# EARTHQUAKE DAMAGE IN UNDERGROUND ROADWAYS

Tim Marchant* and Graham Weir†

**Abstract**

Earthquake damage in underground roadways and mine workings is considered, with particular application to the mines operated by Solid Energy NZ Ltd., on the West Coast of New Zealand's South Island. The scenario considered is the effect on the mine workings of an earthquake, of moment magnitude eight, being generated by a rupture of the Alpine fault.

An empirical relation from the seismology literature is used to relate earthquake magnitude, distance from the epicentre and the peak ground acceleration resulting from the seismic waves. This relation is used to estimate the likely damage at the mine site. Also, the decay scale for Rayleigh (surface) waves is calculated and the implications for the mine workings considered.

The two-dimensional scattering of shear (SH) seismic waves from the mine workings is considered. Analytical solutions relevant to various mine tunnel geometries are presented with the stress and displacement amplification, due to scattering from the mine workings, calculated and discussed.

## 1. Introduction

Solid Energy NZ Ltd operates a number a coal mines on the North and South Islands of New Zealand (NZ), extracting more than three million tonnes of coal each year. These include the Terrace and Spring Creek mines sites, on the West Coast of the South Island, which are the focus of this Mathematics in Industry Study Group (MISG) project.

The Alpine fault is one of the longest faults in the world. It is about 20km deep and extends from Marlborough to Milford Sound, a distance of about 500km. The fault has a return period of 250-300 years with

*School of Mathematics and Applied Statistics, University of Wollongong, NSW 2522, email: tim_marchant@uow.edu.au
†Industrial Research Ltd, g.weir@irl.cri.nz

large earthquakes, of about moment magnitude eight, occurring in the years 1100, 1450, 1620 and 1717. There is a significant chance, of about 40%, that the fault will rupture in the next twenty years resulting in an earthquake of moment magnitude $M_w = 8$ or greater. As the Terrace and Spring Creek mines lie close to the Alpine fault, Solid Energy NZ Ltd. asked the MISG study group to investigate the likelihood of damage to the mine workings, typically about 200 to 400m underground, from a magnitude eight earthquake. This scenario represents a major earthquake. For example, the 1906 San Francisco earthquake was of moment magnitude $M_w = 7.9$, caused by a rupture of the San Andreas fault, while in 2002 a moment magnitude $M_w = 6.7$ earthquake devastated Bam in Iran, which was within about 30km of the epicentre.

Dowding and Rozen**?** summarised the damage to various underground mines from about 70 observations of earthquake damage in mines worldwide. They used an empirical formula to estimate the peak ground acceleration from the earthquake magnitude and the distance from the epicentre. A strong correlation between damage and increasing ground acceleration was found, with only minor damage occurring when the peak ground acceleration is below 0.5g, where g is the acceleration due to gravity. Sharma and Judd**?** added new observations to those used by Dowding and Rozen**?** (about 150 observations in total) and found that damage decreases with increasing depth of the mine workings, related to the fact that Rayleigh waves decay with depth. They also found a strong correlation between mine damage and increasing ground acceleration.

Both St John and Zahrah**?** and Hashash et. al.**?** comprehensively reviewed the design and analysis of underground tunnels and structures subject to seismic events. The papers concentrated on the damage due to ground shaking with the peak ground acceleration and velocities generated by the earthquake considered to be key indicators of damage caused to the structure. Mathematical models, developed using elasticity theory, which estimate the stress (force) and strain (deformation) on underground structures were reviewed. Both analytical and numerical solution techniques were discussed as was the effects of ground-structure interaction.

Pao**?** considered the scattering of compressive waves from a circular cavity. Figures were presented of the stress amplification factor, which is the stress on the cylinder's surface, normalised by the maximum stress resulting from the incident wave propagating in the solid with no cavity present. Their results show a stress amplification of up to three can occur at the cavity boundary with the maximum amplification occurring for incident waves about 25 times longer than the cylinder radius. Moon and Pao**?** considered the scattering of incident plane and spherical elastic waves from a spherical cavity. For a Poisson's ratio of $\nu = 0.3$ the interaction of incident plane waves and the sphere caused a stress amplification of less than two at the cavity boundary.

In 2 the governing equations for seismic waves are presented and the decay length scale for Rayleigh waves is estimated for the most energetic wavelengths incident upon the mine-workings. Also, an empirical relationship is used to determine the peak ground acceleration of the seismic waves incident upon the mine workings and the damage they are likely to cause.

In 3 scattering of incident seismic waves from mine workings with various geometries are considered. Analytical solutions are presented for the scattering of two-dimensional shear (SH) waves. These analytical solutions represent the scattering from some typical mine tunnel geometries and allow the stress and displacement amplification to be estimated. 4 provides a summary of our conclusions and suggestions for future work.

## 2. Governing equations and empirical relations

In this section the equations governing the propagation of seismic waves are briefly described and the properties of the waves incident upon the mine workings are estimated. The decay scale for Rayleigh (surface) waves is calculated and empirical relations are used to estimate the likely damage in the Spring Creek and Terrace mine workings from a moment magnitude $M_w = 8$ earthquake on the Alpine fault.

### 2.1. Governing equations

Assuming that the earth is an isotropic linear elastic medium means that the seismic waves are governed by the partial differential equation

$$\rho_0 \mathbf{u}_{tt} = (\lambda + 2\mu)\nabla^2 \mathbf{u} + (\lambda + \mu)\nabla \times (\nabla \times \mathbf{u}), \qquad (1)$$

where $\mathbf{u}$ is the displacement, $\rho_0$ is the density of the medium, and $\lambda$ and $\mu$ are the Lamé constants (see p37 of Beford and Drumheller **?**). There are three main type of seismic waves, P-waves, S-waves and Rayleigh (surface) waves. For one-dimensional compression waves, or P-waves, we assume that the displacement has the form $\mathbf{u} = u(x, t)\mathbf{i}$, where $u$ satisfies the one-dimensional wave equation

$$u_{tt} = \alpha^2 u_{xx}, \quad \text{where} \quad \alpha^2 = \frac{\lambda + 2\mu}{\rho_0}, \qquad (2)$$

and $\alpha$ is the P-wave speed. For one-dimensional shear waves, or S-waves, the displacement has the form $\mathbf{u} = u(x,t)\mathbf{j}$ and $u$ is described by

$$u_{tt} = \beta^2 u_{xx}, \quad \text{where } \beta^2 = \frac{\mu}{\rho_0}, \tag{3}$$

and $\beta$ is the S-wave speed.

Rayleigh waves propagate along the surface of the Earth and have compression and shear components. We assume that $\mathbf{u} = u_1(x,z,t)\mathbf{i} + u_3(x,z,t)\mathbf{k}$, where $u_1 = \phi_x - \psi_z$ and $u_3 = \phi_z + \psi_x$ The compressive, $\phi$, and shear, $\psi$, components satisfy the two-dimensional versions versions of the P and S wave equations, (2) and (3), respectively. The solution for the two components is

$$\phi = Ae^{-hz}e^{i\theta}, \quad \psi = Ce^{-h_s z}e^{i\theta}, \quad \theta = k_1(x - c_r t), \tag{4}$$

$$h^2 = (k_1^2 - \frac{\omega^2}{\alpha^2}), \quad h_s^2 = (k_1^2 - \frac{\omega^2}{\beta^2}), \tag{5}$$

where the $x$-axis is horizontal, along the Earth's surface, and the $z$-axis is vertical and points downwards. $k_1$ is the wavenumber, $c_r = \omega/k_1$ is the wave speed and $\omega$ is the frequency. The amplitude of the Rayleigh wave decays with depth and $h$ and $h_s$ are the decay rates for the two wave components. Applying a zero-stress boundary condition at the surface $z = 0$ gives the Rayleigh characteristic equation as

$$(2 - \frac{c_r^2}{\beta^2})^2 - 4(1 - \frac{\beta^2}{\alpha^2}\frac{c_r^2}{\beta^2})^{\frac{1}{2}}(1 - \frac{c_r^2}{\beta^2})^{\frac{1}{2}} = 0, \quad \frac{\beta^2}{\alpha^2} = \frac{1-2\nu}{2(1-\nu)}, \tag{6}$$

where $\nu$ is Poisson's ratio. Hence (6) is a transcendental equation for $c_r/\beta$, which depends only on the value of Poisson's ratio.

## 2.2. Length scale for Rayleigh wave decay

As seismic waves propagate away from the earthquake epicentre they undergo attenuation due to damping and geometrical effects. As the damping is frequency dependent the spectral response of the site needs to be determined. Abrahamson and Silva**?** have developed an empirical spectral attenuation relationship based on several hundred recordings from about sixty earthquakes around the world. The empirical model predicts that the peak ground acceleration which occurs at a site decreases as the distance $r$ from the epicentre increases. It also predicts that the period of the seismic wave, for which the ground acceleration is a maximum, increases as $r$ increases. For $r = 40$km, which is the distance of the Spring Creek and Terrace mines from the Alpine fault, the most energetic waves have a period in the range $T \in [0.2, 0.3]$s.

To estimate the wavelength of the incident waves we assume that $\nu = 0.25$ and $\beta = 2$km/s, both of which are reasonable assumptions for rock. Using a period of $T = 0.25$s gives the wavelength of the incident S and P-waves to be $\lambda_s = 500$m and $\lambda_p = 870$m respectively. Using these estimates in (4) and (6) gives the decay constants of the Rayleigh wave as $h = 23$ and $h_s = 11$. To determine the effect of Rayleigh waves on the mine workings the depth at which the wave amplitude has decayed to 10% and the energy to 1% of the surface values is calculated. For the compressional component the depth is $z = 100$m while for the shear component $z = 210$m.

Hence the Rayleigh waves have almost completely decayed away at depths $z > 200$m, where the Terrace and Spring Creek workings are located. Hence Rayleigh waves are unlikely to cause any damage to the underground workings, which will be subject to incident S and P-waves only. Sharma and Rudd (1991) examined 132 cases of earthquakes at mine sites and found moderate or heavy damage rarely occurred below 100m. Hence, it seems Rayleigh waves cause the severe earthquake damage in shallow mines and S and P-waves cause lighter damage only, in the deeper mines.

## 2.3. Attenuation of the incident seismic waves

There exist many empirical relationships in the seismology literature relating earthquake magnitude, distance from the the epicentre and the peak horizontal ground acceleration, see Hu et. al.**?**, for a list of some of these relations. The empirical relation used by Abrahamson and Silva **?** has the form

$$\ln \frac{a}{g} = a_1 + a_2(M_w - c_1) + a_{12}(8.5 - M_w)^2 \tag{7}$$

$$+(a_3 + a_{13}(M - c_1) \ln R, \quad R = (r^2 + c_4^2)^{\frac{1}{2}}, \tag{8}$$

where $a$ is the peak ground acceleration, $g$ is the acceleration due to gravity, $M_w$ is the moment magnitude of the earthquake and $r$ is the distance to the rupture site. The relationship (7) is valid for $M_w < c_1$, for $M_w > c_1$ the coefficient $a_2$ is replaced by $a_4$. The coefficients are found by fitting (7) to the observed earthquake data using a regression analysis. See table 3 of Abrahamson and Silva**?** for a list of the regression coefficients for different incident wave periods.

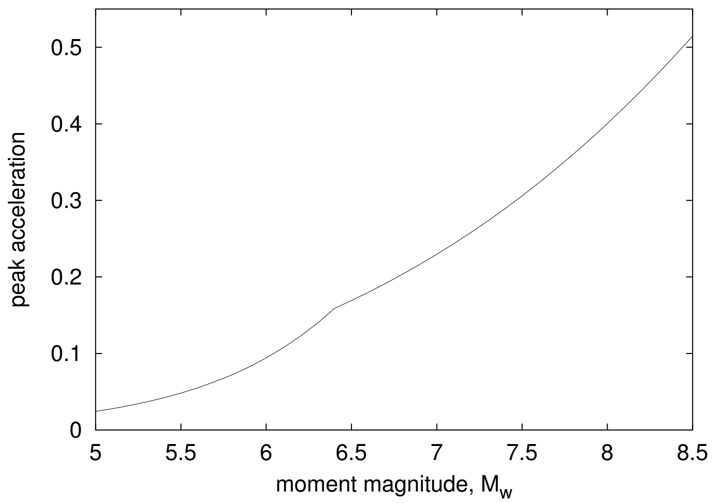


*Figure 1.* The peak ground acceleration, scaled by $g$, versus moment magnitude $M_w$, for $T = 0.2$s and $r = 40$km.

Figure 1 shows the peak ground acceleration versus moment magnitude for $r = 40$km and $T = 0.2$s. Shown is the empirical relationship (7) using the regression coefficients of Abrahamson and Silva**?** corresponding to a wave of period $T = 0.2$s. This period is near the peak of the spectral response curve hence represents one of the most energetic wave periods. The choice of $r = 40$km corresponds to the distance of the Spring Creek and Terrace mines from the Alpine fault.

The peak ground acceleration is a monotone increasing function of the moment magnitude. The curve has a slope discontinuity at the point $M_w = c_1 = 6.4$ because different regression coefficients are used for small and large magnitude earthquakes. For a scenario of $M_w = 8$ the estimate of the peak ground acceleration at the Spring Hill and Terrace mine sites is $a = 0.39g$. Fig. 3 of Dowding and Rozen (1978) plots peak accelerations versus mine damage. They showed that severe damage occurs for $a > 0.5g$, while minor damage occurs when $0.2g < a < 0.5g$, and little or no damage occurs for $a < 0.2g$. Hence it is likely that only minor damage will occur in the Spring Creek and Terrace mine workings as a result of a moment magnitude $M_w = 8$ earthquake on the Alpine fault.

The data used to obtain the empirical relationship (7) comes from about sixty earthquakes of magnitudes ranging from 4.4 to 7.4. No data on more extreme earthquakes, such as the $M_w = 8$ scenario considered here, is included. Extrapolating the data to this scenario must be done with caution, however, we believe that the empirical model gives a reasonable "ballpark" estimate for the peak ground accelerations and note that this estimate is well within the range of accelerations in which light damage occurs.

## 3. Scattering of elastic waves from the mine-workings

During a seismic event the mine workings will be subject to incident S and P-waves as the mines are too deep to be affected by Rayleigh waves. The incident seismic waves will be scattered by mine roadways, tunnels and other workings. In this section we will investigate the increased stress and displacement generated by the scattering of waves from the mine-workings. If the increased stress is excessive, failure of the tunnel or roadway may occur. Analytical solutions, for a number of mine-tunnel geometries, are presented and the stress and displacement amplification is discussed.

We ignore compressive waves and only consider shear (SH) waves incident upon two-dimensional tunnels and cavities. This is done for mathematical simplicity as incident shear waves generate only scattered shear waves whilst the scattered wavefield from an incident compressive wave has both compressive and shear components. Moreover, results in the literature for compressive wave scattering (discussed below) are qualitatively similar to those for shear wave scattering.

The displacement has the form $\mathbf{u} = u(x,y)e^{-i\omega t}\mathbf{k}$ and is governed by the Helmholtz equation

$$\nabla^2 u + k^2 u = 0, \quad \text{where} \quad k = \frac{\omega^2 \rho_0}{\mu}. \tag{9}$$

A boundary condition of zero normal stress is applied at the tunnel or cavity surface. This gives

$$\nabla u \cdot \mathbf{n} = 0, \tag{10}$$

where $\mathbf{n}$ is the normal to the cavity or tunnel. As the problem is linear it is decoupled into incident and scattered components, $u = u^i + u^s$. The scattered wave must satisfy the Sommerfeld radiation condition on the domain boundary,

$$\lim_{r\to\infty} r^{\frac{1}{2}}(u^s_r r - iku^s) = 0. \tag{11}$$

There is a close analogy between the scattering of two-dimensional shear waves and the scattering of acoustic waves.

The first scenario considered is the reflection or scattering of seismic waves from a long roadway or tunnel located at $x = 0$. The solution is

$$u^i = e^{ik(x\cos\alpha + y\sin\alpha)}, \quad u^s = e^{ik(-x\cos\alpha + y\sin\alpha)}, \tag{12}$$

where the wavefield $u = u^i + u^s$ satisfies a condition of no normal stress at $x = 0$ and $\alpha$ is the angle the incident wavetrain makes with the tunnel. The displacement (stress) amplification factor is defined by the ratio of the displacement (stress) of the total wavefield $u$ to the maximum displacement (stress) of the incident wavefield. Hence it represents the amplification of the displacement or stress, due to scattering from the tunnel. We calculate displacement $u(0, y)$ and the tangential stress $\tau_{zy} = \mu\frac{\partial u}{\partial y}(0, y)$ at the tunnel boundary and find that the stress and displacement amplification factor at the boundary is two for all angles of incidence $\alpha$. The magnitude of the tangential stress at the boundary is zero for waves normally incident upon the tunnel ($\alpha = 0$), and increases as $\alpha$ increases. Hence the maximum boundary stress occurs for incident wavetrains propagating nearly parallel to the tunnel ($\alpha$ near $\frac{\pi}{2}$).

The second scenario is the scattering of seismic waves from a circular cavity, which could represent the circular cross-section of a long tunnel or be a two-dimensional approximation to a larger cavern within the mine. Scattering from a circular cavity by plane waves is a classical problem with an exact series solution, see Graff**?**. The solution has the form

$$u_i = e^{iky}, \quad u_s = \sum_{m=-\infty}^{\infty} a_m H_m(kr)e^{im\theta}, \quad a_m = -\frac{J'_m(k)}{H'_m(k)}, \tag{13}$$

where $J_m$ and $H_m$ are Bessel and Hankel functions of the $m$-th kind. The cavity has unit radius and its centre is located at $r = 0$. Note that the wavenumber $k$ is a non-dimensional quantity, as it has been scaled by the radius of the cylindrical cavity.

Figure 2 shows the stress and displacement amplification factor versus wavenumber $k$, at the cylinder boundary. This is found by calculating the displacement $u(1, \theta)$ and the tangential stress $\tau_{z\theta} = \mu\frac{\partial u}{\partial \theta}(1, \theta)$ at the cylinder boundary by summing the series solution (13). Figure 2

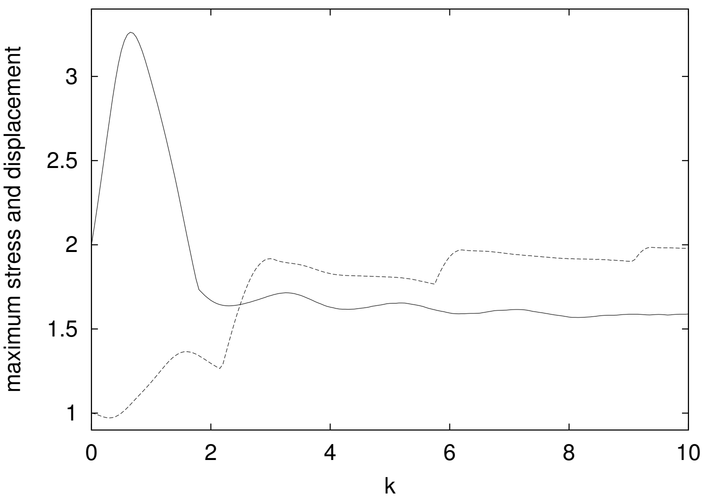


*Figure 2.* The stress (solid line) and displacement (dashed line) amplification factor at the cylindrical boundary versus wavenumber $k$.

shows that in the long wavelength limit ($k \rightarrow 0$) the stress concentration factor is two while is it fairly steady at around 1.5 for shorter waves with $k \in [3, 10]$. The peak of the stress amplification curve is 3.26 at $k = 0.65$. The displacement amplification is unity in the long wavelength limit ($k \rightarrow 0$) while it approaches two in the short wavelength limit ($k \rightarrow 0$).

For the Spring Creek and Terrace mines the wavelength of the incident shear waves is $\lambda_s = 500m$. Hence the peak stress amplification, at $k = 0.65$, corresponds to a radius $r \approx 50m$. This dimension is larger than the radius of a mine tunnel but might correspond to a larger cavern or cavity in the mine. A mine tunnel has a cross-section with radius of about four metres which corresponds to $k = 0.05$, or the large wavelength limit. In this limit the stress amplification factor is two.

Figure 2 is qualitatively similar to those in Pao**?**, who considered the case of the scattering of compressive waves from a circular cavity. For incident compressive waves the results of Pao**?** show that the peak stress amplification of about three occurs at $k = 0.25$, which corresponds to a cavity of $r \approx 35$m in our scenario. Moon and Pao**?** considered the three-dimensional extension to Pao**?**, that of the scattering of compressive waves by a spherical cavity. In this case the peak stress amplification of 1.95 occurs at $k = 0.95$.

The third scenario considered is the two-dimensional scattering of seismic waves from a thin semi-infinite tunnel. Consideration of this scenario allows the displacement and stress amplification at the end of the tunnel to be determined. This is important as development headings (tunnel dead-ends) are areas of substantial human activity in mines.

The incident wavetrain has the form

$$u^i = e^{-ik(x\cos\alpha + y\sin\alpha)}, \tag{14}$$

hence the incident waves make an angle $\alpha$ with the positive $x$-axis. The tunnel is located along the negative portion of the $x$-axis, hence the stress-free boundary condition becomes

$$u_y = 0, \quad -\infty < x \leq 0, \quad y = 0. \tag{15}$$

This is the classical Sommerfeld diffraction problem, and the solution for the scattered wavefield is

$$u^s(r,\theta) = \mp \frac{ie^{i\frac{\pi}{4}}}{\pi^{\frac{1}{2}}}[e^{-ikr\cos(\theta+\alpha)}F(p_2) - e^{-ikr\cos(\theta-\alpha)}F(p_1)], \tag{16}$$

$$\text{where} \quad p_1 = (2kr)^{\frac{1}{2}}\cos((\theta-\alpha)/2), \quad p_2 = (2kr)^{\frac{1}{2}}\cos((\theta+\alpha)/2), \tag{17}$$

$$F(v) = \int_v^\infty e^{it^2}dt, \tag{18}$$

see Graf**?**. The function $F(v)$ is related to the Fresnel sine and cosine integrals and is simple to calculate numerically. The two solution branches correspond to the solution for $y$ positive (minus sign) and $y$ negative (plus sign).

Figure 3 shows the displacement of the wavefield $u = u^i + u^s$ versus $-kx$, along the surface of the tunnel, $y = 0$. The angle of incidence $\alpha = \frac{\pi}{2}$, hence the seismic waves are normally incident upon the tunnel. The upper curve shows the the displacement along along the upper edge of the tunnel while the lower curve shows the displacement along the lower edge of the tunnel. The upper edge reflects the incident waves so on this side of the tunnel the solution consists of both reflected and diffracted components. As $-kx \to \infty$ the displacement approaches two, the appropriate displacement amplification factor for reflection from an infinitely long tunnel. The peak displacement of 2.35 is greater than that for a infinitely long tunnel, hence the diffracted wave from the tip of the tunnel increases the displacement slightly. The lower edge of the tunnel is in the shadow zone so the solution here represents a diffracted

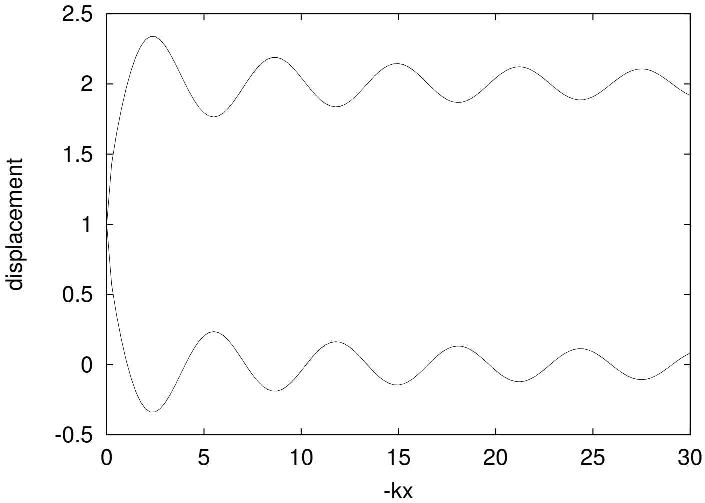


*Figure 3.* The displacement versus $-kx$ for $\alpha = \frac{\pi}{2}$. The upper (lower) curve is the displacement along the upper (lower) edge of the tunnel

wave only. As $-kx \to \infty$ the displacement approaches zero, again the correct value for an infinitely long tunnel. The peak displacement in the shadow zone is $u = 0.34$ at $-kx = 2.35$, hence the diffracted wave does not cause a large displacement on the lower edge of the tunnel.

Taking the limit as $x \to 0$ gives $u^s \to cx^{\frac{1}{2}}$, where $c$ is a constant. Hence the tangential stress, which is proportional to $u^s_x$, is singular at $x = 0$. So for a tunnel of zero width, the stress at the end of the tunnel is infinite. This result indicates that the stress at the end of a real tunnel, with finite width, is likely to be very large.

The presented solutions indicate that stress and displacement amplification, due to wave scattering, is limited to a factor of two or three. In general this amplification is unlikely to be sufficient to be the cause of extensive damage. However, sharp corners, bends and development headings in the mine tunnel network are locations where larger stresses will occur and are points of possible failure during an earthquake.

A possibility not considered in this section is mine damage due to the focussing of seismic waves. This may occur if the mine lies in a band of rock with spatially varying material properties. A numerical study using a finite-element package would be the most suitable approach for investigating this.

## 4. Conclusions and recommendations

The study has considered the likely impact of a moment magnitude eight earthquake, from a rupture of the Alpine fault, on the Terrace and Spring Creek mines. The decay length scale for Rayleigh waves has been identified and it is shown that the underground mine-workings are too deep to be affected by Rayleigh waves. Observations of earthquake induced mine-damage also indicate that deep mines are largely protected from earthquake damage.

An empirical relation is used to estimate that peak ground acceleration at the mine-sites and it is found that a magnitude eight earthquake is only likely to cause light damage within the mine. Analytical solutions are used to estimate the stress and displacement amplification which results from the scattering of shear waves from the mine tunnels and workings. It is found that the stress amplification is generally limited to a factor of only two or three and hence is unlikely to cause serious damage. However, sharp corners, bends and development headings have been identified as possible failure points in the mine tunnel network. Consideration should be given to reinforcing or lining the tunnels at these locations to prevent damage.

For the Terrace and Spring Hill mines, it is concluded that serious damage is unlikely to occur to the underground workings, located at a depth below 200m, as a result of a magnitude eight earthquake on the Alpine fault. However, Rayleigh waves from the earthquake are likely to cause extensive damage to the surface-based mine operations and mine portal area. This may mean that unharmed mine-workers are unable to exit the mine due to damage to the mine portal. Moreover, earthquake damage to the mine ventilation system may have serious consequences for any miners trapped below. Hence earthquake proofing the mine portal area and ventilation system is very important and represents a a good topic for future investigation and study.

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