DROPLET COOLING OF GALVANISED IRON

1. Introduction

This problem was presented to the 1989 Mathematics-in-Industry Study Group by the Research and Technology Centre, BHP Coated Products Division. The problem is concerned with the solidification of a layer of molten zinc on a steel substrate. The crucial issues which need to be investigated are the formation of 'spangles' on the zinc layer, and how the size of spangles are influenced by spraying a fine mist of high speed water droplets at the layer of zinc. Under present operations, the zinc layer cools in air after it is passed through the bath of molten zinc and the air jet strippers. It is proposed to cool the zinc layer by a bank of nozzles which spray a cloud of microscopic water droplets at high speed towards the surface.

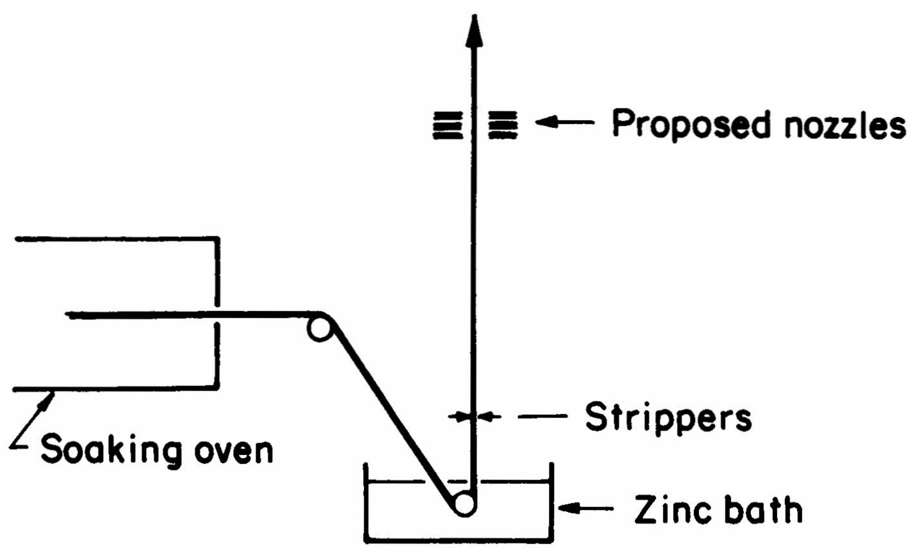

Figure 1: A sketch of the proposed process line

The proposed physical situation is sketched in Figure 1. The following values may be taken as representative of the production of galvanised iron by water cooling:

speed of strip	1.5 $m.s^{-1}$
thickness of strip	10^{-3} m
thickness of zinc layer	10 $\times 10^{-6}$ m
temperature of zinc bath	450° C
fusion temperature of pure zinc	420° C
speed of water droplets at nozzles	200 $m.s^{-1}$
diameter of water droplets	50 $\times 10^{-6}$ m
distance of nozzles from zinc	0.1 m
thickness of spray region	0.1 m

Under normal manufacturing conditions, the layer of molten zinc on the steel strip solidifies about 3 m above the zinc bath. For about the last 0.5 m below the point where solidification is complete, the zinc surface takes on a dull finish associated with the breakthrough of zinc crystals to the surface. The vertical region where solidification takes place is called the mushy region in this Report. The zinc solidifies from the inside to the outside leaving a pronounced pattern called 'spangles' on the zinc. The spangles are regions where solidification has taken place from a single nucleation site. Typically, the spangles have a diameter of about 5×10^{-5} m or less, and at the boundary between the spangles can be found a very small valley. The small variations in coating thickness from spangle to spangle and between spangles is called spangle relief. Spangle relief does not cause a problem for many applications of galvanised iron, but can be unwanted for some applications (such as car panels) where a very smooth surface is required.

The Coated Products Division of BHP is therefore interested in techniques which would produce smaller spangles and a smoother surface. It has been found that spraying water onto the surface can achieve a spangle size of about 10^{-3} m; even smaller spangles with a diameter of about 10^{-4} m can be obtained when a proprietary nucleation agent is mixed in with the water.

In developmental experiments, it has been found by BHP that if the mushy zone evident on the surface of the zinc is significantly *above* the nozzle region, then the resultant surface takes on a frosty appearance, as if there were numerous small pin pricks on the zinc surface. If the visible mushy zone corresponds approximately to the top of the mist spray region, the zinc layer surface was found to be smooth. Conversely, if the zinc was allowed to solidify before mist spraying then there was a change in spangle size or smoothness. This often occurs at both edges of the strip where cooling is greater. Other experiments also revealed that blocked nozzles could cause big drops to hit the zinc layer surface leaving occasional impact 'craters' on the surface. It is noteworthy that, for some applications (*e.g.* non-skid surfaces), it may be desirable to have a frosty or pitted surface. The crucial issue for the mathematicians therefore was to obtain a better understanding of the whole process.

A thorough investigation of this industrial process involves many features including the problem of solidification of the zinc layer, surface tension effects, wave propagation on a highly viscous thin layer, and the impact of high speed droplets with mushy zinc. A full treatment of all these effects during the Study Group was impossible, although some consideration was given to all of them. A good way to examine and classify the importance of these effects was to look at the time scales on which the various processes operate. These effects and their associated time scales are now examined.

2. Heat transfer calculations

It is of interest to investigate the cooling of the galvanised iron under normal conditions without the water spray operating. In particular, we shall use two simple calculations to estimate the vertical extent of the zone where solidification of the molten zinc takes place. Before doing so, the physical behaviour of a solidifying material is briefly reviewed.

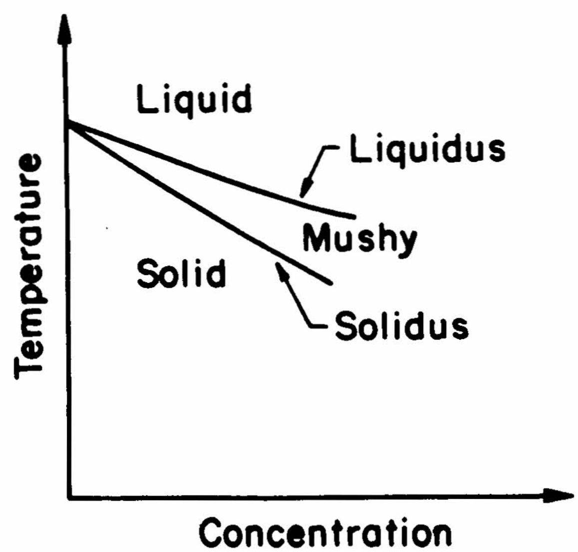

Figure 2: Typical solidification behaviour in the presence of impurities

For most of this work, it has been assumed that the molten zinc is pure. This is not likely to be the case. The solidification of a fluid containing impurities is a delicate phenomenon which is still imperfectly understood. In general, the presence of impurities leads to the situation depicted in Figure 2 in which solidification at a certain concentration of contaminant starts at the *liquidus* temperature and is completed at the *solidus* temperature. Impurities generally lower the solidus and liquidus temperatures below that of the pure material. There is another significant complication in that the impurities are transported by diffusion in the liquid ahead of the solidification front. Thus, solidification in the presence of impurities involves a consideration of diffusion processes for both heat and impurities, as well as the moving boundary aspects of the solidification front. It is also possible for liquid to be *supercooled*, or remaining in liquid state below the liquidus temperature.

The solidification front is typically unstable when the liquid is supercooled, and consequently the front forms a dendritic structure. The dendrites and the molten material trapped between them form what is commonly called a "mushy zone" which is neither completely solid nor completely liquid. A good review of modelling work on mushy zones has been given by Fowler (1987).

For the case of a very thin layer of zinc on steel strip, solidification usually is initiated at a nucleation site at the interface of the zinc and steel. Thereafter, crystals of solid zinc grow away from the nucleation site in all directions and eventually either collide with crystals from another nucleation site or reach the free surface. This is called "equiaxed growth". Each nucleation site produces a spangle. The evidence for the inside-to-out growth comes from experimental studies described by the BHP representative. If a solidification boundary moves from the zinc-steel interface to the free surface, then it follows that the still-to-be-solidified zinc is supercooled and presumably contains impurities. A basic description of equiaxed crystal growth has been given by Kurz & Fisher (1984).

We now turn to some simple estimates for the vertical extent of the solidification zone.

Constant heat transfer

During the time interval τ_1, suppose there is constant heat transfer Q per unit area at the free surface, and that the temperature is reduced from the bath temperature T_0 to the fusion temperature T_f. If the contribution of the zinc layer to the sensible heat is neglected and if thermal diffusion throughout the steel is so fast that the temperature T at a vertical location is almost independent of depth in the strip, then a simple heat balance gives

$$\frac{dT}{dt} = \frac{Q}{\rho_s c_s \delta_s}$$

[The notation which is used is given at the end of this report.]

When the strip has reached the fusion temperature T_f of pure zinc, we therefore have the estimate

$$\tau_1 = (T_f - T_0)\rho_s c_s \delta_s / Q. \tag{1}$$

Suppose that solidification of the zinc layer takes place during the time interval τ_2. This gives the simple estimate

$$\tau_2 = \rho_z \delta_z L_z / Q. \tag{2}$$

It follows that

$$\tau_1 + \tau_2 = [(T_f - T_0)\rho_s c_s \delta_s + \delta_z \rho_z L_z]/Q \tag{3}$$

and, inserting the values $\tau_1 + \tau_2 = 2$ s and the material properties given in the Appendix, it follows that

$$Q = 8.7 \times 10^4 \text{ Wm}^{-2}, \tag{4}$$

$$\tau_1 = 1.25 \text{ s}, \tag{5}$$

$$\tau_2 = 0.75 \text{ s}. \tag{6}$$

The above calculation indicates that, if the strip speed is 1.5 m.s^{-1}, the vertical extent of the zone where solidification takes place is about 1.1 m. This calculation is almost certainly an underestimate, since it is likely that more efficient heat transfer will take place near the bottom of the vertical strip, thereby leading to a smaller zone for sensible heat loss and a longer solidification zone.

Newton's law of cooling

This hypothesis just raised can be tested using a slightly more realistic heat transfer boundary condition at the free surface of the zinc. Specifically, the boundary condition at the edge of the zinc is taken to be Newton's law of cooling

$$k_z \frac{dT}{dy} = -H(T - T_a)$$

where y is a co-ordinate measured outwards from the steel-zinc interface. A simple heat balance for the stage when sensible heat is removed gives

$$\rho_s c_s \delta_s \frac{dT}{dt} = -H(T - T_a)$$

which may be integrated to give

$$T - T_a = (T_0 - T_a)e^{-Ht/(\rho_s c_s \delta_s)}.$$

The time interval τ_1 for the strip to reach the fusion temperature T_f is now

$$\tau_1 = -\frac{\rho_s c_s \delta_s}{H} \log[(T_f - T_a)/(T_0 - T_a)]. \tag{7}$$

The calculation to give the length of the zone where solidification takes place requires more care. We assume that the movement of the front is governed by approximately steady state diffusion processes (see Hill & Dewynne, 1987, section 7.3 for details) and write down the following system of equations for the moving boundary problem:

$$\begin{aligned} \frac{d^2T}{dy^2} &= 0, \quad Y(t) < y < \delta_z, \\ k_z \frac{dT}{dy}(\delta_z, t) &= -H(T - T_a), \\ T(Y(t), t) &= T_f, \\ -k_z \frac{dT}{dy}(Y(t), t) &= \rho_z L_z \frac{dY}{dt}. \end{aligned}$$

The solution satisfying the conditions at the strip edge δ_z and the moving front $Y(t)$ is

$$T(y, t) = A(t) + B(t)y$$

where

$$\begin{aligned} A(t) &= [HT_aY - (k_z + H\delta_z)T_f]/(HY - (k_z + H\delta_z)) \\ B(t) &= H(T_f - T_a)/(HY - (k_z + H\delta_z)) \end{aligned}$$

whilst the moving boundary condition gives the equation

$$-k_z B = \rho_z L_z \frac{dY}{dt}$$

which may be integrated to give

$$0.5\ \rho_z L_z Y[HY - 2(k_z + H\delta_z)] = -k_z H(T_f - T_a)t.$$

Finally, the solidification front will have moved all the way through the zinc layer after an interval τ_2 when $Y(\tau_2) = \delta_z$, and this gives

$$\tau_2 = \frac{\rho_z L_z \delta_z^2}{2k_z(T_f - T_a)}(1 + \frac{2k_z}{H\delta_z}). \tag{8}$$

Again, it is known that $\tau_1 + \tau_2 = 2$ s. We assume that $2k_z/H\delta_z >> 1$ and insert values from the Appendix for the other constants to obtain

$$H = 790\ \mathrm{W/(^oCm^2)},$$

$$\tau_1 = 0.35 \text{ s},$$
$$\tau_2 = 1.65 \text{ s}.$$

It may be now confirmed that $2k_z/H\delta_z >> 1$.

The results of this calculation also confirm that the zone where solidification takes place is surprisingly large indeed if a slightly more realistic cooling law is used. Basically, therefore, solidification takes place shortly after the zinc emerges from the bath, and it is not until the last part of the process that the crystals of zinc growing from the inside to the out are evident to the eye.

3. Surface tension effects

The enhanced solidification processes discussed in this Report also involve very fine granules of a proprietary nucleation agent. Although the exact nature of this agent was not disclosed at the MISG, there is a distinct possibility that the granules could cause important surface tension effects. It would be possible for granules to dissolve and spread across the surface almost instantly, thus causing a significant reduction in the surface tension around the granule. The consequence would be a reduction in the depth of the molten zinc layer in the region surrounding the granule. A two dimensional calculation is now presented to show how quickly surface tension could cause an appreciable reduction in the depth of the layer.

Consider the situation sketched in Figure 3 where the nucleation agent has caused the surface tension in a region of width $2a$ to be $\sigma_0/2$ in comparison to the undisturbed value of σ_0. A velocity profile in the y direction will develop as a result of the surface tension; the time for this profile to develop fully into Couette flow is of the order of

$$\tau_3 = \delta_z^2/(2\nu_z) \approx 100 \times 10^{-6} \text{ s}$$

The vorticity equation in the zinc layer is

$$\frac{\partial \omega}{\partial t} = \nu_z \frac{\partial^2 \omega}{\partial y'^2}$$

which has solution

$$\omega = \omega_0 \operatorname{erfc}(y'/2\sqrt{\nu_z t})$$

provided $t << \tau_3$. Here, ω_0 is determined by its value at the free surface as follows:

$$\omega_0 = -\frac{du}{dy} = \Sigma/\mu_z \approx \frac{\sigma_0}{2a\mu_z} \qquad (9)$$

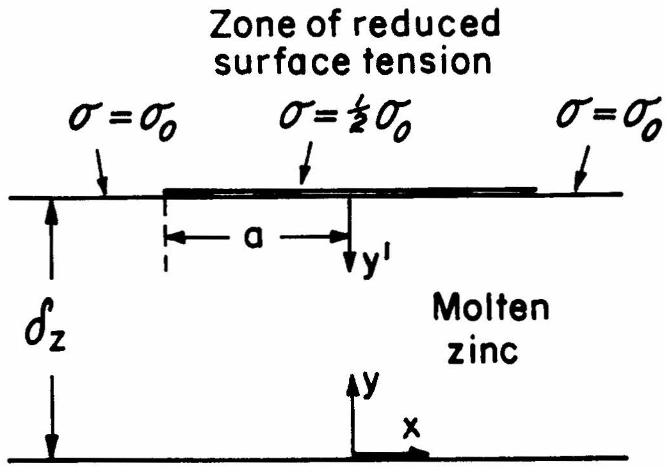

Figure 3: The physical situation investigated in Section 3

where Σ is the shear stress at the surface and the reduced value $\sigma_0/2$ of the surface tension applies in a region of width a.

The speed of the molten zinc as a response to the surface tension is therefore

$$u = 2\omega_0\sqrt{\nu_z t}\ \mathrm{ierfc}(y'/2\sqrt{\nu_z t});$$

and the volume flux out of a control surface of length $2a$ surrounding the original site of the granule is

$$2\int_0^{\delta_z} u\, dy' = 4\omega_0\nu_z t\ \mathrm{i}^2\mathrm{erfc}(0).$$

Here, $\mathrm{i}^n\mathrm{erfc}$ is the nth integral of the complementary error function and $\mathrm{i}^2\mathrm{erfc}(0) = 1/4$.

Hence, if h denotes the depth of the molten zinc, it follows that

$$\text{rate of change of volume} = 2a\frac{dh}{dt} \approx \omega_0\nu t = -\frac{\sigma_0\nu_z t}{2a\mu_z}$$

where equation (9) has been used for ω_0.

This equation can be simplified and integrated to give

$$h = h_0 - \frac{\sigma_0 t^2}{8a^2\rho_z},$$

from which it follows that the time required for the zinc layer to halve its thickness under the action of surface tension is

$$\tau_4 = 2a(\rho_z h_0/\sigma_0)^{1/2}. \tag{10}$$

This produces the typical value for τ_4 of 61×10^{-6} s when a is 100×10^{-6} m and the other physical values are as given in the Appendix.

The calculation produced in this Section indicates that surface tension might have dramatic consequences in the extremely thin layers of molten zinc under consideration. In practice, drawing down of the surface by reduced surface tension would be affected by the growth of the solid crystals of zinc from below. To conclude, we note that it would be worthwhile for BHP to carry out a close examination of galvanised iron cooled with drops and the nucleation agent. This would determine whether the granules of the agent remain undissolved on the surface of the galvanised iron. If so, it might be worthwhile to improve the model calculation produced in this Section by considering such aspects as the time dependent spreading of the nucleation agent on the surface and the 3 dimensional axisymmetric nature of the induced fluid velocity.

4. Waves on a shallow viscous fluid

In this Section, we consider another illustrative calculation which might be considered a precursor to full numerical solutions discussed in the next Section. Specifically, suppose that an impact by a water droplet has caused the surface of the zinc to deform, and it is desired to find out how long it would take for the surface to return to its original condition neglecting any thermal effects. The required theory is that of wave propagation in a very shallow viscous fluid with surface tension effects included. As a general comment, we mention that shallow water wave theory is a highly developed field, although most attention has been devoted to inviscid fluids without surface tension. Non-linear effects have been widely studied, for example, in the Korteweg - de Vries equation for which Whitham (1974) and others have given comprehensive details. The effects of viscosity and surface tension have not been as widely studied. Lighthill (1978, section 3.5) reviews results for viscous dissipation, but does not present work which is applicable to our problem of waves on a very shallow viscous liquid.

Consider the situation shown in Figure 4 in which the surface of the molten zinc is initially in a non-equilibrium state, and we seek to find out how long it would take to recover the equilibrium position if surface activity is ignored. The starting point is the set of equations

$$\rho_z[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}] = -\frac{\partial p}{\partial x} + \mu_z\frac{\partial^2 u}{\partial y^2}, \tag{11}$$

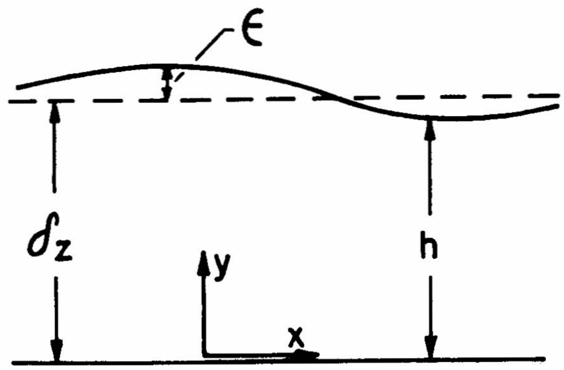

Figure 4: Definition sketch for waves on a shallow fluid

$$0 = \frac{\partial p}{\partial y}, \tag{12}$$

for viscous flow in a shallow region, together with the surface tension condition

$$p = -\frac{\sigma}{R} \approx -\sigma \frac{\partial^2 h}{\partial x^2}, \tag{13}$$

the continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \int_0^h u \, dy = 0 \tag{14}$$

and the approximate boundary conditions

$$u(x, 0, t) = 0, \quad \partial u / \partial y(x, h, t) = 0.$$

In general, these equations require a numerical solution, but some some simplification is possible in certain cases as is now shown. A solution satisfying the boundary conditions is sought in the form

$$u = W(t) \, \frac{y}{h}(\frac{y}{h} - 2).$$

This enables the integral in the continuity equation to be evaluated. Further, if the acceleration terms in equation (11) are neglected [see the end of this section for discussion of this point], the governing equations become

$$0 = \sigma \frac{\partial^3 h}{\partial x^3} + \frac{2\mu_z W}{h^2}, \tag{15}$$

$$\frac{\partial h}{\partial t} = \frac{2}{3}\frac{\partial}{\partial x}(Wh). \tag{16}$$

The term W can now be eliminated from (15,16) to give

$$-\frac{\partial h}{dt} = \frac{\sigma}{3\mu_z}(3h^2\frac{\partial h}{\partial x}\frac{\partial^3 h}{\partial x^3} + h^3\frac{\partial^4 h}{\partial x^4}). \tag{17}$$

This equation also requires a numerical solution in general, but further simplification is possible when the wave amplitude ϵ is small in comparison with the depth h. In this case, the ratio of the terms on the RHS is ϵ/h, so that (17) reduces to the governing equation for the shallow viscous capillary waves

$$-\frac{\partial h}{dt} = \frac{\sigma\delta_z^3}{3\mu_z}\frac{\partial^4 h}{\partial x^4} \tag{18}$$

where the term h^3 has been approximated by the mean term δ_z^3.

Equation (18), which represents a gross simplification, allows a simple solution which gives some insight. It is straightforward to show that the damping rate of the Fourier coefficient in

$$h = \delta_z + \epsilon \cos kx \, e^{-\gamma t}$$

is

$$\gamma = \sigma\delta_z^3 k^4/3\mu_z. \tag{19}$$

Thus, in this approximation, the viscous waves are standing waves which are very highly damped at large k (corresponding to small wavelength λ).

The numerical values given in the Appendix may be used to calculate the relaxation timescale $1/\gamma$ in the following table:

λ	$1/\gamma$
10^{-3} m	8.2×10^{-3} s
5×10^{-4} m	5.1×10^{-5} s
10^{-4} m	8.2×10^{-7} s

Thus the time required for these small wavelength waves (say of wavelength slightly smaller than 5 $\times 10^{-4}$ m) to decay is much faster than the thermal processes associated with the solidification of the zinc in the absence of water drops (Section 2), and approximately the same as that for the surface tension effects described in Section 3. If surface activity is important, the relaxation timescale will be longer. This is a complicated problem which has had some attention for wavelengths much less than the depth (Hansen & Ahmad, 1971), but apparently not for our case of wavelength greater than the depth.

We must note however the limitations at small wavelengths of the theory in this Section. If $1/\gamma$ is taken as an appropriate timescale, the acceleration terms in (11) are of order $\rho_z W \gamma$ compared to the order $\mu_z W/\delta_z^2$ of the viscous term. Acceleration terms should therefore be included when γ is of ν_z/δ_z^2, or for timescales less than about 2×10^{-4} s. Thus the last two lines of results in the above table are not strictly applicable because of the neglected acceleration terms. We also note that detailed numerical work would be required if equations (15,16) need to be solved, or even if equation (17) were to be solved.

5. Drop and particle impact studies

The problem discussed in this Report is special in that it involves a combination of impact of fluid drops onto a fluid surface *and* solidification. The scales of the problem are also rather special, involving a consideration of high speed microscopic drops onto a very shallow fluid layer.

There is quite a large literature on drop and particle impact studies, and we discuss some of this literature briefly under the heading of experimental work and simple mathematical models, and computational work.

Experimental work and simple mathematical models

The first extensive description was given by Worthington (1963). Some other papers which have dealt with impact studies include: Edgerton & Killian (1954) (photography), Sahay (1944) (breaking up of water drops falling into oil), Hobbs & Kezweeny (1967) (breaking up of rebounding splash), Levin & Hobbs (1971) (splashing onto solids and liquids), and Macklin & Metaxas (1976) (splashing onto liquid layers).

Special mention is made of a paper by Araki & Moriyama (1981) which deals with the deformation behaviour of a liquid droplet impinging onto a hot metal surface. The concern of their paper is heat transfer, specifically how do droplets spread out over a hot surface after impact, and what are the implications for cooling the surface? Araki & Moriyama provide a semi-empirical mathematical model for the impact process, and confirm the utility of the model for drops which do not break up (or equivalently for which the Weber number of the drop is not too large).

In general, however, these experimental studies are not directly applicable to the problem which is central to this Report.

Computational work

Many computational studies have been made of the impact of drops on either solid or liquid surfaces. Some remarkable computations were done over 20 years ago using the Marker-In-Cell method. Thus Harlow & Shannon (1967) studied the impact of a drop onto a layer of fluid. In most of their work, the fluids were the same and they neglected viscosity, gravity and surface tension, although these effects can be included into the MIC method as shown by Daly (1969). More recently, Nishikawa, Amatatu & Suzuki (1988) looked at very high speed impacts with solid surfaces, and Monaghan (1986) studied impacts of asteroids with the earth.

The methodology used by Monaghan deserves special mention. It is called Smoothed Particle Hydrodynamics, and the essence is to regard the incident drop or particle as composed of individual particles which interact with near neighbours. The method relies on a knowledge of the equation of state for the material, and a smoothing process to give quantities such as fluid density from the particle distribution. The method appears to be highly suited for free surface problems such as discussed in this report, although work would be required to handle the solidification of the molten zinc as caused by the water droplets. Further details and use of the method can be found in a recent paper by Monaghan (1988).

Bubbles

Finally, in this Section, we mention that experimental and computational studies of bubbles might have some relevance to the mist spray impingement problem under consideration. A good review on this work on bubbles has been given by Blake & Gibson (1987).

6. Discussion

The preceding 4 Sections describe the bulk of our deliberations on this problem at the MISG. Some attention was also given to the properties of the cloud of water droplets, particularly to the speed at which the droplets hit the wall. A simple calculation for this is now presented.

The equation of motion for a droplet is

$$m\frac{dv}{dt} = -C_D\frac{1}{2}v^2A\rho$$

where C_D is the drag coefficient, A the cross-sectional area of the droplet, m its mass, v its speed and ρ the density of air. If the initial speed is V_0, the speed at any subsequent time is given by

$$v = \frac{V_0}{1 + A\rho C_D V_0 t/2m}$$

from which it may be estimated (using the data given in Section 1) that the droplets have their speed reduced to about 1/3 of their initial speed by the time they reach the zinc layer. Thus the droplets still have a substantial speed (of greater than 50 ms^{-1}) on impact with the molten zinc.

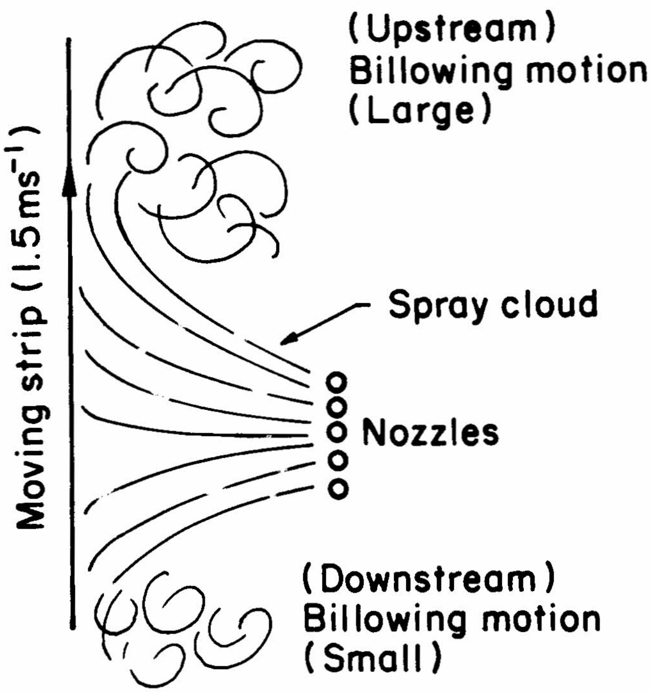

Figure 5: The behaviour of the cloud of water droplets

A characteristic feature of the cloud of droplets is that it is *not* unidirectional; rather there is a billowing motion, with some of the water droplets moving 'downstream' against the motion of the galvanised iron, and most of the water droplets moving 'upstream'. The shape of the spray is sketched in Figure 5. The droplets which move downstream might be particularly significant. These droplets could be expected to provide a mechanism for solidification of the outer surface of the thin zinc layer, whilst at the same time, they would not necessarily have substantial momentum normal to the wall compared to that momentum possessed by droplets in the centre of the cloud. Such an observation might explain an intriguing feature of the trials carried out by BHP; namely that when the bank of nozzles was near the point where solidification had finished, the surface of the zinc that was produced was smooth. Conversely, when the nozzles were directed lower where more of the zinc coating was molten, then the resultant surface took on a frosty or pitted nature.

This downstream billowing motion of droplets might have the effect of causing surface solidification to move completely down to the zinc crystals growing from the zinc-steel interface. Remelting of the solid zinc would not occur as all of the zinc layer is reckoned to be approximately at the fusion temperature.

These ideas could be confirmed by close observation of galvanised iron cooled by water droplets. It might be expected that impurities would be driven ahead of the solidification fronts, and that consequently there would be a zone below the surface of the zinc where the concentration of impurities would be higher than elsewhere. This would be evidence of cooling from the inside to the outside *and* from outside to the inside.

If the bank of nozzles is too low, then presumably insufficient cooling is caused by the droplets in the downstream billowing motion, so that the droplets in the bulk of the nozzle zone punch a hole in the molten zinc, then leave behind a tiny pin prick. Any waves which move outwards from any individual droplet would be strongly damped according to the theory of Section 4.

Further information on this process requires detailed study of impact studies involving freezing. These studies would require considerable skill and labour, but an attack using the Smoothed Particle Hydrodynamics code would seem to have a reasonable chance of success.

Acknowledgements

The moderator for this problem (Noel Barton) would like to acknowledge the assistance of the BHP representative, Cat Tu, and the contributions and enthusiasm of the following people: Rodney Carr, Paul Cleary, John Harper, David Jenkins, Mark McGuinness, and Chee Ng.

Appendix: nomenclature, physical data

Nomenclature

a	diameter of region affected by drop (m)
A, B	functions of time (°C, °C m^{-1})
c	specific heat (Jkg^{-1} °C^{-1})
C_D	drag coefficient
h	depth of molten zinc (m)
H	heat transfer coefficient ($Wm^{-2o}C^{-1}$)
k	thermal conductivity of heat ($Wm^{-1o}C^{-1}$)
k	wave number (m^{-1})
L	latent heat of fusion (Jkg^{-1})
p	pressure (Pa)
Q	heat transfer coefficient (Wm^{-2})
R	radius of curvature of surface (m)
t	time (s)
T	temperature (°C)
u, v	velocity components (ms^{-1})
W	velocity component (ms^{-1})
x	coordinate parallel to the zinc layer (m)
y, y'	coordinates normal to the zinc layer (m)
Y	position of solidification front (m)

γ	damping rate (s^{-1})
δ	thickness (m)
ϵ	wave amplitude (m)
μ	viscosity ($kgm^{-1}s^{-1}$)
ν	kinematic viscosity (m^2s^{-1})
ρ	density (kgm^{-3})
σ	surface tension (Nm^{-1})
Σ	shear stress (Nm^{-2})
τ	time interval (s)
ω	vorticity (s^{-1})

Subscripts

s steel
z zinc

Physical values

c_s	450 $\mathrm{Jkg^{-1} {}^oC^{-1}}$
k_z	100 $\mathrm{Wm^{-1} {}^oC^{-1}}$
L_z	10^6 $\mathrm{Jkg^{-1}}$
μ_z	3×10^{-3} $\mathrm{kgm^{-1}s^{-1}}$
ν_z	0.46×10^{-6} $\mathrm{m^2\ s^{-1}}$
ρ_s	8×10^3 $\mathrm{kg\ m^{-3}}$
ρ_z	6.5×10^3 $\mathrm{kg\ m^{-3}}$
σ	0.7 $\mathrm{Nm^{-1}}$

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