Bubble Bursting in Molten Glass

Pilkingtons

Bubbles generated in glass making should, hopefully, all burst through the free surface whilst the glass is still molten. Pilkingtons find that certain bubbles, approximately one in a thousand, can approach the surface and sit underneath it for up to an hour. They would like to understand the mechanism behind this process. In this short report we suggest that the presence of surface tension gradients provide a plausible explanation for this phenomenon.

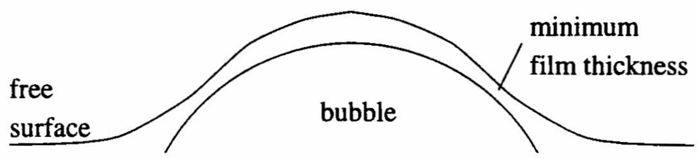

Figure 1: Possible effect of surface tension gradients

When a bubble approaches a free surface a thin liquid film forms above the bubble as shown in Figure 1. This film drains under the effects of gravity, surface tension and possibly surface tension gradients before the bubble eventually breaks through. Figure 2 shows how gravity and surface tension effects may compete to prolong this draining process.

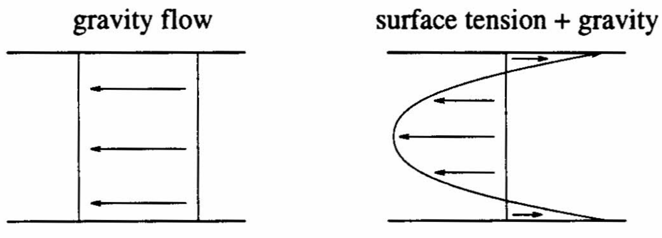

Figure 2: Geometry of the problem

Typically, as the film drains, the thickness reaches a minimum in a ring away from the bubble centre line, as shown in Figure 1. This variation in thickness provides a possible mechanism for surface tension variation since SO_2, which is known to affect surface tension, diffuses from the bubble to the surface. The non-uniform film thickness means that the SO_2 concentration will also be non-uniform at the surface giving rise to surface tension gradients.

Assuming, for simplicity, that the surface of the bubble can be modelled as a rigid boundary and applying lubrication theory to the thin film of molten glass above the bubble leads to the following equation for the leading order film thickness $h = h(r,t)$

$$h_t + \frac{1}{r}\frac{\partial}{\partial r}\left\{ r\left[\frac{h^3}{3\mu}\left(C(\sigma\nabla^2 h)_r - Bh_r\right) - \frac{M}{2\mu}h^2\frac{\partial s}{\partial r}\right]\right\} = 0 \ , \tag{1}$$

where $s = s(r,t)$ denotes the leading order concentration of SO_2. The non-dimensional Capillary number C, Bond number B and Marangoni number M are given by

$$C = \frac{\sigma_0\delta^3}{\mu_0 U}, \qquad B = \frac{\rho g R^2\delta^3}{\mu_0 U}, \qquad M = \frac{\Delta\sigma\delta}{\mu_0 U}$$

where μ_0, ρ, σ_0 and $\Delta\sigma$ denote characteristic values of viscosity, density, surface tension and surface tension variation respectively, R is the bubble radius and g the acceleration due to gravity. The parameter $\delta \simeq 10^{-2}$ denotes the aspect ratio of the film while U denotes the as yet unspecified velocity scale. Setting $C = 1, B = 1$ and $M = 1$ in turn yields the appropriate surface tension, gravity and surface tension gradient velocity scales for the flow; the corresponding timescales are given by

$$\begin{aligned} T_C &= \frac{h_0}{U_C} = \frac{\mu_0 h_0}{\sigma_0\delta^3} \sim 10^4 s \\ T_B &= \frac{h_0}{U_B} = \frac{\mu_0 h_0}{\rho g R^2\delta^3} \sim 10^6 s \\ T_M &= \frac{h_0}{U_M} = \frac{\mu_0 h_0}{\delta\Delta\sigma} > 10^3 s \ . \end{aligned} \tag{2}$$

The value for T_M is a lower bound since there was no accurate estimate for $\Delta\sigma$. If the fluid were to drain purely by gravity, *i.e.* with no surface effects, then the gravity time scale would be $O(\delta^2)$ less then the present value making it $\sim 10^2$s. The true draining time scale must lie somewhere between the pure gravity case and the lowest value obtained from (2).

A more accurate model of the thin film would treat the surface of the bubble as a free (rather than rigid) boundary. This makes the analysis somewhat more complicated since now in order to close the leading order problem we must consider the first order approximation. For a thin, two-dimensional, symmetric sheet this leads to the following equations for the leading order film thickness $h = h(x,t)$ and velocity $u = u(x,t)$;

$$h_t + (uh)_x = 0 \tag{3}$$

$$4(\mu h u_x)_x + Ch(\sigma h_{xx})_x + Bh\sin\alpha - Ms_x = 0 \ , \tag{4}$$

where $s = s(x,t)$ again denotes the leading order distribution of SO_2. These equations are valid in a segment of bubble in which α, the angle of the segment with respect to horizontal, does not vary significantly. Since they are higher order corrections to (1) they do not significantly affect the time scales.

This work is based on surface tension forces moving fluid in the opposite direction to gravity forces and thus retarding the draining. A simple check on the time scales shows that this can make the bursting time last from between a minute (with pure gravity) up to a few hours (with surface tension and surface tension gradients). Clearly then this provides a plausible mechanism for long lasting bubbles. It is possible that other factors not considered here may also act to increase bursting times. For example a locally high viscosity, due to an inhomogeneity in the glass would have a similar effect.

T.G. Myers, S.K. Wilson, J. Dewynne, A.A. Lacey