EFFECT OF DEPOSITION OF COMBUSTIBLE MATTER ONTO ELECTRIC POWER CABLES

1. Introduction

Electric power cables are used in plant installations according to selection criteria set out in AS3000 and AS3008. The bases for selecting and installing the correct size and type of cable are governed by the current flow, the length of installations and type of cable insulation to be used. "Current rating" of cables is determined by the temperature reached in the conductor under steady state operation. This temperature is of course determined by the rate of least conduction through the insulator. Cables placed in a conduit, or adjacent to each other on a tray are "derated" since the heat generated will be harder to dissipate than that for a single cable. The geometry of the cable arrangements determines the quantitative value of the derating. Standard tables are available for this purpose and are applied to each installation.

A problem not hitherto considered is the collection of combustible (and at the same time insulating) dust on the installations. This could achieve considerable thickness over long periods and lead to two possible problems which are intimately related.

Firstly, the layer of dust could ignite if the critical ignition condition were to be reached. This depends, as we shall see later, on the ambient temperature, the thickness and configuration of the dust layer, the electric heating by the current *etc.* Secondly, the dust layer, if not extremely reactive, might cause failure of the cable by overheating since the extra insulation of the dust layer is not allowed for in standard tables, nor is the heat generated by normal decomposition of the dust. One can thus envisage two extreme types of failure, ignition of the dust before cable failure and cable failure before the dust ignites due to its insulating and thermogenetic properties.

The primary question raised at the Mathematics-in-Industry Study Group (MISG) was whether any effects of this type could occur for a reasonable thickness of dust layer; *i.e.* would it be km, m or cm for a reasonable cable installation?

2. The problems formulated

The most promising formulation with possibility of mathematical solution is shown in Figure 1. It is a cable of infinite length inside a conduit with the annular space between insulator and conduit filled with combustible dust of uniform density.

The cylindrical symmetry of the problem renders it attractive, as well as the fact that related problems involving ignition of materials by annular sheet sources of energy (representing friction between rotating cylinders) have already been solved (Gray & Wake (1984), Gomez *et al.* (1985)). It is also well known that the infinite cylinder gives results which are bounds for finite cylinders, erring on the side of safety.

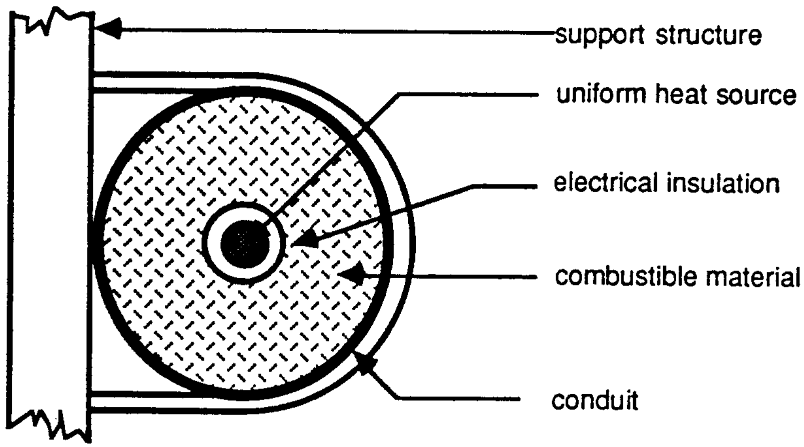

Figure 1: Representation of a cable inside a conduit filled with combustible insulating material.

Other problems considered included that caused by a high transient current overload, associated with the switching of heavy plant. This would give a thermal spike of short duration but large magnitude. The effect of this on a dust layer could be calculated using hot spot theory, but this was not thought to be practicable for a one week period.

A second problem considered concerned the wedge of material formed between two or more adjacent cables, again by dust collection. Treatment of this was again thought to be theoretically possible, but not practicable within the time available. It was thus decided to work on the single cable at the centre of a conduit. The central position of the cable is clearly an extreme as far as stability is concerned, probably a minimum, but it remains an interesting problem to show this.

The energy conservation equations in the conductor, PVC insulator and outer dust annulus take respectively the following forms (1-3)

$$\frac{d^2\theta}{dx^2} + \frac{1}{x}\frac{d\theta}{dx} + \frac{EI^2R}{RT_a^2k_c\pi} = 0 \qquad (0 < x < a), \tag{1}$$

$$\frac{d^2\theta}{dx^2} + \frac{1}{x}\frac{d\theta}{dx} = 0 \qquad (a < x < b), \tag{2}$$

$$\frac{d^2\theta}{dx^2} + \frac{1}{x}\frac{d\theta}{dx} + \delta e^{\theta} = 0 \qquad (b < x < 1), \tag{3}$$

where θ is a dimensionless temperature rise in the assembly, referred to ambient temperature ($\theta = 0$), x is a dimensionless radial distance from the centre of the cable and δ is a dimensionless measure of the heat production rate by the decomposition and/or oxidation of the dust. At the interfaces between the three layers, energy flux and temperature continuity conditions are applied. Analytical solutions of equations (1-3) are available and the continuity conditions plus the boundary condition $\theta = 0$ along with the symmetry condition $d\theta/dx = 0$ at $x = 0$ serve to determine all the integration constants.

The form of the solution in the reactive layer is

$$\theta = \ln\frac{2F^2Gx^{F-2}}{\delta(1+Gx^F)^2} \tag{4}$$

where F and G are integration constants.

If θ_0 is the central (maximum) temperature in the assembly, application of the boundary and continuity conditions gives us an exact parametric relationship between δ and θ_0:

$$e^{\theta_0} = \frac{[(F+k'+2) + b^{-F}(F-k'-2)]^2 b^{F-2}}{4F^2k''}, \tag{5}$$

$$\delta = \frac{2F^2(F-k'-2)(F+k'+2)b^{-F}}{[(F+k'+2) + (F+k'-2)b^{-F}]^2}, \qquad |F| > |k'+2|. \tag{6}$$

The constants k' and k'' are defined as

$$k' = -\frac{EI^2R}{2R'T_a^2k_i}, \qquad -\infty < k' \leq 0, \tag{7}$$

$$k'' = \exp[-\frac{EI^2R}{2R'T_a^2k_i}\ln(b/a) - \frac{EI^2R}{4R'T_a^2k_c}], \tag{8}$$

where

E = activation energy of the chemical reaction of the dust,
R'= universal gas constant,
I = current,
R = conductor resistance/unit length,
a = radius of conductor,
T_a= ambient temperature,
λ= thermal conductivity of dust layer,
k_i= thermal conductivity of PVC,
k_c= thermal conductivity of copper,
b= radius of insulator annulus,
$b + h$ (= 1)= radius of dust layer.

Both a and b are scaled with respect to the radius of the annulus of dust $(b + h)$.

A typical graph of δ *versus* θ is shown in Figure 2. This curve would be calculated for a particular pair of values of k' and k'' as well as given radii for conductor, insulator and dust. It is seen that for $\delta < \delta_{cr}$ (the critical value of δ) there are two solutions for the temperature profile and the time dependent solution represents a diverging temperature *i.e.* ignition.

The critical values of δ and θ_0 are calculated for a given set of the parameters k', k'' and cable dimension. and this enables the construction of a table such as Table 1 (at the end of this report). Table 1 is calculated for $k'' = 0.5$. Other values of k'' affect θ_0^{cr} through the formula

$$\theta_0^{cr} \; = \; \theta_0^{cr}(0.5) \; - \; \ln(k''/0.5); \tag{9}$$

δ_{cr} is independent of k''. The thickness of the layers are expressed in units of the radius of the dust layer.

To use this table we have to calculate δ for a given material from the formula

$$\delta \; = \; \frac{\rho Q Z \ell^2 E e^{-E/R'T_a}}{\lambda T_a^2 R'} \tag{10}$$

where

ρ = density,
Q = heat of reaction/unit mass,
Z = pre-exponential rate factor,
E = activation energy,
R' = universal gas constant,

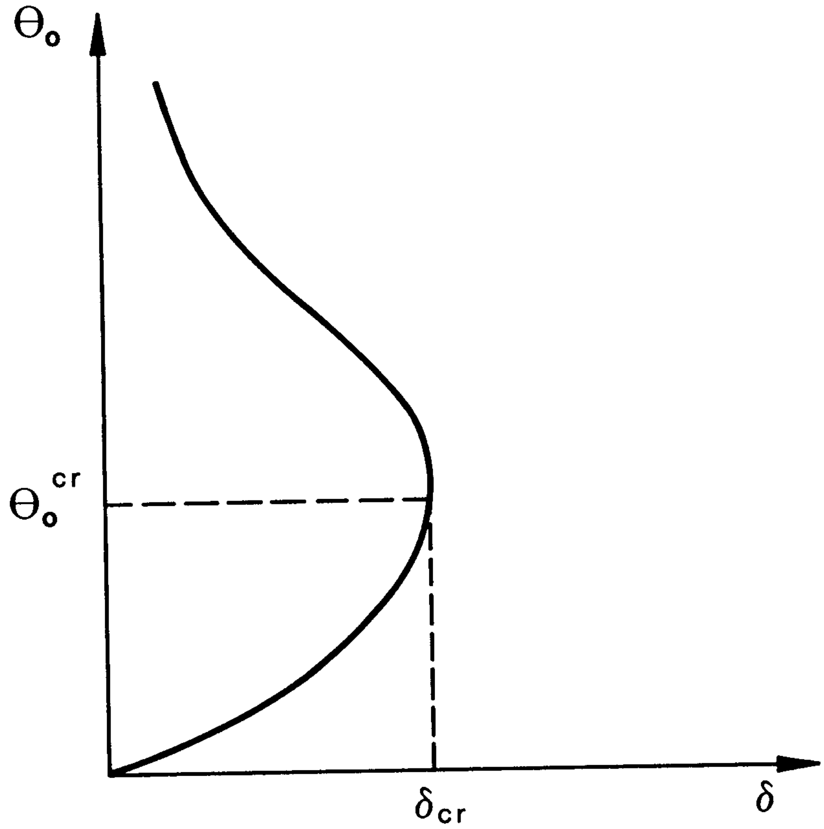

Figure 2: Schematic of the relationship between the heat production rate δ and the central temperature θ_0.

T_a = ambient temperature,
λ = thermal conductivity of dust,
ℓ = characteristic distance.

Experiments carried out at Macquarie University by B.F. Gray *et al.* using various dusts in an equicylinder of gauze of radius 7.5cm determined the critical ambient temperature for each. For an equicylinder, $\delta_{cr} = 2.78$. For ℓ, T_a and E known we thus have

$$\begin{aligned} 2.78 &= \frac{\rho Q Z}{\lambda} \frac{\ell_0^2 E}{R' T_a^2} e^{-E/R'T_a} \\ &= \chi \, \frac{\ell_0^2 E}{R' T_a^2} e^{-E/R'T_a}. \end{aligned}$$

So

$$\chi = 2.78 \frac{RT_a^2}{\ell_0^2 E} . e^{E/R'T_a} \quad (11)$$

enabling effective experimental measurement of the group χ. Equation (10) thus becomes

$$\delta = \frac{\chi \ell^2 E e^{-E/R'T_a}}{R'T_a^2} \quad (12)$$

where ℓ and T_a are now appropriate for the configuration in question, *i.e.* an annulus at an ambient temperature of 30° C or 40° C.

As an example, let us consider bagasse or sawdust, for which $E = 125$ kJ/mole and $T_a^{cr} = 470$ K for a 7.5cm radius equicylinder.

For this material, then $E/R' \sim 15000$ K, so equation (12) gives us, at criticality

$$2.78 = \frac{\chi (7.5)^2 \; 15000 \exp(-15000/470)}{(470)^2}$$

giving

$$\chi = 5.3 \; 10^{13} \mathrm{Kcm}^{-2}$$

We substitute this figure in equation (12) and obtain a value of δ for a given thickness of dust on the cable; *e.g.* for a dust layer nine times thicker than the PVC insulation (still only 1.17cm radius) then $\delta = 1.7 \times 10^{-8}$. This is considerably less than the critical value for this case ($h = 9b$ in Table 1) of $\delta_{crit} = 6.0 \times 10^{-4}$. We can increase the dust layer until the calculated value of δ from equation (12) equals the critical value of δ computed from equations (5,6). In this case, this occurs around 15cm sawdust around the cable, an amount which could easily occur in practice.

For coal dust, where $E/R' \simeq 5000$, similar calculations give $\chi = 3.37 \times 10^5$ and $k' = -1.63$. The critical depth of dust in this case, for the same cable is of the same order of magnitude.

Even in the subcritical cases, however, where neither dust is close to ignition, we would expect the insulating effect of the dust to cause melting of the PVC insulation; *e.g.* for the cable carrying a current of 14 amps with a layer of dust nine times thicker than the PVC, the temperature at the conductor centre can easily be calculated to be around 300° C for an ambient temperature of 20° C. This assumes a thermal conductivity for the dust of 5×10^{-4} W.cm^{-1}.K^{-1} and that of the PVC to be 1.67×10^{-3} W.cm^{-1}.K^{-1}. With no dust, the corresponding calculation gives a temperature of 36° C above ambient for this case. With a dust layer of equal thickness to that of the PVC the temperature rise is 77° C.

Clearly, even for ignitable materials, well before ignition takes place, the insulating effect of the dust will cause the PVC to melt unless very considerable de-rating is done.

3. Possible Sources of Error

A possibly important source is the density of the dust layer. In the above calculations it is implicitly assumed to be the same as that of the samples tested for criticality at Macquarie University. The density achieved by natural settling over a period is unknown, but is likely to be less than that used in the experiments. If it were 10% of this, both χ and δ would be reduced by this factor.

We do not know how the conductivity of the dust layers depends on its density, and the above calculation assumes it is independent of density. The thermal conductivity of air is approximately 1.5×10^{-5} W.cm^{-1}.K^{-1} which is somewhat smaller than that for either sawdust or coal at the packing density used. Thus we might expect a very porous layer of dust or still air to make the system even less stable than calculated above.

In conclusion, the author of this report (Brian Gray) would like to acknowledge the contributions to it by Dr J. Dewynne, Mr M. Hood, Prof. G.C. Wake, and Dr R. Weber.

References

B.F. Gray & G.C. Wake, *Combustion and Flame* 55 (1984), 23-30.

A. Gomez, G.C. Wake & B.F. Gray, *Combustion and Flame* 61 (1985), 177-187.

h	9b	4b	$\frac{7}{3}b$	$\frac{3}{2}b$	b	$\frac{2}{3}b$	$\frac{3}{7}b$	$\frac{1}{4}b$	$\frac{1}{9}b$
k' = 0 k" = 1	1.4 2.1	1.3 2.3	1.3 2.7	1.3 3.4	1.2 4.5	1.2 6.6	1.2 11	1.2 24	1.2 92
k' = 2	6.4 0.17	5.1 0.34	4.3 0.61	3.7 1.1	3.3 1.8	2.9 3.4	2.6 6.9	2.3 18	2.1 79
k' = 5	13 6.0×10^{-4}	9.8 7.5×10^{-3}	7.9 3.7×10^{-2}	6.3 0.13	5.3 0.39	4.4 1.1	3.6 3.2	3.0 11	2.4 63
k' = 10	25 1.4×10^{-8}	18 5.3×10^{-6}	14 1.9×10^{-4}	11 2.6×10^{-3}	8.7 2.2×10^{-2}	6.9 0.14	5.4 0.79	4.0 4.7	2.9 43
k' = 15	36 2.2×10^{-13}	26 2.6×10^{-9}	20 7×10^{-7}	15 4×10^{-5}	12 1×10^{-3}	9.4 1.5×10^{-2}	7.1 0.18	5.2 2.0	3.4 29
k' = 20	48 3.0×10^{-18}	34 1.1×10^{-12}	26 2.3×10^{-9}	20 5.4×10^{-7}	16 4.1×10^{-5}	12 1.6×10^{-3}	8.9 3.8×10^{-2}	6.2 0.77	4.0 19
k' = 25	59 3.7×10^{-23}	42 4.6×10^{-16}	32 7.0×10^{-12}	25 7.0×10^{-9}	19 1.6×10^{-6}	14 1.5×10^{-4}	11 7.8×10^{-3}	7.4 0.30	4.5 13
k' = 30	71 4.6×10^{-28}	50 1.8×10^{-19}	38 2.0×10^{-14}	29 8.6×10^{-11}	23 6.0×10^{-8}	17 1.4×10^{-5}	12 1.5×10^{-3}	8.5 0.11	5.0 8.4
k' = 35	82 5.4×10^{-33}	58 6.7×10^{-23}	44 5.8×10^{-17}	34 1.0×10^{-12}	26 2.2×10^{-9}	20 1.2×10^{-6}	14 3×10^{-4}	9.6 4.2×10^{-2}	5.5 5.5
k' = 40		66 2.4×10^{-26}	50 1.6×10^{-19}	38 1.2×10^{-14}	29 7.8×10^{-11}	22 1.1×10^{-7}	16 5.7×10^{-5}	11 1.6×10^{-2}	6.0 3.5

Table 1 Critical values of $\theta_\circ^{cr}$ (first line) and δ^{cr} (second line) for various values of the parameter k'