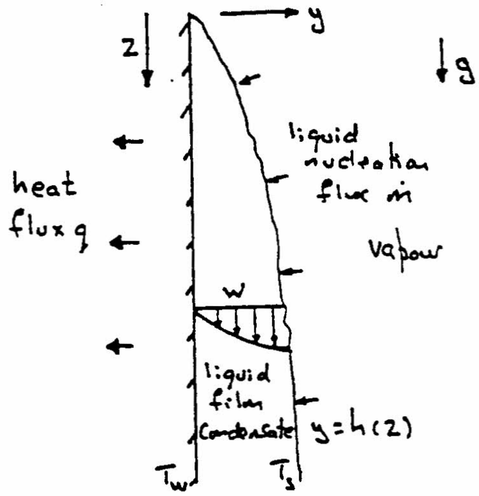

Figure 3. Flow of a laminar condensate film over a vertical plate

Under conditions where the plate is cooled sufficiently below the saturation temperature of the vapour then nucleation of the liquid droplets in the vapour occurs on the vertical wall. A simple mathematical model can be constructed on the assumptions of a one-dimensional, steady, laminar film in which heat is transferred solely by heat conduction. A coordinate system is taken with the z-ordinate vertically down the plate and a y-ordinate perpendicular to the plate. Thin film theory applied in the absence of any significant shear from the vapour phase gives the film velocity profile as

$$w = \frac{(\rho_l - \rho_v)g(2h - y)y}{2\mu_l} \qquad (1)$$

where ρ_l and ρ_v are the liquid and vapour densities, μ_l the viscosity of the liquid, g the acceleration due to gravity and h=h(z) the film thickness. Integrating across the film gives the mass flow rate per unit width as

$$Q = \rho_l \overline{w}_l h = \rho_l \int_0^h w\,dy = \frac{\rho_l(\rho_l - \rho_v)gh^3}{3\mu_l} \qquad (2)$$

The thickening of the fluid film is due to condensate according to the mass balance relationship

$$\frac{dQ}{dz} = \dot{m} \qquad (3)$$

where $\dot{m}$ is the mass flux of liquid condensate per unit area and condensation is controlled by the local thermal conditions.

Assuming heat transfer in the condensate film is by conduction only leads directly to the local temperature profile in the liquid as

$$T_l = T_w + (T_s - T_w)\, y/h \qquad (4)$$

where T_s is the saturation temperature of the vapour and T_w the wall temperature. The heat flux per unit width q into the plate is given by Fick's law as

$$\frac{dq}{dz} = k\frac{(T_s - T_w)}{h} \qquad (5)$$

where k is the thermal conductivity of the wall material. A heat balance is maintained in a steady state between the heat released by condensation at the film interface and the conduction of heat through the wall according to

$$\frac{dq}{dz} = \lambda \dot{m}, \qquad (6)$$

where λ is the latent heat per unit mass of the vapour.

The growth of the liquid film can be determined from the differential relationship

$$gh^3\frac{dh}{dz} = \frac{\mu_l k(T_s - T_w)}{\rho_l(\rho_l - \rho_v)}, \qquad (7)$$

obtained by eliminating $\dot{m}$ between equations (5) and (6) and using the expression (2). Integrating directly then gives the film thickness as

$$h = \left[\frac{4k\mu_l(T_s - T_w)z}{\lambda\rho_l(\rho_l - \rho_v)g}\right]^{\frac{1}{4}} \qquad (8)$$

on taking $z=0$ as the start of the film.

Of practical importance is the prediction of a heat transfer coefficient. Within a length L of the plate, the total heat flux through the wall per unit width is given by

$$\bar{q}L = \int_0^L dq = \int_0^L \lambda\frac{dQ}{dz}dz, \qquad (9)$$

where $\bar{q}$ is the average heat flux per unit area. An averaged heat transfer coefficient $\bar{\alpha}$ can be usefully defined as

$$\bar{q} = \bar{\alpha}(T_s - T_w), \qquad (10)$$

to relate the heat flux through a length L of the plate to the temperature difference across the film. Evaluating $\bar{q}$ from (9) and substituting into (10) gives the heat transfer coefficient as

$$\bar{\alpha} = \frac{k}{3}\left[\frac{\rho_l(\rho_l - \rho_v)g}{3Q_L\mu_l}\right]^{\frac{1}{3}}, \qquad (11)$$

where $Q_L = Q_l(z = L)$ and provides the basis for the Nusselt predictions displayed in Figure 2. Real fluid effects are considered by McNaught(1987) with the result of adjusting some of the flow parameters and provide a better agreement between the theory and experiment, particularly at the higher flow rates.

Extended Mathematical Model

The Nusselt theory can be usefully extended by the inclusion of surface tension forces acting on the film surface, the interface shear effect of the core vapour flow and the varying wall geometry in the spanwise direction. The channel geometry is more realistically generalised as cylindrical but of arbitrary cross-sectional shape. A three-dimensional configuration is used as defined in Figure 4 involving a curvilinear coordinate system (n,s,z) with z measured vertically down the channel wall, s a local orthogonal coordinate measured along the wall and n a mutually orthogonal coordinate measuring distance from the wall and in the plane of a cross-section.

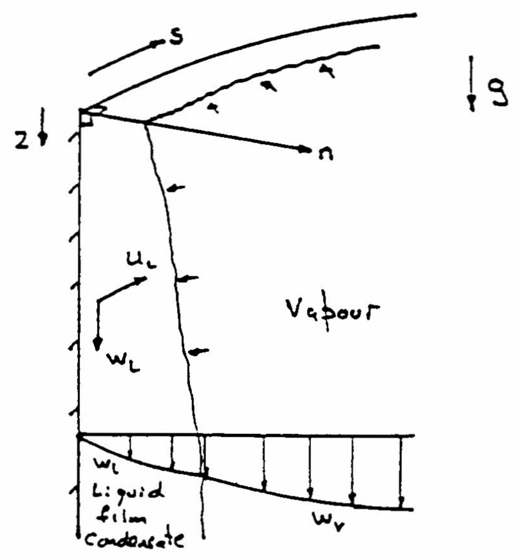

Figure 4. Flow over a curved vertical plate

Retained are the existing assumptions of steady, laminar flow of a thin liquid film and negligible heat convection within the film. Typically for low flow rates the radius of curvature of the condenser wall is large compared to the film thickness. Consequently, the flow quantities are expected to posses a slow variation with respect to the spanwise coordinate s and this can be used to simplify the three-dimensional formulation.

Under the above assumptions the governing Navier -Stokes equations for the liquid phase reduce in a leading approximation to the approximate equations

$$0=-\frac{\partial p_l}{\partial z}+\rho_l g+\mu_l\frac{\partial^2 w_l}{\partial n^2}\ , \tag{12}$$

$$0=-\frac{\partial p_l}{\partial s}+\mu_l\frac{\partial^2 u_l}{\partial n^2}\ , \tag{13}$$

and

$$0=-\frac{\partial p_l}{\partial n}\ . \tag{14}$$

In the above subscript l denotes properties associated with the liquid condensate with p_l as the pressure in the condensate.

The continuity equation can be included in the form

$$\frac{\partial(\bar{u}h)}{\partial s}+\frac{\partial(\bar{w}h)}{\partial z}=\frac{\dot{m}}{\rho_l}. \tag{15}$$

where $\bar{u}$ and $\bar{w}$ are liquid condensate velocities averaged over the film thickness and $\dot{m}=\dot{m}(z,s)$ is the mass flux of condensate released onto the film surface.

A no-slip boundary condition at the channel walls requires

$$u_l=w_l=0 \quad \text{on} \quad n=0\ . \tag{16}$$

At the liquid - vapour interface, matching of the stresses yields

$$\mu_l\frac{\partial w_l}{\partial n}=\tau \quad \text{and} \quad \mu_l\frac{\partial u_l}{\partial n}=0 \quad \text{on} \quad n=h(z,s)\ , \tag{17}$$

on taking no cross-flow in the vapour phase and denoting the shear exerted at the interface by τ which is usefully assumed locally constant. A pressure difference across the interface is supported by the action of surface tension as specified by

$$p_l(z,s)=p_l'+\rho_v gz+\sigma K(z,s;h) \quad \text{on} \quad n=h(z,s) \tag{18}$$

where $K(z,s;h)$ is the surface curvature of the interface and σ the coefficient of surface tension. The pressure in the vapour phase has been taken as hydrostatic with an initial pressure specified as p_l'

The velocity components in the condensate can be readily solved as

$$w_l = \frac{(\rho_l - \rho_v)g'(2h-n)n + n\tau}{2\mu_l} \quad , \tag{19}$$

and $$u_l = \frac{(\rho_l - \rho_v)g''(2h-n)n}{2\mu_l} \quad , \tag{20}$$

on writing $$g' = g - \frac{\sigma}{(\rho_l - \rho_v)}\frac{\partial \mathrm{K}}{\partial z} \tag{21}$$

and $$g'' = -\frac{\sigma}{(\rho_l - \rho_v)}\frac{\partial \mathrm{K}}{\partial s} \; . \tag{22}$$

The expression (19) can be compared with with the Nusselt theory (1) whilst expression (20) gives the cross flow velocity driven by the surface tension effects. Performing the averaging of the velocities (19) and (20) over the film thickness and substituting into the continuity equation (15) gives the governing equation for the film interface from the first-order partial differential equation

$$g'h^2\frac{\partial h}{\partial z} + \frac{\tau\mu_l}{(\rho_l - \rho_v)}h\frac{\partial h}{\partial z} + g''h^2\frac{\partial h}{\partial s} = \frac{\dot{m}\mu_l}{\rho_l(\rho_l - \rho_v)} \; . \tag{23}$$

Discussion

The result (23) generalises the Nusselt theory; the Nusselt result essentially balances the first (gravity) term directly with the right-hand side condensate term. The present analysis includes the two further contributing effects of interface shear and surface tension and consideration is now given to their relative importance.

Interface curvature in the downflow direction is likely to be small and physically it is expected that

$$\left| \frac{\sigma}{(\rho_l - \rho_v)}\frac{\partial K}{\partial z} \right| << g \tag{24}$$

and thus in (21) $g' \approx g$ to a good approximation. The principal effect of surface tension is through span-wise curvature of the liquid condensate interface generated by the cross-sectional curvature of the channel. This effect is quantified by the third term in equation (23) and is comparable to the gravity flow term where

$$g''h^2\frac{\partial h}{\partial s} = O(g'h^2\frac{\partial h}{\partial z}) \quad ,$$

i.e. $$\sigma\frac{\partial K}{\partial s}\frac{\partial h}{\partial s} = O\left(g'(\rho_l - \rho_v)\frac{\partial h}{\partial z}\right), \tag{25}$$

which is expected appropriate near sharp corners of any channels cross-section. The principal effect of the action of surface tension is to locally increase the film thickness in regions of high curvature. Locally, the relative heat flux removed through conduction ($\propto 1/h$) will be reduced whilst the flux of condensate falling vericALLY will be increased ($\propto h^3$). A detailed global analysis would be required to asses the effect of a specific geometry and would provide a basis for a separate project.

The effect of the interface shear can also be assessed. Neglecting surface tension effects in this instance, equation (23) can be rewritten in the form

$$gh^2\frac{\partial h}{\partial z} + \frac{\tau\mu_l}{(\rho_l - \rho_v)}h\frac{\partial h}{\partial z} = \frac{\dot{m}\mu_l}{\rho_l(\rho_l - \rho_v)} = \frac{C}{h}, \tag{26}$$

on taking the Nusselt theory for condensate, where C is an appropriate local constant. Integrating equation (26) gives

$$\frac{gh^4}{4} + \frac{\tau\mu_l}{(\rho_l - \rho_v)}h^3 = Cz \tag{27}$$

on taking the film initial postion to correspond to $z=0$.

It should be noted that the two terms on the left-hand side of equation (27) have a relative magnitude that involves the film thickness h. Near to entry to the channel the liquid film will be very thin and if high interface shear exists such that locally $\tau >> (\rho_l - \rho_v)gh/\mu_l$ then the gravity term will not be as influential as the shear term. The effect of the shear is then to restrict the film growth as $h \propto z^{\frac{1}{3}}$ compared with $h \propto z^{\frac{1}{4}}$ for a gravity dominated flow. Schematically the situation is as shown in Figure 5 where the flow behaviour can be identified into three regions.

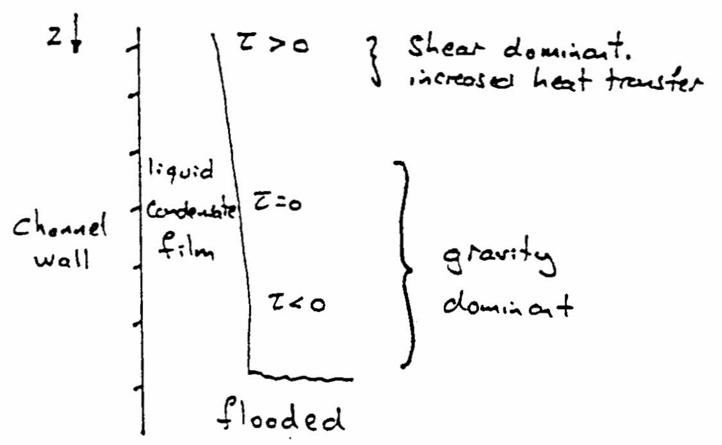

Figure 5. Schematic development of a film on a channel cross-section

It is reported that for plate-fin heat exchangers the inlet vapour velocity is high whilst the interfacial velocity is small relative to the vapour velocity. Thus on entry, and possibly extending for some distance downstream, the interfacial shear dominates over the gravity forces, resulting in an enhanced heat transfer from the vapour to the wall by conduction. As the film thickens and its interface velocity shear decreases, the shear effects are reduced and the film thickens more rapidly. The interfacial shear reduces to zero and towards the bottom of the drainage channel will reverse as the condensate flow velocity exceeds the core vapour flow velocity. This analysis suggests that at low flow rates, the region of flow influenced by the interfacial shear is extended, which in turn would provide the mechanism for an enhanced heat transfer coefficient.

Acknowledgements

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Professor BJ Azzopardi is currently investigating the effects of cross-sectional shape on heat exchangers as part of a research programme in the Dept. of Chemical Engineering, Nottingham.

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