Optimal Blower Design

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1 Introduction

The 'slitblower' consists of a channel into which high-speed air is introduced through two nozzles slightly inclined to the channel (see figure 1). Hydrodynamic forces from the air then stretch the filament, and transport it to a conveyor belt. Although around 25 filaments are transported simultaneously, they are so thin and far apart that they do not affect the airflow to lowest order. Hence we may approximate the behaviour to the two-dimensional single filament problem indicated in figure 1 and calculate the filament response *a posteriori*.

The channel is open to the atmosphere at the top and bottom. The pressure in the inlet area is maintained at a constant value which is usually around 5 atmospheres. The width of the channel is 2mm, with depth 25cm, and length which typically takes a value 20-40cm. A pressure difference of around 4 atmospheres, i.e. 4×10^5 Nm^{-2}, implies velocities of order $(\Delta p/\rho)^{\frac{1}{2}}$, i.e. 600ms^{-1}. Hence the Reynolds number based on the channel width is 10^5. Other data provided includes the width of the inlet tubes at the end, which is 0.25mm, and the angle of incidence of the inlet tubes to the channel, which is 20°. Furthermore experimentally-determined values of the mass flows are around 500 kg/hr through the inlet tubes, with an additional 50 kg/hr flowing through the top of the tube and mixing with the injected air.

The aim is to find a model for the air flow in the slitblower that is consistent with the experimental data that has already been obtained for the static pressure at the wall of the channel and the total pressure in the centre of the channel (see figure 2). We wish this model to explain how a reverse flow region may be set up in the channel. This is a potentially catastrophic problem, which results in filaments being blown out of the top of the slitblower. The ultimate goal of the model is therefore to predict the inlet pressure that will maximise the total shear stress on the filament subject to the condition that there is no reverse flow.

In addition, we wish to predict the effect of changing the geometry of the system. In particular, it is known that a backward-facing step at the end of the channel has the effect of reducing the possibility of reverse flow, and moreover has a positive effect on the friction forces.

2 Gasdynamics

2.1 Equations of Motion

As a result of the high pressure difference, velocities approaching or exceeding the speed of sound are likely, and so it is necessary to give a brief discussion of some elementary results from compressible flow (see [1, 2, 3] for further details). Here, a key assumption is that there is no dissipation, so the flow is isentropic. This assumption is not valid in the boundary layers or in the region where the two jets collide, as there is turbulent entrainment of the air above the inlet region. The assumption of isentropy permits the system of equations consisting of continuity of mass and Euler's equations to be closed with the equation

$$p/\rho^\gamma = p_0/\rho_0^\gamma. \tag{1}$$

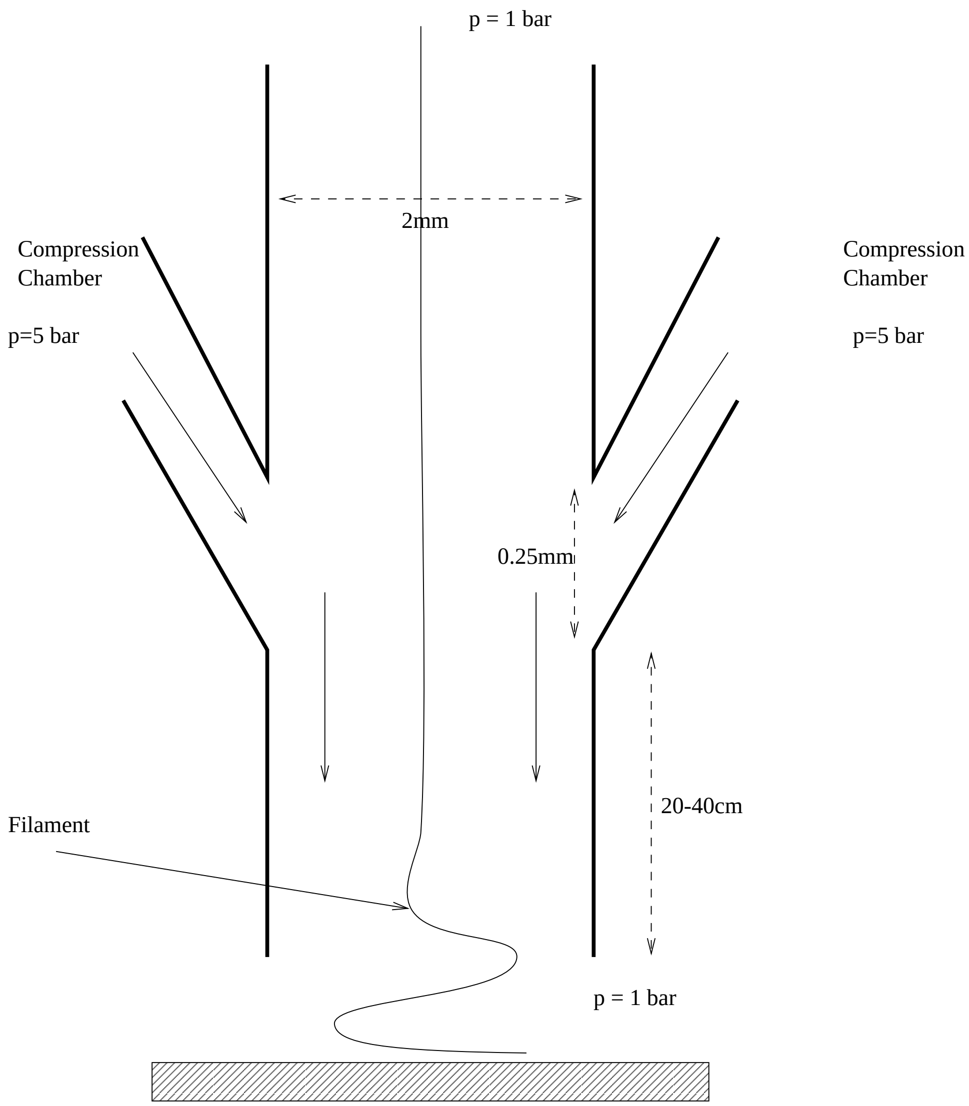

Figure 1: Filament Transport Through the Slitblower

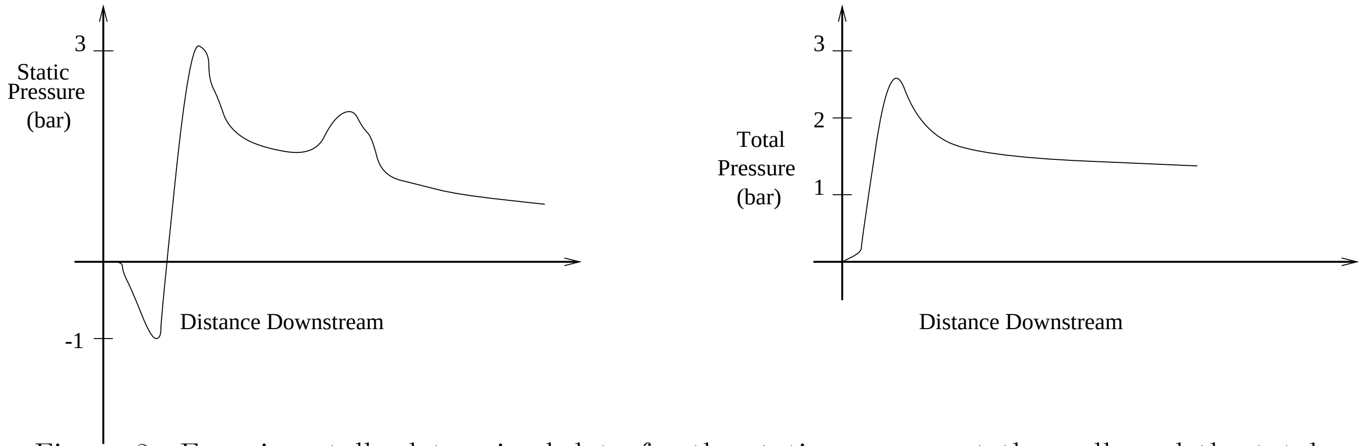

Figure 2: Experimentally determined data for the static pressure at the wall, and the total pressure at the centreline

Here, p is the pressure, ρ is the density, γ is the ratio of specific heats (which is 1.4 for air) and the suffix 0 denotes the stagnation value of a variable, which is its value if the fluid is isentropically brought to rest. It follows from this expression that the speed of sound, a, given by $a^2 = dp/d\rho$, may be expressed in terms of the density by

$$a^2 = \gamma \rho^{\gamma-1} \frac{p_0}{\rho_0^\gamma}. \tag{2}$$

For irrotational flow, we may also apply Bernoulli's equation which when combined with (1) becomes

$$\frac{a^2}{\gamma-1} + \frac{1}{2}|\boldsymbol{u}|^2 = a^2 \left(\frac{1}{\gamma-1} + \frac{1}{2} M^2 \right) = \frac{a_0^2}{\gamma-1}, \tag{3}$$

where $\boldsymbol{u}$ is the velocity. Hence, given the stagnation values of a fluid the flow variables, namely a, $|\boldsymbol{u}|$, p, and ρ may all be determined from the Mach number, $M = |\boldsymbol{u}|/a$.

2.2 Expansions and Contractions

For the purposes of this problem the flow of a gas round a corner is particularly important. The cases of expansion and contraction, for both subsonic and supersonic flow must be considered separately.

Firstly we consider a contraction in the flow. Schematic flow diagrams for the subsonic and supersonic case are indicated in figure 3. The key difference is that in the case of supersonic flow there is a 'shock' that emanates from the corner, across which there is a discontinuity in the pressure and velocity. An explicit solution may be derived in this case, with the change in the flow variables across the shock determined by the Rankine-Hugoniot conditions.

For an expansion flow, if the flow is supersonic then there is an expansion fan, known as a Prandtl-Meyer expansion. The flow near the corner consists of two regions of uniform flow, separated by a fan-shaped region of expansion with characteristics that expand radially from the corner (see figure 4). The equations of motion are therefore the continuity equation, the two components of the Euler equation, and the isentropic condition (1). From these [2] a relation may be deduced between the angle through which the flow is turned, θ, and the Mach number, namely

$$\lambda\theta = \arctan(\lambda \cot \mu) - \arctan(\lambda \mu_1). \tag{4}$$

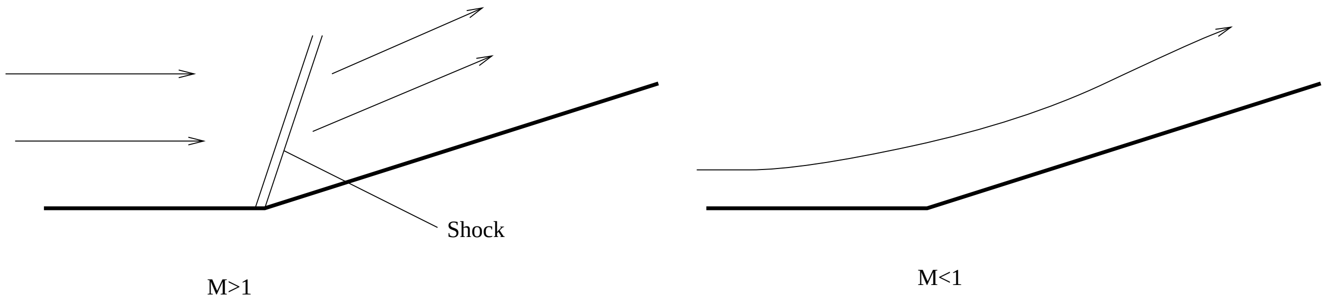

Figure 3: Supersonic and subsonic flow through a contraction

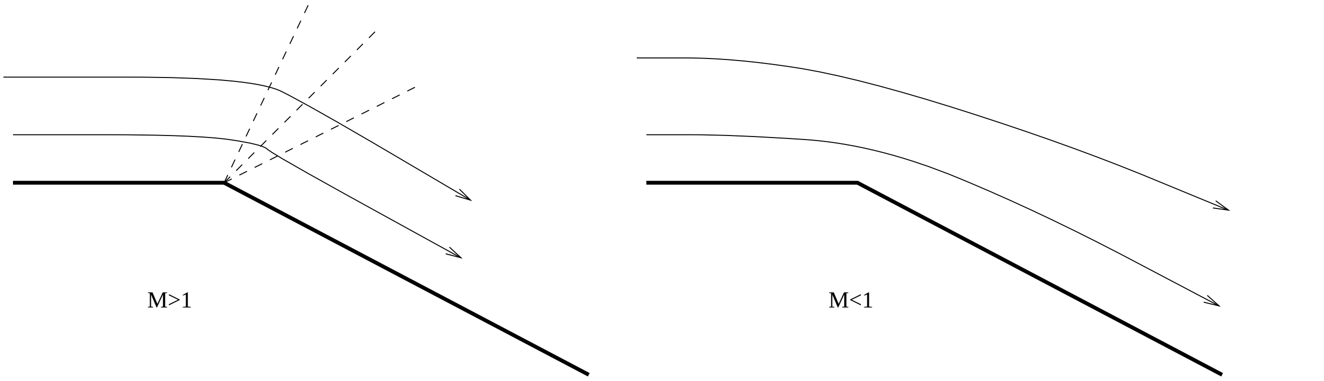

Figure 4: Supersonic and subsonic flow through an expansion

Here, $\lambda^2 = (\gamma - 1)/(\gamma + 1)$, μ is the Mach angle, $\arcsin M^{-1}$, and μ_1 is the Mach angle before the expansion.

2.3 Nozzle Flow

As both the channel and the inlet tubes are long and thin, we may model the flow as nozzle flow everywhere except in the boundary and shear layers and a small region downstream of the exit from the inlet tube. If we label the axis of the tube as the x-axis, we may write the conservation of mass equation as

$$\rho u A = M = \text{const.} \tag{5}$$

If we combine (1), (2), (3) and (5) we obtain

$$\frac{1}{A}\frac{dA}{dx} = \frac{M^2 - 1}{M(1 + \frac{\gamma-1}{2}M^2)}\frac{dM}{dx}. \tag{6}$$

This shows that the Mach number can only be equal to one at a point where the cross-sectional area takes a stationary value. It may also be shown that this stationary value must be a minimum, i.e. the 'throat' of a nozzle. This implies that supersonic flow can only be achieved via a convergent-divergent nozzle, and moreover that increasing the initial pressure above that necessary for a Mach number of 1 does not increase the Mach number at the throat, but only the pressure. This phenomenon is known as 'choking'.

2.4 Nozzle Flow with Friction

In the presence of friction at the walls, the momentum equation is replaced by (see e.g. [4])

$$uu_x = -\frac{p_x}{\rho} - 2\frac{\eta}{A}u^2 f, \tag{7}$$

where f is the friction factor and η is the perimeter of the channel. The friction factor depends on the Reynolds number and the surface roughness of the channel, and is weakly dependent on the Mach number and turbulence. However it is usually reasonable to assume that it is constant, and for Reynolds numbers of the order of 10^5 is usually taken to be around 0.005 (for a more detailed discussion of the value of the friction factor, see [5]).

On combining (7) with the mass conservation equation (which now implies that ρu is constant, as the cross-sectional area is constant) and (1), we obtain

$$-a^2\frac{du}{u} + udu + \frac{2u^2 f\eta dx}{A} = 0.$$

The analogous equation for frictionless nozzle flow with a varying cross-sectional area is, from (1), (3) and (5),

$$-a^2\frac{du}{u} + udu - a^2\frac{dA}{A} = 0.$$

These equations are the same provided

$$-\frac{dA}{dx} = 2M^2 f\eta. \tag{8}$$

Hence it follows that flow down a nozzle with parallel walls and friction is equivalent to flow down a suitably converging channel with no friction.

3 The Flow in the Slitblower

3.1 Choking in the Entry Nozzle

For simplicity, we assume the flow is symmetric, and consider one half of the flow. The first problem is to determine whether the flow ever becomes supersonic. It follows from (1), (2) and (3) that

$$\left(\frac{p}{p_0}\right)^{-\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{2}M^2 + 1, \tag{9}$$

and hence the pressure ratio corresponding to a Mach number of 1 is approximately 0.53. As the pressure at the bottom of the channel is atmospheric, and the pressure in the compression chamber is typically around 5 atmospheres the flow certainly becomes supersonic at the throat. Therefore as the flow goes through the throat it undergoes a Prandtl-Meyer expansion in the region B on figure 5. It must be emphasised that figure 5 is still very speculative but the scenario it presents does not contradict any of the experimental or numerical evidence.

Near the downstream edge of the inlet region, C, there is another Prandtl-Meyer expansion, of 20°. In both regions, as the effective cross-sectional area increases, the Mach number increases. However the part of the flow nearest the upstream corner, B, does not feel the effect of the second expansion, and so the flow near the channel boundary, henceforth referred to as the 'outer' flow (region E) is at a higher Mach number. Furthermore the 'inner' flow, which exits closer to the upstream corner and then travels downstream through the centre of the channel (regions D and F), meets the jet emanating from the other slit, and therefore must undergo a compression. This leads to a shock in this region. Downstream of the inlet area, these two regions (E and F) mix. It is important to determine which of the two regions, the expanded 'inner' region or the shocked 'outer' region, contains most of the air in the downstream flow. Hence it is necessary to calculate the flow variables after the Prandtl-Meyer expansions.

3.2 The First Prandtl-Meyer Expansion

We may combine the Prandtl-Meyer equation (4) with the Bernoulli equation in the form of (9) to determine the angular deflection, and subsequent Mach number, of flow exiting the inlet at B into a region where the pressure is one atmosphere. (The assumption that the pressure immediately upstream of the incoming jet is one atmosphere is a first approximation — in reality turbulent entrainment of the upstream air results in a downstream velocity in this region, and hence the static pressure is not exactly equal to the atmospheric pressure upstream. However the mass flow upstream of the jet is around a tenth of the total mass flow, and so may be ignored for the purposes of this calculation). Given the Mach number, the other flow variables may be calculated using (1) and (3) or (9). Hence immediately after the expansion we have $M = 1.71$, and $\rho u/\rho_0 a_0 = 0.43$. It then follows from (4) that the angle through which the flow is turned in this region is 18°.

3.3 The Second Prandtl-Meyer Expansion

Near the downstream corner, C, there is a second Prandtl-Meyer expansion. In this case the condition that the flow expands into a region of given pressure is replaced by the condition that the deflection angle must be 20°. Hence from (4) it follows that the Mach number following this expansion will be 2.45. This then implies, from Bernoulli's equation and mass conservation, that $\rho u/\rho_0 a_0 = 0.23$. In addition, the experimentally determined Schlieren pictures indicate that the inner region is roughly 2.3 times as thick as the outer region, so on combining these figures it follows that the mass flow in the inner region is roughly 5 times that in the outer region. Therefore the inner part of the flow will dominate the behaviour of the mixed region, as $\rho u A$ is much larger in this region.

4 The Flow Downstream of the Inlet Area

As the two jets meet there is a sudden contraction in the flow, and this is achieved via a shock (see figure 5). The shock will curve slightly downstream into the expansion fan. As the shock approaches the channel walls, the shock becomes weaker and the angle through which the flow is turned approaches zero. In the limiting case for a weak shock, the inclination of the shock to the flow tends to the Mach angle, which for $M = 2.45$ is 24°.

All these calculations have led to the scenario presented in figure 5. However this does not take into account the turbulent boundary layers on the walls or the mixing layer in the centre

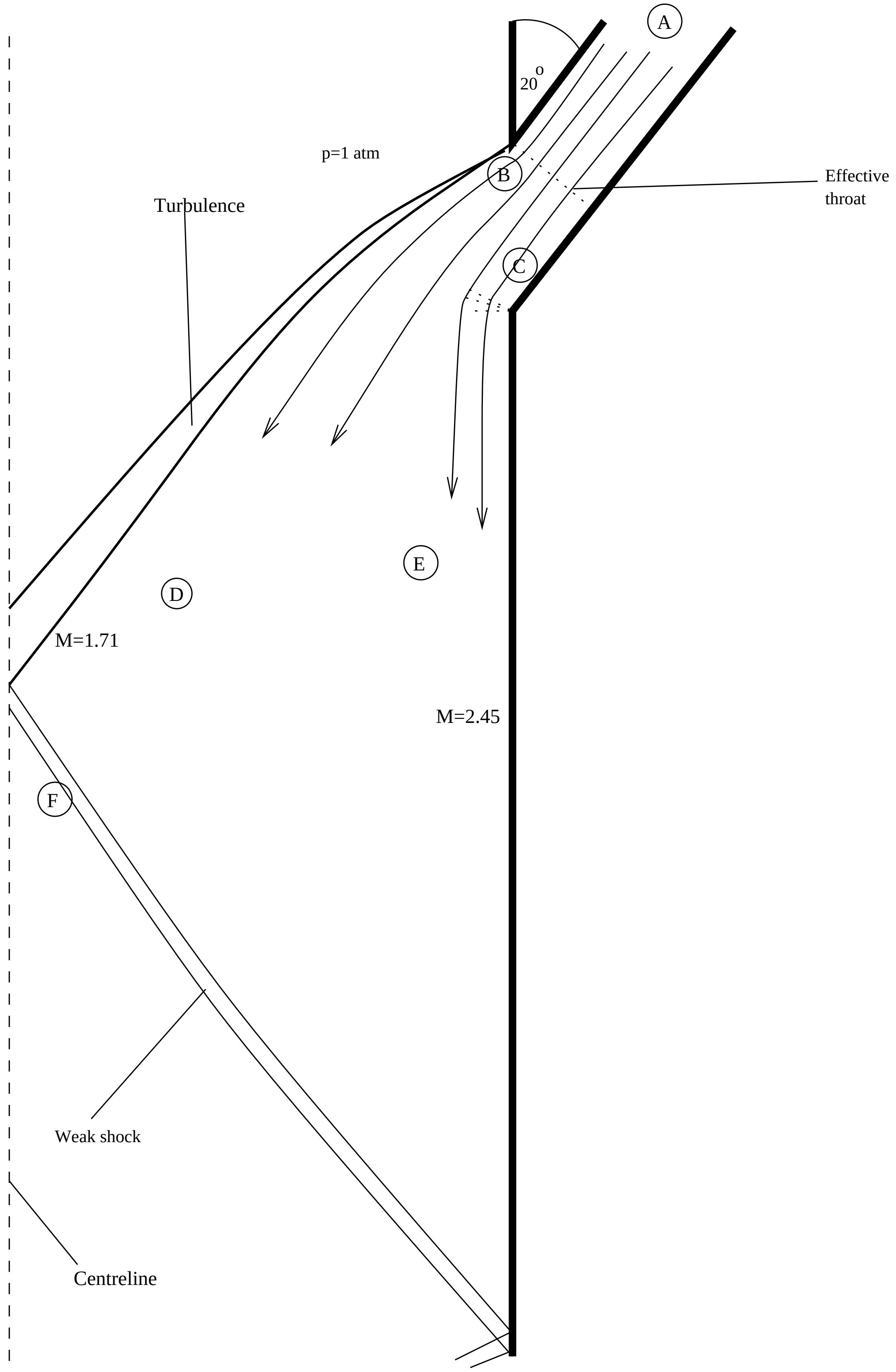

Figure 5: A schematic diagram of the jets exiting the inlet region

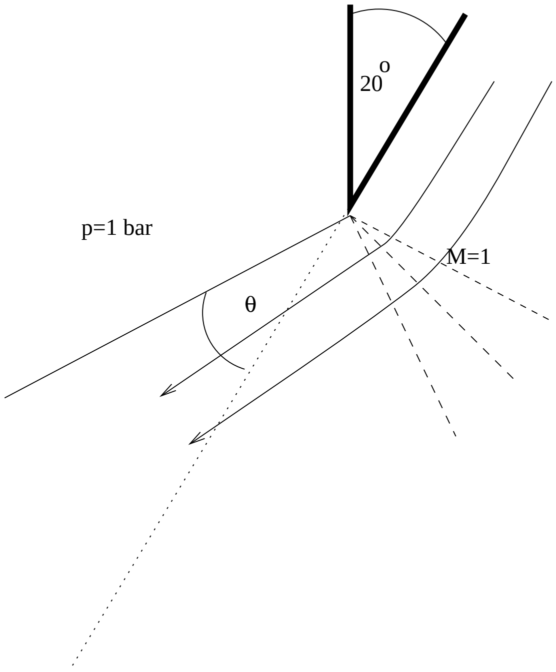

Figure 6: The first Prandtl-Meyer expansion, near the upstream corner

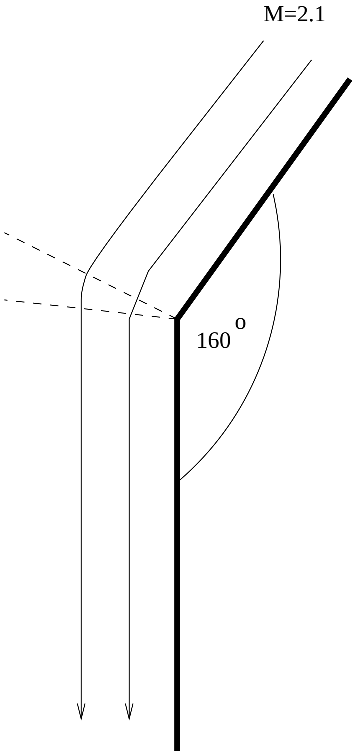

Figure 7: The second Prandtl-Meyer expansion, near the downstream corner

of the tube. Indeed, it is observed that the total pressure decreases to approximately 1.5 bar down the length of the tube (see figure 2). Therefore it is certainly necessary to incorporate friction into this model.

Note that because of the second Prandtl-Meyer expansion there is a drop in the wall pressure immediately downstream of the inlet area, at E, in accordance with figure 2. The pressure then rises as a result of mixing with the higher pressure flow that exits at the upstream end of the inlet area and then collides with the other jet (F). The wall pressure then declines due to the effect of friction until it meets the backward-facing step. The total pressure rises in the mixing region due to turbulent entrainment of the upstream air, and then decreases, due to the effects of friction.

5 Choking at the End of the Channel

If the Mach number reaches one at the end of the channel then the flow is choked at the end of the channel. This will lead to large pressures, and hence reverse flow upstream of the inlet region. This gives an upper bound on the pressure in the compression chamber before reverse flow occurs, and hence on the mass flow.

On combining (6) and (8), we may relate the the Mach number to the distance downstream via

$$\frac{M^2-1}{M(1+\frac{\gamma-1}{2}M^2)}\frac{dM}{dx} = -2M^2\frac{f\eta}{A}.$$

On integration of this we obtain the relation between the critical length for choking, L^*, and the initial Mach number, namely [1]

$$L^* = \frac{A}{f\eta}\left[\frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\log\left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right)\right]. \qquad (10)$$

Similarly, from Bernoulli's equation we may obtain a relationship between the initial value of the total pressure, P_0, and the value at choking, P_0^*, in terms of the initial Mach number. Hence

$$\frac{P_0}{P_0^*} = \frac{1}{M}\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}. \qquad (11)$$

Hence the minimum distance before choking may be inferred in terms of the ratio of the initial total pressure to the stagnation value. From the experimental data the total pressure is 2.1 bar after the initial adjustment due to entrainment. Although the flow out of the channel must reach zero velocity and atmospheric pressure downstream of the channel exit, this transition will not happen isentropically, as a result of turbulent entrainment. Therefore instead of using stagnation values, we use the experimental data which suggests that at the end of the channel the total pressure is approximately 1.4 bar. From (11) we therefore have an initial Mach number of 0.43, and then from (10) we obtain a critical length of 36cm before choking occurs.

The maximum mass flow may be evaluated directly from (1) and (3), as they combine to give

$$\rho q = \frac{M a_0 \rho_0}{(1+\frac{\gamma-1}{2}M^2)^{\gamma-\frac{1}{2}}}.$$

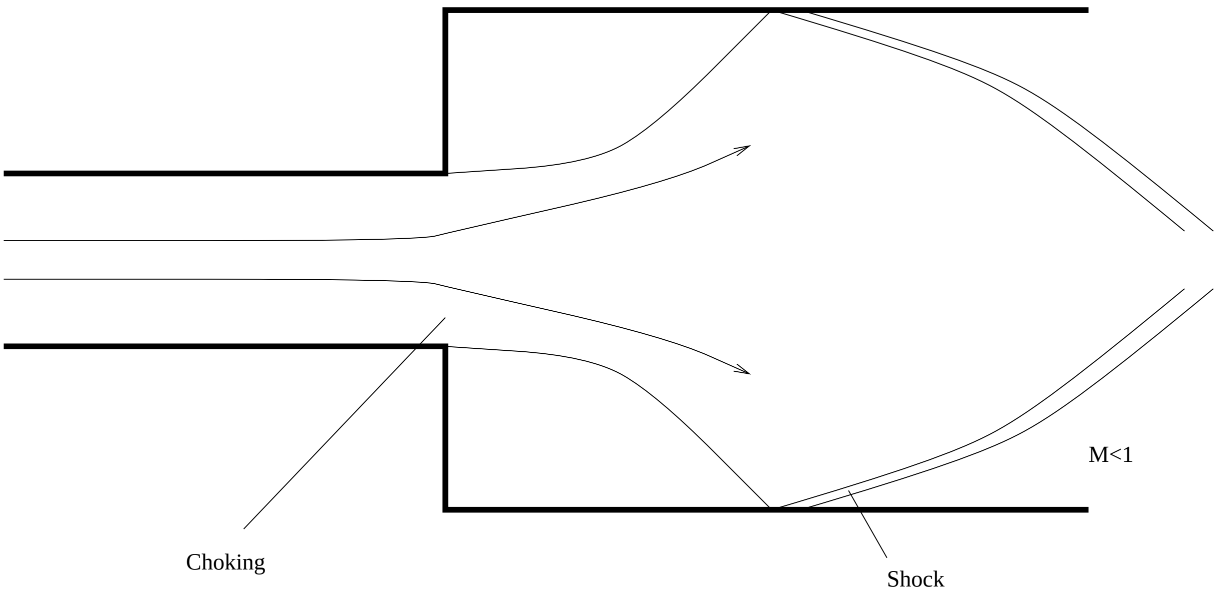

Figure 8: Premature choking caused by the backward-facing step

Therefore for subsonic flow ρq is a monotonic function of M, so the maximum mass flow is given by $M = 1$. Assuming that the flow is isentropically brought to rest downstream, it then follows that the maximum mass flow is given by $\rho q = 316$kg m^{-2} s^{-1}, which for a channel 2mm wide and 25 cm deep is equivalent to 570 kg/hr.

The maximum channel length will depend on the initial pressure. Therefore for a given channel geometry there is a window of initial pressures that do not result in backflow, as too large a pressure will result in choking and to small a pressure will result in subsonic flow everywhere, which will always lead to some backflow.

6 The Effect of the Backward-Facing Step

The introduction of a backward-facing step will have the effect of increasing the turbulence, and hence the frictional force on the filament. In addition, expansion of the flow will result in a 'premature shock'. The flow will become supersonic as it expands out, and then as it hits the walls there will be a shock and it will become subsonic, thus permitting a longer channel length before the flow is choked, and hence a larger mass flow. It seems likely that a slowly diverging channel would have a similar effect.

In addition, the backward-facing step means there is more room for the filament to oscillate, enabling larger oscillations. This is desirable for the way in which the filament is collected onto a conveyor belt.

7 Possible Areas for Future Work

The finding of a finite range for the possible values of the imposed pressure in the compression chamber should be confirmed by further experimental work. In addition, a more detailed picture of the flow field in the region where the jets meet could be obtained by solution of the two-dimensional problem by use of a hodograph transformation. The flow is still not well understood in the region where the jets meet.

A more detailed description of how upstream flow results from choking would also be of interest. One possible model would be to consider jet impingement into a region that is blocked at one end, as if a plug had been inserted in the channel downstream of the inlet region.

There is a well-established analogy between supersonic flow and flow in a shallow channel [2], with the transverse wave speed analogous to the speed of sound. Given the difficulties and expense associated with experiments for the slitblower, it would be advisable to investigate the behaviour of shallow water waves in a similar geometry to that indicated in figure 1.

8 Postscript

The above report has been discussed with Prof. T.V.Jones of the Engineering Department, Oxford University. Experimental evidence appears to broadly confirm the schematic picture of the flow field indicated in figure 5. The Schlieren picture in figure 9 of the interaction of two supersonic jets shows the formation of a shock wave as the jets collide, with the shock curving away from the reflection as in figure 5.

Prof. Jones shares the concern that the flow upstream of the injection region may be significant, and that further work is needed to understand the influence of the flow in this region.

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