PROBLEM 9

TELEMETRY PROBLEMS WITH PACEMAKERS IN A NOISY ENVIRONMENT

1. INTRODUCTION

This report covers the investigation of problems relating to the telemetry of pacemakers in a noisy environment. An attempt was made to suggest schemes that improve system performance. We define system performance to be improved noise immunity with high reliability and increased message throughput.

Three aspects were considered, with implementation schemes that incur minimal protocol changes over systems in existence.

1. The present scheme is writing to the pacemaker and echoing each bit back within a 2 ms frame. If any echoed bit is not in agreement with that sent, the message is re-transmitted from the start of the block. Because the return link is poor in terms of signal to noise ratio, echoing bits up this link degrades system performance, particularly as the noise power increases. An ARQ (automatic repeat request) scheme is suggested as a solution to this problem.

2. When data is to be read from the pacemaker, reply is via two 4ms frames each of 6 bits. A CRC (cyclic redundancy check) is performed on the returned data (8 bits) which checks for three or less errors, and the redundancy bits are appended to the end of the 9 bits. If any errors are detected, a retransmission is required. Using a FEC (forward error correction) scheme to correct errors, together with a CRC to check the

integrity of data, throughput can be significantly improved especially in a noisy environment. The scheme we suggest is to encode data bits plus CRC with a 23,12 Golay code, and send the data using four by 4ms frames. The Golay code suggested is a perfect code, triple error correcting.

3. The final aspect we considered is the possible system performance improvement using soft decision quantization at the programmer of received data. This system gives gains of the order of 1.75 dB over current practice.

2. CODING ON THE DOWN-LINK TO THE PACEMAKER

The proposed strategy is to use an ARQ (automatic repeat request) scheme for writing to the pacemaker.

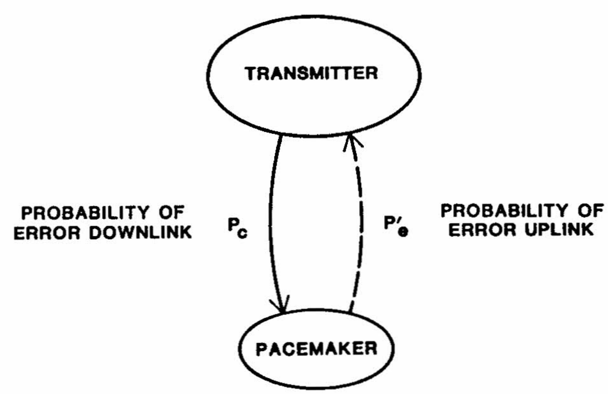

Figure 1. Illustration of the downlink and uplink error probabilities

At present, each bit is sent by an echo to the pacemaker when writing is validated. If the echo invalidates the information bit, the whole message frame is sent again. Because the P_e' is in general much greater than P_c, the throughput of the system is hampered by the return link.

To implement the ARQ scheme a good (n,k) block code is used in the error detection mode. Here, n is the number of bits in the block and k is the number of message bits.

We define the following error probabilities

P_c = probability a received n bit word contains no errors,

P_d = probability of a detectable error pattern,

P_{e_o} = probability of a n bit block containing an undetectable error pattern.

Clearly we have

$$1 = P_{e_o} + P_d + P_c.$$

The probability that a decoding error is made is given by

$$\begin{aligned} P(E) &= P_{e_o} + P_d P_{e_o} + P_d^L P_{e_o} + \ldots \\ &= P_{e_o} \frac{1}{1-P_d} \\ &= \frac{P_{c_o}}{P_c + P_{e_o}} \end{aligned}$$

For a random channel with bit error rate p, we have

$$P_c = (1-p)^n.$$

It has been shown that linear block codes exist such that the probability of undetected error has the upper bound (Massey, 1978).

$$P_{e_o} \leq [1-(1-p)^k]2^{-(n-k)}$$

Thus an arbitrarily high system reliability may be generated using a powerful enough error detection code.

Compare this with the results of ARQ systems in Lin et al. (1984). At this stage, we have not been able to obtain a closed form throughput for the full duplex system used at present. However, an estimate may proceed as follows: the expected number E of 2 ms frames sent per block can be obtained using the current echoing process as follows. Define

n = the number of frames in a block,

p_S = probability of individual frame success,

p_F = probability of individual frame failure.

Clearly, we have $P_S + P_p = 1$.

Let E_i be the expected number of frames needed to be sent to fill a block with exactly i failures. Then E is given by

$$E = \sum_{i=o}^{\infty} E_i$$

where

$$E_o = np_s^n,$$

$$E_1 = P_F P_s^n(2nP_s^{n-1} + (2n-1)P_s^{n-2} + \ldots + n+1),$$

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$$E_i = P_F^i P_6^n((i+1)nP_s^{in-i} + \ldots + i+n).$$

In the actual calculation of E, one would terminate the process of calculating the E_i once the value P_F^i is small enough to make further contributions by E_i negligible.

3. SOFT DECISION: AN IMPROVEMENT OF REPEAT STRATEGY PRESENTLY USED

The error probability of a coded system is bounded by (Viterbi & Omura, 1979)

$$P_E \leq \sum_{k=2}^{M} e^{-w_k d}$$

where w_k are the weights of codewords which are independent of quantization. For the 3 times repeat strategy, the weight of codeword is 3. The constant d depends on the quantization technique and is given by

$$d = -\log_e \sum_{Y} \sqrt{p_o(y)p_1(y)}$$

where

$p_o(Y) \equiv p(y_n|x_{mn} = +\sqrt{\xi_s}) = p(y_n|V_{mn}=o)$,

$p_1(y) \equiv p(y_n|x_{mn} = -\sqrt{\xi_s}) = p(y_n|V_{mn}=1)$,

ξ_s is energy per symbol,

y_n is nth component of received vector,

x_{mn} is nth component of mth message,

V_{mn} is channel bit.

For the additive white Gaussian noise (AWGN) channel, we have

$$\begin{aligned} d &= -\log_e \int_{-\infty}^{\infty} \sqrt{p_o(y)p_1(-y)}dy. \\ &= -\log_e \int_{-\infty}^{\infty} \exp[-(y-(2\xi_s/N_o)^{\frac{1}{2}})^2/4].\ \exp[-(y + (2\xi_s/N_o)^{\frac{1}{2}})^2/4]\frac{dy}{(2\pi)^{\frac{1}{2}}} \\ &= \xi_s/N_o. \end{aligned}$$

Now it can be shown that $d = \log_e(4p(1-p)^{\frac{1}{2}}$ for a BSC where $p = Q(\sqrt{\frac{2\xi_o}{N_o}})$ and Q() is the Gaussian integral function. Also using the assumption $\xi_o/N_o << 1$, we have

$$d \sim \frac{2}{\pi} \frac{\xi_s}{N_o}.$$

Thus $\pi/2 \sim 2dB$ more signal power is necessary for the BSC to perform as well as the infinite quantities channel. Although the assumption made is $\xi_s/N_o << 1$, in practice this condition may hold even when the assumption is relaxed somewhat.

Rather than use infinite quantization, multilevel quantitization is more practical. For example, an 8 level quantized signal will give performance close to infinite quantization (Heller & Jacobs). This system requires an AGC (automatic gain control) circuit in the receiver. The channel model is displayed in Figure 2.

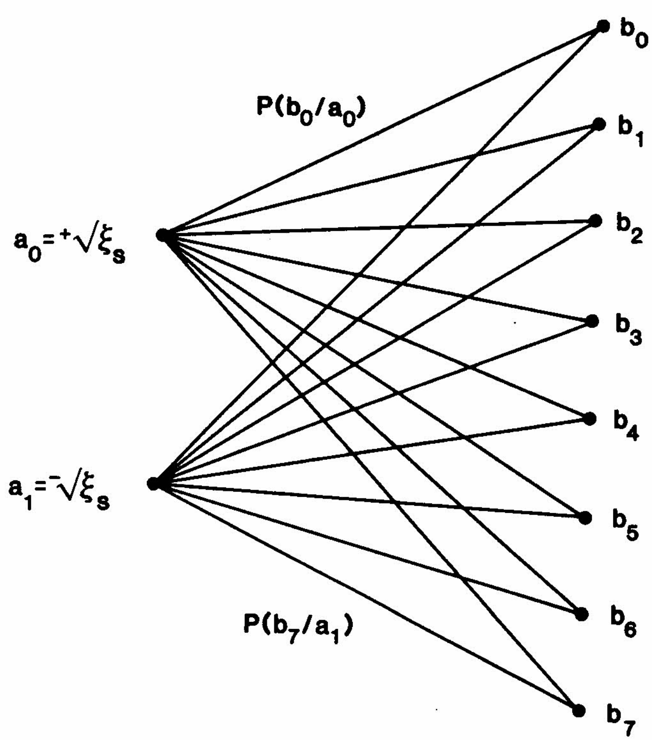

Figure 2. Illustration of the channel model

The likelihood functions are given by

$$P_N(\underline{Y}|\underline{X}) = \prod_{n=1}^{N} p(y_n|x_{mn}) \quad m=1,2,\ldots,M$$

$$= p(y_n=b_j|x_{mn}=a_k)$$

$$= \frac{1}{\sqrt{\pi N_o}} \int_{z\varepsilon B_j} e^{-(z-a_k)^2/N_o} dz$$

where B_j is the jth quantization level. This now gives the information to construct

$$d = -\log_e \sum_y [p_o(y)p_1(y)]^{\frac{1}{2}}$$

to obtain a new P_E bound

$$P_{E_\infty} \text{ level} < P_{E_8} \text{ level} < P_{E_{BSC}}$$

4. CODING OF REGISTER CONTENTS ON THE RETURN LINK

We shall consider using a binary linear code, C. Some coding theory definitions are as follows (Viterbi & Omura (1979), Gallager (1968), Lin & Costello (1983), MacWilliams & Sloane (1978)).

1. We define distance d as the Hamming distance between two codewords.

2. Let w(v) be the number of non-zero coordinates in v.

3. $d_{min}=\{w(\underline{c}):\underline{c} \varepsilon C, \underline{c} \neq 0, w(\underline{c}) \leq w(\underline{c}^1) \text{ for } \underline{c}^1 \varepsilon C \}$.

Let $\underline{y}$ be the received vector. Then $\underline{y} = \underline{c} + \underline{e}$ where $\underline{c}$ is a codeword and $\underline{e}$ is the error vector. Using maximum likelihood decoding, $\underline{y}$ can be correctly decoded, provided

$$w(e) < y/2d_{min}$$

The average probability of error becomes,

$$P_E \le \begin{cases} \displaystyle\sum_{k=\frac{d_{min}+1}{2}}^{L} \binom{L}{k} p^k (1-p)^{L-k} & d_{min} \text{ odd} \\ \displaystyle\sum_{k=\frac{d_{min}}{2}}^{L} \binom{L}{k} p^k (1-p)^{L-k} & d_{min} \text{ even} \end{cases}$$

where p is the channel crossover probability for a BSC. If we decode using the full power of the code to correct errors, we can no longer use the same code to detect errors.

At present, a CRC check of four redundancy bits is envisaged to be used on eight data bits. This CRC will detect only patterns of three or less errors. If an error is detected, a retransmission occurs.

A FEC (forward error correcting) scheme on this link will enhance the throughput on noisy channels. A possible format for the return data is to use four by 4 ms frames. At present two by 4 ms frames are used. The 4 frames gives us access to 24 bits. In this space, it is feasible to use a 23,12 Golay code (MacWilliams & Sloane 1978). This code will allow correction of any pattern of 3 bits of error, since $d_{min} = 7$.

The data section of this code is 12 bits. This allows us to concatenate a 4 bit CRC (cyclic redundancy check) to this data. The codeword can thus be decoded and a CRC check can be effected on the decoded data block of 12 bits. Now a further integrity check on the data is achieved.

The Golay code may be cyclicly encoded using the polynomial

$$g(x) = 1 + x^2 + x^4 + x^5 + x^6 + x^{10} + x^{11}$$

thus implying minimal hardware overhead. The Kasami decoder (Lin & Costello, 1983), for example, is a suitable decoder with error correcting capability to the full power of the code.

For the 23,12 Golay code, the probability an error occurs in decoding is

$$P_E \leq \sum_{k=4}^{L} \binom{L}{k} p^k (1-p)^{L-k}$$

where L is the length of the block. In the present method of CRC, an error is detected and retransmission is necessary for probability of detected error P_α

$$P_\alpha \sim \sum_{k=1}^{3} \binom{L'}{k} p^k (1-p)^{L'-k}$$

where L' is the length of present block. A retransmission for the hybrid scheme above is necessary where there is an error in decoding and the CRC check finds an error in the decoded information.

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