Sunroof Boom

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June 10, 1999

1 Further background to the problem

We summarize here some further information about sunroof boom that was supplied by Jaguar Research at the Study Group, some of it from a Jaguar Technical Report [1].

For some Jaguar Saloons (the Series 3, XJ40 and XJ82 models) the frequency of the boom is around 25 Hz (noticeably lower than the frequency mentioned in the problem description paragraph) and it occurs at speeds from 20 mph to 70 mph when the sunroof is fully open, and is most serious between 25 mph and 65 mph. The greatest internal noise level, reached at 40 mph, is 120 dB, comparable to the sound level near an aeroplane engine. This is approximately 15 dB louder than the figure for competitor cars at the same speed. Experimental spectrum measurements show that the excess noise is almost entirely a pure tone at 25 Hz, with no noticeable overtones or sidebands. The term "sunroof boom" will be used here to refer specifically to this very intense resonance.

Flow visualization has been carried out in a wind tunnel, but did not reveal anything conclusive — visualization with smoke just produced a grey blur, with no visible vortices or other structure, and tuft visualization also showed nothing of significance. There is no measurable mean flow in or out through the aperture.

The sunroof panels in these cars open by sliding backwards into a space inside the car. The open sunroof aperture is then essentially a rectangular hole with rounded corners, extending across most of the width of the car. It will be convenient to introduce some notation: we choose (x, y, z) axes fixed in the car, with x in the direction of the air flow, y across the width of the car, and z vertical, and we let the aperture size be a by b (in the x, y directions).

Jaguar have experimented with varying the extent of the opening a, and with the use of flow deflectors (also called "flips") mounted across the roof of the car upstream of the front lip of the aperture (*ie* on section AB of Figure 3). Both of these have significant effects reported in [1] which we outline here. When *no* deflector is used, it is found that at speeds from 20 mph to 40 mph boom sets in at small roof openings, disappears as the roof is opened further and reappears at higher apertures. If we let $a_1(U)$ and $a_2(U)$ be the openings at which the first and second onset of boom occur, then the general behaviour is as shown in Figure 1. As U rises, the opening a_1 giving the first onset of boom increases linearly with U for U up to about 40 mph. However, a_2 varies much less, and eventually decreases and joins with a_1 for $U \approx 40$ mph. (The opening at which the first onset of boom disappears is not recorded.) When a deflector is used, the lower booming region is not observed: there is simply

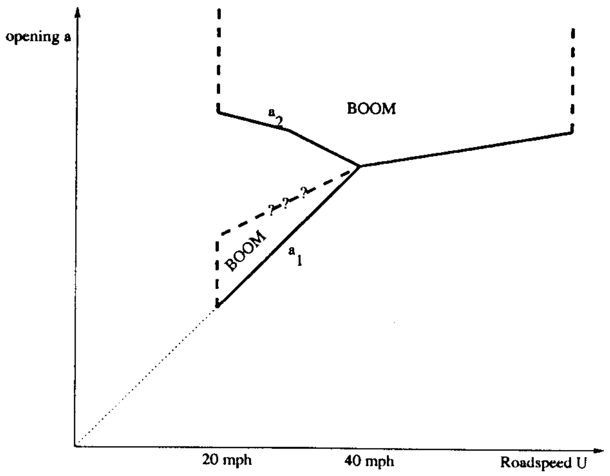

Figure 1: Typical behaviour of a_1 and a_2, the openings for first and second onset of boom as functions of road speed U.

a single threshold $a_d(U)$ above which boom occurs, and a_d is generally somewhat greater than a_2 (roughly as shown in Figure2) and can be raised further by using various patterns of cutouts in the deflector.

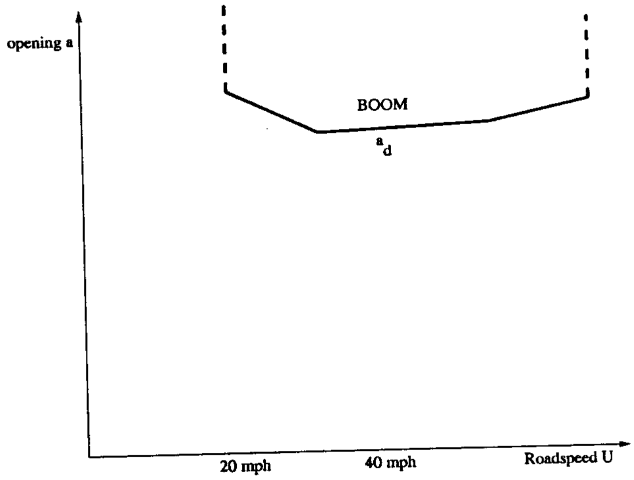

Figure 2: Typical behaviour of a_d, the opening at which boom occurs when a deflector is used.

The main method of avoiding boom in practice is to use a deflector, and to use an opening that is below the minimum of $a_d(U)$, rather than the full opening available.

1.1 Assorted additional information

Boom occurrence is independent of whether the climate-control system is on or off, and of whether or not the rear vents are sealed up, and there is no known structural resonance at 25 Hz, and no known source of 25 Hz oscillations in the incident flow.

Some non-competitor cars reported as also suffering from sunroof boom were the Vauxhall Omega, Range Rover Classic and Ford Escort.

When boom occurs, its frequency varies very little with a.

2 The Helmholtz resonator

It is natural to surmise from the information presented that there is an acoustic mode of frequency 25 Hz, which becomes linearly unstable for a certain range of speeds in the Jaguar Saloons mentioned. The most obvious candidate is the Helmholtz resonator mode (like the note obtained by blowing across the top of a bottle) and calculation of the Helmholtz resonator frequency for the car produces a value close to 25 Hz as we shall see. We outline the theory of the Helmholtz resonator here, but the classical reference is Rayleigh [2], Chapter XVI, and a more recent but less detailed exposition is in Lighthill [3], Chapter 2.5.

The mode of oscillation is one in which air flows alternately in and out of a closed cavity through a narrow aperture, with associated pressure fluctuations in the cavity: in our case the cavity is the interior of the car, and the narrow aperture is the open sunroof. Although the flow of air past the car is crucial to the *instability* of the mode, it can be ignored for the purpose of analysing the mode itself and calculating its frequency. (The Mach number is about 0.05 at 40 mph). We describe both a lumped-parameter and a numerical method of calculating the frequency, and comment on the natural damping of this mode in the absence of flow.

2.1 Lumped parameter model

At a frequency of 25 Hz, the acoustic wavelenth in air is 13 m, several times the maximum linear dimensions of the interior of the car (say 3 m long). This is typical of a Helmholtz resonator, and allows us to approximate the mode of oscillation by regarding the air in the cavity as being at a *uniform* pressure $p(t)$ relative to atmospheric. If the volumetric flow rate out of the cavity through the aperture is $Q(t)$, then the fact that the cavity is closed, and that the air in it expands and contracts adiabatically, is represented by

$$dp/dt = -\gamma p_0 Q/V \tag{1}$$

where V is the volume of the cavity, γ the ratio of specific heats of air, p_0 atmospheric pressure. Assuming (because of the narrowness of the aperture) that the air velocities in the bulk of the cavity are negligible compared with those in the immediate vicinity of the aperture, Newton's law for the "plug" of air at the aperture is

$$p = \rho_0 I \, dQ/dt \tag{2}$$

where I is the "inertance" or "inductance" of the aperture, and ρ_0 the undisturbed air density. If the aperture were in the form of a tube of length L and cross-sectional area A, then $I \approx L/A$. A more accurate value for I

would add end-corrections to L, proportional to the diameter of the tube. But in our case, with no tube, the end-corrections are everything. Rayleigh shows that the inertance of a circular aperture of area A in an infinite flat plate is $I = \sqrt{\pi}/(2\sqrt{A})$, and that this is a good approximation to the inertance of an elliptical aperture. (Lighthill quotes a similar figure of $0.8\sqrt{A}$ as the end-correction to L.)

This pair of equations give us a simple harmonic oscillator, with frequency ω_0 given by

$$\omega_0^2 = c_0^2/(IV) \tag{3}$$

where $c_0 = \sqrt{\gamma p_0/\rho_0}$ is the sound speed.

The most difficult quantity to estimate here is the inertance: the exterior flow is fully 3-dimensional, while the interior is much more 2-dimensional because the aperture extends over almost the full width of the car. It will therefore be best to estimate the inertance as the sum of a 3-dimensional inertance for the exterior of the aperture and a 2-dimensional inertance for the interior. The exterior value will be approximated by one half of Rayleigh's approximation (since that includes 3-dimensional flow on *both* sides of an aperture). Analysis of the 2-dimensional Helmholtz resonator (by Michael Ward) shows that the inertance due to the 2-dimensional interior flow will be approximately $\log(D/a)/(\pi b)$ where D takes a certain value scaling with the diameter of the cavity in the (x, z)-plane, and which could be calculated numerically if required. This gives

$$I \approx I_{\text{ext}} + I_{\text{int}} \approx \frac{\sqrt{\pi}}{4\sqrt{ab}} + \frac{\log(D/a)}{\pi b}. \tag{4}$$

If we treat the car as a box 3 m×2 m×1 m (in the x, y, z directions) with a=0.4 m, b=2 m, and estimate say D=1.5 m, then the resonant frequency calculated from the above is $f_0 = \omega_0/(2\pi)$=25.5 Hz. (! The closeness is fortuitous of course, but it arose without any retrospective reguessing of the data values.)

From this description we can see that the Helmholtz resonator mode has some similarities to the lowest mode of an organ pipe: in each case a mass of air oscillates on a spring whose stiffness is due to the adiabatic compressibility of air. There are also some differences between the cases: the organ pipe mode is essentially one-dimensional, and the whole of the air participates in the motion, whereas in the Helmholtz resonator the kinetic energy is almost entirely confined to the air near the aperture. This is why the organ pipe frequency depends on the pipe *length*, while a Helmholtz resonator of comparable size has a lower frequency depending on the cavity *volume* and aperture inertance.

2.2 Numerical calculation

A 2-dimensional *Fastflo* calculation was carried out, with some representative data for the car shape, sunroof aperture and interior geometry. The geometry of the windscreens and roof was taken as in Figure 3, where representative values supplied by Jaguar are

- l_1 =0.35 to 0.50 m (this is smaller for Jaguar Saloons than for competitor cars)
- $\alpha_1 = 25°$ to 35°
- l_2 =0.25 m
- $l_3 + l_4$ =1.15 m
- $l_3 = a$ takes values up to 0.44 m
- $\alpha_2 = 25°$ to 40°

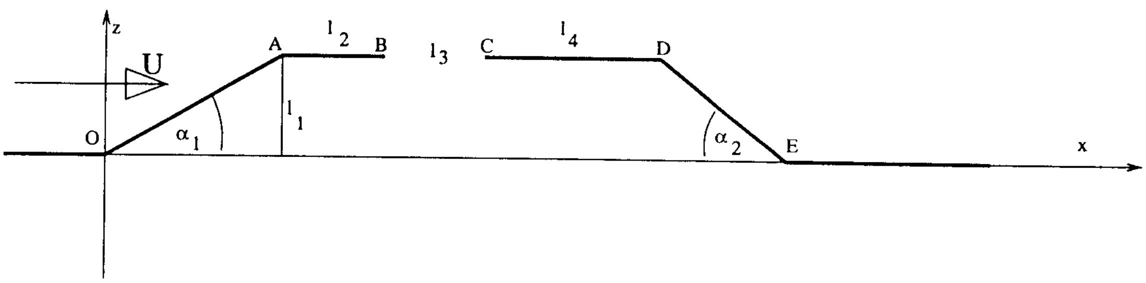

Figure 3: Representative geometry of windscreens and roof

The geometry of the interior (essential to give a realistic area of the 2D cavity) was chosen as in Figures 4 and 5, but is not based on actual measurements.

To find the modal frequency ω, the acoustic pressure is represented as $p_a(\mathbf{r}, t) = p(\mathbf{r})\sin(\omega t)$, and then the wave equation for p_a reduces to Helmholtz' equation for p, $\nabla^2 p + k^2 p = 0$, where $k = \omega/c_0$ is the wavenumber.

On the rigid boundaries $\partial p/\partial n = 0$, and on the (artificial) boundary introduced to reduce the problem to a bounded domain $p = 0$. The problem has an infinite spectrum of eigenvalues k^2, of which we are interested in the *lowest*, corresponding to the case in which $p(\mathbf{r})$ has constant sign in the region. The calculation gives a frequency of 30 Hz, and Figures 4 and 5 show the pressure contours and velocity vectors at an arbitrary instant of the cycle.

FASTFLO

Plot of pressure contours

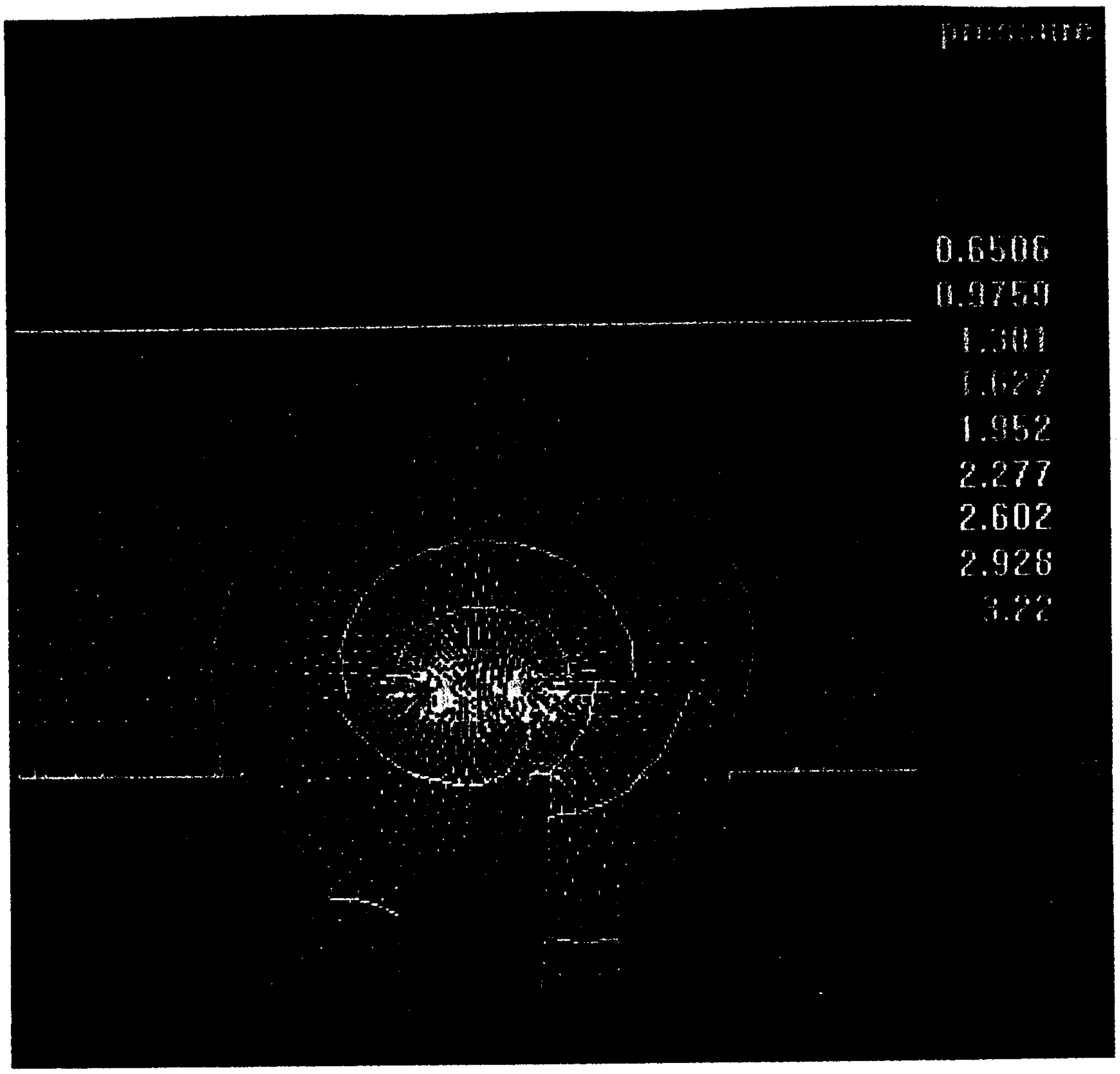

Figure 4: Presure contours for acoustic mode

The proportion of the oscillation outside the body of the car is quite large, and is not well represented by the 2D approximation. Consequently the frequency calculated is likely to be too high. The velocities are seen to be nearly orthogonal to the mean flow past the car, and the maximum velocities are near the lips B and C of the aperture, as is to be expected from the fact that the exact solution would have a singularity there since the lips are assumed sharp.

2.3 Damping of this mode

The mode in reality is not conservative. One reason for this is that the walls of the cavity are not perfectly rigid and so will invalidate (1) and absorb some of the energy. We do not know how much this structural damping differs between Jaguar Saloons and the other cars.

A second reason is that the sound energy will be radiated away from the cavity, and this is an effect that we

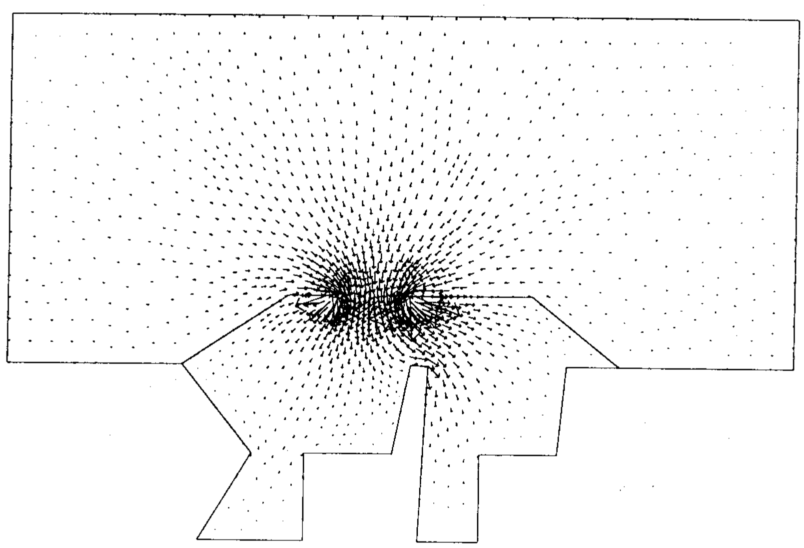

Figure 5: Velocity vectors for acoustic mode

can quantify. Rayleigh shows that (2) will be modified to

$$p = \rho_0 I\, dQ/dt + \rho_0 \omega_0^2 Q/(2\pi c_0). \tag{5}$$

If we then write the second order equation for Q in standard form as $\ddot{Q} + 2\zeta\omega_0\dot{Q} + \omega_0^2 Q = 0$ then the damping factor is $\zeta = 1/(4\pi\sqrt{I^3 V})$. For the numerical values mentioned in section 2.1 this is $\zeta \approx 0.055$, corresponding to a "quality factor" $1/(2\zeta) \approx 9$.

One way to measure the combined structural and radiation damping would be to generate sound at 25 Hz in the car (when stationary) with a loudspeaker, and then measure the decay rate when the source is switched off.

3 Flow-induced instability of the Helmholtz resonator mode

We now move on to address the main effect involved in sunroof boom, the fact that the Helmholtz resonator mode becomes unstable in certain flow regimes, and its amplitude builds up to a point determined by nonlinear effects. We wish to understand why this build-up occurs in certain cars and not others. We do not know where the flow over the roof separates and reattaches for the cars in question. This is likely be an important piece of information, particularly since the Jaguar's low aspect ratio may give it a smaller region of separated flow than other cars.

The Jaguar technical report [1] mentions two possible instability mechanisms:

- *vortex interaction*: the vortices shed from the front lip (B in Figure 3) will pass the rear lip (C) at a certain point of the resonator cycle, and if this is correctly phased then instability can occur;
- *flow deflection*: vertical oscillation of the air in the aperture deflects the horizontal mean flow causing a modulated air flow into the cavity at the rear lip, which can cause instability.

We describe these in turn.

3.1 Vortex-excited acoustic resonance

The flow of air past the cavity with an open sunroof is typical of those where vortex-excited acoustic resonance occurs. This is associated with the periodic shedding of vortices from the upstream lip of the aperture. The resonance excited would in this case be the lowest acoustic mode of the cavity.

A musical analogy may help to distinguish vortex-excited resonances from other kinds. Consider the trumpet and the flute. Despite its loudness, the trumpet is a passive device, generating no new sound energy, but modifying the sound supplied by the trumpeter in two ways :

- it acts as an impedance-matching device, allowing the small cross-section of air vibrated by the trumpeter at high pressure to excite a much larger area at lower pressures, which at musical frequencies enhances efficiency of radiation
- it is frequency selective.

The flautist, on the other hand, does not supply sound energy. Instead, it is vortex shedding at the opening of the flute that supplies energy to the sound field.

Conditions for sustaining such a resonance are:

- There must be a forcing function supplying energy to the acoustic field at the resonant frequency, and within a limited phase range. The energy supply is needed to balance losses of acoustic energy through radiation and friction. The hypothesis is that this occurs when vortices from the leading lip of the aperture are swept past the rear lip.
- The resonance itself must feed information back to the forcing mechanism so that the frequency and phase will be kept correct. It is proposed that this can occur by the acoustic oscillation synchronizing the shedding of vortices from the front lip, a process called *locked vortex shedding*.

3.2 Vortex shedding

Vorticity builds up in the boundary layer on the roof upstream of the aperture, and is shed as a vortex sheet from the front lip. This quickly rolls up into discrete vortices (of one sign only), which are then advected with the main flow across the aperture. It is reasonable to expect that this transit velocity will be about one half of the road speed.

The vortex shedding is periodic, and with it is associated a dimensionless number, the Strouhal Number

$$\mathrm{St} = Lf/v \tag{6}$$

where L is a characteristic length, f is the shedding frequency, and v is a characteristic speed of the mean flow. Strouhal numbers are determined by the geometry, and are commonly in the range 0.1–0.2. Although there is therefore a natural frequency of shedding, forced vibration of either the shedding body or the surrounding air can force shedding at a different frequency. It is harder to reduce the frequency than to increase it.

3.3 Locked vortex shedding

When vortex shedding occurs in a strong acoustic field which synchronizes the shedding, the character of the flow changes markedly. This is shown in the pictures in Figure 8 (taken from [4]) of an experiment in a wind tunnel with vortex shedding from plates about 12 mm thick. In the upper picture, there is no sound, and the smoke visualization shows vortices shed with imperfect periodicity, and some variation along their axes. In the lower picture, the only difference is that there is now a considerable sound field. The picture shown is exactly reproduced each sound cycle, and will appear stationary if illuminated with light flashing stroboscopically at the sound frequency.

In Figure 6, an intense acoustic resonance is excited at certain spacings of the plates. The acoustic mode is transverse to the plates, and is a modification of the fundamental cross mode of the wind tunnel duct. The geometry has some similarity to the opening of the sunroof.

3.4 Power transferred to the sound field

According to Howe's low Mach number modification of Lighthill's theory of aerodynamic sound generation [5], the power transferred from the mean flow to the sound field is locally

$$P = \rho_0 \mathbf{u}.(\mathbf{v} \wedge \boldsymbol{\omega}) \tag{7}$$

where $\mathbf{u}$ is the acoustic velocity, $\mathbf{v}$ the mean velocity, and $\boldsymbol{\omega}$ the vorticity. To get the total energy available to drive the resonace, this has to be integrated over time and space.

FIGURE 11. Photograph of the flow around the straight plates at a spacing of $x/t = 3.75$ (30 mm) at the instant in the sound cycle corresponding to maximum upwards acoustic particle velocity; upstream flow velocity = 22.9 m s^{-1}.

Figure 6: Acoustically synchronized vortex shedding across an aperture

Because **u** is oscillatory, if $\mathbf{v} \wedge \boldsymbol{\omega}$ is a constant, the net transfer of energy over a cycle is zero. It is only where the flow is perturned, so this product changes significantly over an acoustic cycle, that energy can be supplied to the resonance.

The energy transferred can be positive or negative, depending on the phase of the sound cycle at which the vortex passes the rear lip. If the mean air speed is U=20 m/s (about 45 mph), vortices will travel at about $U/2$=10 m/s across the gap, taking about 0.04 s if the opening is a =0.4 m. That is one period of the oscillation. At 40 m/s it would take just half a period, and so would pass 180° out of phase, reversing the sign of the energy transfer. We cannot tell, without further analysis, at what phase the energy transferred would be a maximum.

Figure 7 shows the likely pattern of resonance/nonresonance that would arise from this mechanism. It is based on the periodicity of the energy transfer function, and is subject to an uncertainty of phase. The assumption is made that resonance first occurs when the vortices pass the rear lip 0.7 cycles after leaving the front lip. The boundaries of the resonance bands are then lines in the (U, a)-plane with boom for $2af_0/U$ in the intervals (0.7,1.2), (1.7,2.2) *etc*, and no boom for (1.2,1.7), (2.2,2.7) *etc*. For a fixed opening, there is a maximum speed at which resonance can occur, and a succession of bands as the speed is decreased.

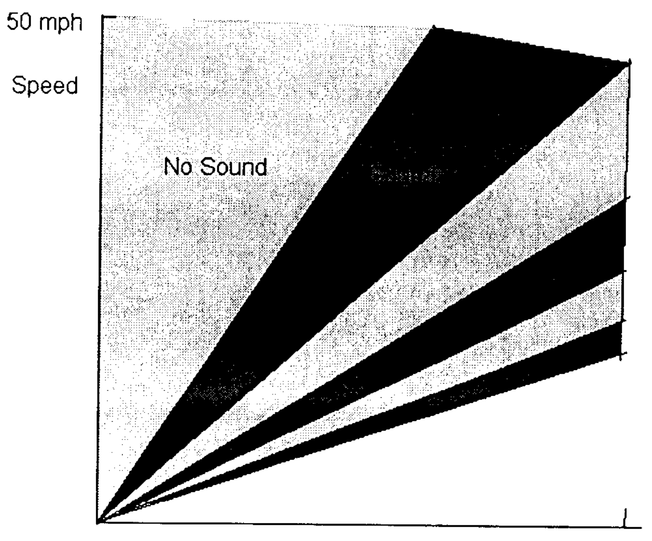

Figure 7: Illustration of regions where boom will and will not be heard

The considerations so far have concerned the existence of the resonance, but from Jaguar's perspective the important question is, How loud will it be? This depends on the driving energy, which in turn depends on how close the vortices pass to the rear lip. If they pass very close, the advective velocity will change substantially just where the acoustic velocity is the greatest, giving a larger power transfer integral. If they pass further away, then not only is the triple product smaller, but the non-acoustic terms change more slowly, and the integral is then substantially diminished by the oscillatory effect of the acoustic component of the triple product.

Another factor is the strength of the synchronization of the vortex passage past the rear lip in the sound field. This will be imperfect in any case because the edges are not straight, and the speed of the mean flow is not uniform.

3.5 Flow deflection

To describe this, we return to the lumped parapeter model of section 2.1. Write $Q = \dot{X}$ so X is the volume change of the air in the cavity. Then on the simplest view of the matter, the whole mean flow pattern across the cavity will oscillate up and down with X — the acoustic velocity is just superimposed on the mean flow pattern.

Then the effect we now wish to include is that when, say, $X < 0$ the horizontal mean flow across the cavity will be deflected downwards and so there will be an additional flow into the cavity at the rear lip. In fact if we denote this additional flow by Q_a and count it positive outwards, then we expect Q_a to be an increasing function of X, generally nonlinear, but for simplicity let us take $Q_a = \alpha X$ for a constant $\alpha > 0$. Then the system of 2.1 modified to take

$$dp/dt = -(\gamma p_0/V)(Q + Q_a) \tag{8}$$

gives a third order equation for X,

$$\rho_0 I\, d^3X/dt^3 = -(\gamma p_0/V)(dX/dt + \alpha X). \tag{9}$$

This has oscillatory solutions with exponentially growing amplitude, proportional to $\exp(\alpha t/2)$ for α small. In other words, interpreting the growth as a negative damping factor, the damping factor is $-\alpha/(2\omega_0)$. Thus if this exceeds the damping mechanisms in section 2.3 then instability is to be expected.

Note that it is essential to have the cavity present in order to see this effect of alternately charging and discharging the cavity: there was no need to consider this effect in [4]. Note also that this instability mechanism is in no way incompatible with the *existence* of synchronized vortex-shedding: the difference between the two mechanisms is that the flow deflection mechanism is governed simply by the mean flow at the rear lip and its sensitivity to vertical displacement, while the vortex interaction mechanism is governed by the timing of the passage of discrete vortices past the rear lip.

4 Review of Jaguar data

The data in [1] show that at low speeds (around 20 mph) the vortex travel time between the two critical openings $a_1(U)$ and $a_2(U)$ is one period of the Helmholtz resonator. For instance, for a Series 3 at U=21 mph the first onset of boom (with no flow deflector) is at opening a_1=125 mm, and the second at a_2=310 mm. Assuming that the vortices travel at speed $U/2$, the travel time for a_1 as a multiple of the resonator period is $2a_1 f_0/U$=0.67 periods. The corresponding figure for a_2 is $2a_2 f_0/U$=1.65 periods, almost exactly one period greater. This is in good agreement with the hypothesis that vortex interaction with the rear lip is responsible for the instability. Moreover, experiments on a flow-excited resonator [6] and a theoretical analysis of it using the describing function method to analyse the vortex interaction mechanism [7] show very good agreement in the amplitude and frequency of the resonance, in a case where the graphs of sound level as a function of air speed are very reminiscent of those in the Jaguar Technical Report [1].

However, the vortex interaction method does not fully explain everything that is happening: a_2 does not increase linearly with U, and at say 30 mph $a_2 - a_1$ corresponds to much less than a full period of the resonator. One of the main differences between the experiments in [4],[6] and the flow over a car is that the experiments are carried out with the mean flow as near 2-dimensional as can be arranged.

5 Experimental observations: 3-dimensional aspects of the flow

A Ford Escort (JND's) also exhibits sunroof boom, but only in a narrow speed range around 50 mph. It is possible therefore that the mechanism in this case is somewhat different from that in the Jaguar Saloons where resonance occurs over a wide range of speeds. However, in the absence of a Jaguar, experiments were conducted in the Escort, both around the Edinburgh ring road, and statically with the car parked facing into a gale part way up Arthur's Seat. The sunroof aperture dimensions are a =0.45 m, and b =1.2 m.

The flow appeared to separate at the top of the windscreen (A in Figure 3) with a reattachment point somewhere along CD, behind the rear lip. At the centre of the front lip of the aperture there was flow upwards out of the car. There are strong conical vortices with vertices at the leading corners of the aperture, driving air down into the aperture along the sides. Also the air flow within the roof cavity may be important (*ie* the space between the metal roof of the car and the inner skin). There was flow into this cavity along the sides of the aperture (driven by the conical vortices) and flow out of this cavity at the rear lip of the aperture, some of this outflow going down into the car and some up and out.

This suggests that the view of vortices forming at the front lip, propagating directly across the aperture and impinging on the rear lip is a serious oversimplification. (However, whatever form the mean flow takes, the acoustic disturbances to it are likely to propagate down the shear region at approximately one half of the road speed, and so the linear dependence of a_1 on U in Figure 1 is accounted for, even if vortex shedding is not the primary means by which the flow drives the resonance.)

It was also pointed out that vorticity generated at the front roof pillars (*ie* the sides of the windscreen) sweeps up over the car and will drive secondary flows over the aperture. The effect of such flows can be seen in the pattern of dust deposited on a car roof.

6 Conclusions recommendations and suggestions

In the absence of CFD or experimental flows over the car it is difficult to know exactly what features of the air flow cause boom in certain cars. However, a likely mechanism is that the air flow causes the vortices shed from the front lip of the aperture to impinge directly on the rear lip. Our chief recommendation is that Jaguar test this hypothesis by performing smoke visualizations with stroboscopic illumination, the frequency of illumination taken from the measured acoustic field in the car interior, and the phase in the acoustic cycle adjustable. This should establish the exact conditions where resonance occurs and how near the vortices pass the trailing edge. If this is true, then one way to anticipate the problem at the design stage before wind tunnel testing would be to use a CFD package to track vortex motion in the air flow. (Prof Richard Hillier, Imperial College, would be a good person to consult about this.)

Remedial action should be directed to either disturbing the synchronization of shedding at the front lip, or deflecting the path of the vortices away from the rear lip. The deflectors tested by Jaguar presumably do the first of these. Several suggestions were made for deflecting the vortices away from the rear lip:

- bleed off some of the air from the car's air inlets and feed it out through the front lip of the aperture;
- use a curved deflector (higher in the middle of the car than at the sides) with the aim of making the vortices roll up in a way that raises the vortex paths.

Another benefical action would be to have all edges and corners on the rear lip as rounded as possible.

7 Contributors

Many of those present at the study group contributed comments and suggestions to this problem. Those particularly involved were Jon Chapman, John Coats, Jeff Dewynne, Alistair Fitt, Julie Guneratne, John Hinch, Sam Howison, Andrew Lacey, John Ockendon, Colin Please, Domingo Salazar-Gonzales, Graham Veitch, and the compilers of this report.

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