LONG TERM FOREIGN CURRENCY EXCHANGE RATE PREDICTIONS

1. Introduction

The motivation of this work is to predict foreign currency exchange rates between countries using the long term economic performance of the respective economies. The predictions are long term - approximately one year in advance - and it is recognised that short term predictions (on a daily basis or shorter) require different approaches to those presented here.

The problem was presented to the Study Group by World Values Ltd. This company has provided advice for many years on foreign currency exchange rates using a model entitled "the currency evaluator", the derivation of which reflects the extensive experience of the principals of the company. In brief, points are awarded to each country in respect of its economic performance in eight key economic properties. The points are summed for each country over all the properties, and a comparative ratings or performance table is constructed. Minor adjustments are made if necessary, to this table in the light of practical experience – for example, if a country *always* scores well with respect to certain economic properties, then the effect of those properties might be discounted somewhat in predictions. The ratings table is then used to make predictions for exchange rates one year ahead. Some details of this process have been published by World Values Ltd (1989).

The World Values model is not optimized in the usual mathematical sense in which parameters are determined by fitting predictions to data. Rather, the model has been tuned according to practical experience. The World Values model is successful, but has room for improvement. The opinion of the delegates at the Study Group was that it was more important for them to suggest alternative mathematically rigorous frameworks for foreign currency predictions than to suggest incremental improvements to the World Values model.

The eight key economic properties used by World Values Ltd to make long term foreign currency exchange rate predictions are described in Section 2. These properties have been identified through years of experience, rather than through rigorous economic modelling.

Prior to the Study Group, a small amount of computer modelling was undertaken in an attempt to improve the World Values forecasts. The results of this exercise, incorporating some suggestions made at the Study Group, are presented in Section 3. Under specific criteria which are described later, the preliminary results are superior to those achieved by the World Values model.

Our preliminary model possessed some shortcomings from the point of view of computational efficiency. Accordingly an improved model was suggested at the Study Group and is described in Section 4. This model is linear and is appropriate when we anticipate that the economic performance of a country has a more or less constant effect on the exchange rate. The parameters in the model are the same for all countries at all times. In our opinion, this linear model should be constructed first in any subsequent computer modelling.

In fact, however, it is reasonable to expect that economic performance over time has a variable effect on exchange rates. That is, stochastic properties should be considered. Accordingly, a more general model incorporating variability was developed at the Study Group and is described in Section 5. This model is based on the Kalman filtering approach and would be more elaborate to implement than the linear model described in Section 4.

The contentious issue of the short term effect versus long term effect of interest rates was discussed at the Study Group. These discussions were inconclusive. For completeness, however, we have included a discussion on this topic in Section 6. The discussion incorporates a practical suggestion to explore the short term importance of interest rates.

2. Notation and key economic properties

All models for foreign currency exchange rate prediction rely on a large number of properties and variables. The following notation has been adopted:

Indices

m, n	refer to countries $(m, n = 1, \ldots, 8)$
i	refers to time periods $(i = 1, \ldots, I)$. In general, we have used quarterly data and forecasts.
j	refers to economic properties $(j = 1, \ldots, J)$

Exchange rates

p^i_{mn}	$=$	predicted value of 1 unit of currency m in terms of currency n at time t_i
P^i_{mn}	$=$	$\log p^i_{mn}$
q^i_{mn}	$=$	actual (market) value of p^i_{mn}
Q^i_{mn}	$=$	$\log q^i_{mn}$

Economic properties - general notation

$$
\begin{aligned}
e_{mj}^{i*} &= \text{economic property } j \text{ of country } m \\
&\quad\ \text{at time } t_i \\
e_{mj}^{i} &= \text{dimensionless form of } e_{mj}^{i*} \\
&= \frac{1}{\sigma_j^{i*}}(e_{mj}^{i*} - \bar{e}_j^{\,i*}) \\
\text{where } \quad & \bar{e}_j^{i*} = \tfrac{1}{8}\textstyle\sum_{m=i}^{8} e_{mj}^{i*} \\
& \left(\sigma_j^{i*}\right)^2 = \tfrac{1}{8}\textstyle\sum_{m=1}^{8} \left(e_{mj}^{i*} - \bar{e}_{mj}^{i*}\right)^2
\end{aligned}
$$

These scaled (dimensionless) properties have the advantage that e_{mj}^{i}

- has zero mean for i, j fixed as m varies
- has a similar numerical range for all i, j as m varies

Actual economic properties

$$
\begin{aligned}
e_{m1}^{i*} &= \text{\% increase p.a. of c.p.i.} \\
e_{m2}^{i*} &= \text{\% increase p.a. of real money } (M_3) \\
e_{m3}^{i*} &= \text{long term interest rates} \\
e_{m4}^{i*} &= \text{\% increase p.a. of average wages} \\
e_{m5}^{i*} &= \text{current account balance as \% of GDP} \\
e_{m6}^{i*} &= \text{trade balance as \% of imports} \\
e_{m7}^{i*} &= \text{financial reserves: the number of months} \\
&\quad\ \text{of last months imports that financial reserves will buy} \\
e_{m8}^{i*} &= \text{\% increase p.a. of G.N.P.}
\end{aligned}
$$

Further descriptions of these eight properties are given by World Values Ltd (1989); and we note that preliminary smoothing procedures are sometimes applied before the properties are used in forecasts. For use in Section 6, we define

$$
\begin{aligned}
e_{m9}^{i*} &= \text{short term interest rates} \\
&\quad\ (\textit{e.g. } \text{90 day bank bill}).
\end{aligned}
$$

We also use the concept of 'value' of currencies, but only in a relative sense. Let

$$v_m^i = \text{'value' of currency } m \text{ at time } t_i$$

so that

$$p^i_{mn} = v^i_m / v^i_n$$

The following identities are ensured by short term market forces:

$$p^i_{mn} = 1/p^i_{nm} \tag{1}$$

$$p^i_{mn} = p^i_{mk} p^i_{kn} \tag{2}$$

3. Preliminary model

For our preliminary model, we hypothesize that the present value of a currency is related linearly to the eight dimensionless economic properties four quarters earlier:

$$v^{i+4}_m = v^i_m \{1 + \sum_{j=1}^{8} \alpha_j e^i_{mj}\} \tag{3}$$

The forecast period of four quarters is the same as in the World Values model. This leads to the predicted exchange rates

$$p^{i+4}_{mn} = p^i_{mn} \frac{1 + \sum_{j=1}^{8} \alpha_j e^i_{mj}}{1 + \sum_{j=1}^{8} \alpha_j e^i_{nj}} \tag{4}$$

In this model, the linearity inherent in the hypothesis (3) has been lost. Moreover, there are a number of parameters to be determined, namely eight coefficients α_j $(j = 1, \ldots, 8)$ and seven initial exchange rates p^o_{mn}(m fixed, $n = 1, \ldots, m-1, m+1, \ldots 8$). From these seven initial values, all other initial values can be determined using identities (1,2).

Minor modifications were made to the model to predict p^1_{mn}, p^2_{mn} and p^3_{mn}, given initial values p^0_{mn}. These modifications took the form (for k=1,2,3)

$$p^k_{mn} = p^0_{mn} \frac{\{1 + \sum_{j=1}^{8} \frac{k}{4} \alpha_j e^0_{mj}\}}{\{1 + \sum_{j=1}^{8} \frac{k}{4} \alpha_j e^0_{nj}\}} \tag{5}$$

The unknown parameters were chosen to minimize the least squares expression

$$E = \sum_{i=0,\ldots,I} \sum_{m,n \epsilon M} \frac{(q^i_{mn} - p^i_{mn})^2}{({q_{mn}}^i)^2} \tag{6}$$

where M is an index set accounting for currency cross rates

$$M = \{(m,n) \text{ such that } m = 1, \ldots, 7, n = m+1, \ldots, 8\}$$

The choice of weights in the least squares expression (6) should depend on the estimate of p^i_{mn} rather than q^i_{mn} because the presence of observational errors inside the weights leads to biased estimates and inflated variances. The choice of q^i_{mn} was made out of convenience, bearing in mind the time constraints of the MISG. In any subsequent investigation, the weights must be carefully chosen. The minimization algorithm used was the LMM (Levenberg, Marquadt and Morrison) algorithm for non-linear least squares. In this algorithm, derivatives of p^i_{mn} with respect to the parameters are required, although these details are not included here.

To use the preliminary model for predictions, results available at time t_I were used to predict values at t_{I+4}. This procedure was repeated for $I = 6, \ldots, 13$.

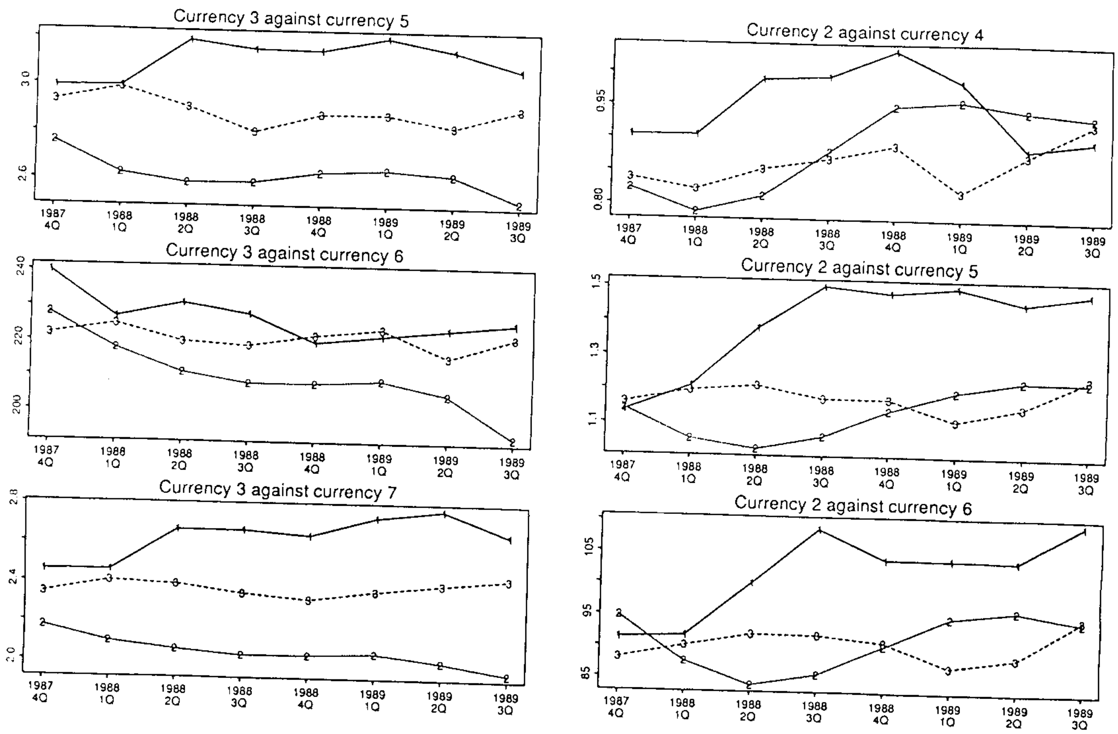

Figure 1: Illustrative one year forecasts of foreign currency exchange rates compared with actual values. Legend: 1 - actual exchange rates; 2 - World Values Ltd forecast; 3 - method of Section 3.

Figure 1 shows sample predictions of the preliminary model compared with actual exchange rates and the predictions of the World Values model. Objective comparisons between competing models were made using the mean square errors

$$\mathcal{E}_{mn} = \frac{1}{8} \sum_{I=10}^{17} (p_{mn}^{I} - q_{mn}^{I})^2 \tag{7}$$

which are tabulated below. In terms of this criterion, the predictions made using our preliminary model were superior to the World Values predictions on 25 out of the 28 cross rates.

mn	2	3	4	5	6	7	8
1	0.24638	0.03613	0.11343	0.06963	1021.53	0.03788	1.00271
2		0.00104	0.00971	0.08369	174.63	0.10041	0.68421
3			0.03847	0.25675	358.43	0.36115	1.89157
4				0.06042	149.37	0.08583	0.40526
5					55.31	0.00474	0.02421
6						0.000004	0.000009
7							0.29705

Table 1: Mean square error $\mathcal{E}_{mn}$ of the predicted foreign currency exchange rate by the World Values model.

mn	2	3	4	5	6	7	8
1	0.04516	0.00331	0.00624	0.03367	109.88	0.03097	0.38264
2		0.00255	0.01178	0.06695	159.80	0.05990	0.66391
3			0.03896	0.06645	76.71	0.07804	0.50907
4				0.03772	98.68	0.03571	0.37955
5					29.42	0.00058	0.00635
6						0.000001	0.000009
7							0.04188

Table 2: Mean square error $\mathcal{E}_{mn}$ of the predicted foreign currency exchange rate by the model described in Section 3.

Certain shortcomings of the preliminary model must be pointed out however. The model provides estimates for the parameters α_j and the initial values p_{mn}^0 at each time $t_I (I = 6, \ldots, 13)$. These parameters do not always converge with increasing I, as is shown, for example, in the results in figure 2 for α_8. Besides the choice of weights in the least squares expression (6), there is another factor which leads to biased estimates and inflated variances. Since the index set M includes cross-rates for currencies, there is the possibility of correlations in residuals if exchange rates do not reflect market values, for example, through government manipulation of certain currencies.

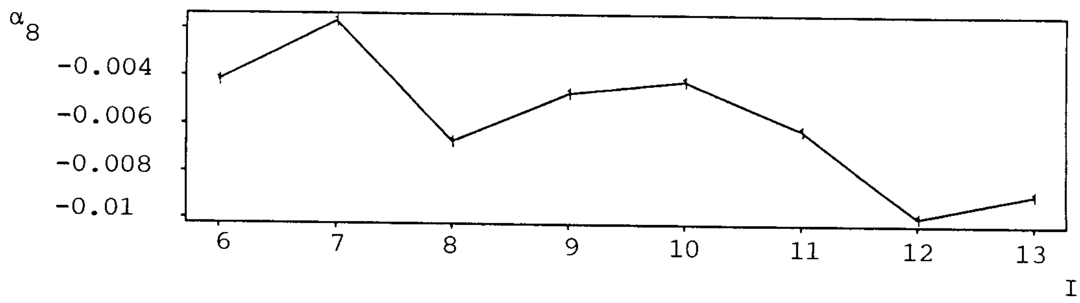

Figure 2: Estimation of the parameter α_8 for $I = 6, \ldots, 13$.

4. Improved linear model

The model described in the previous section has the limitations that parameters are determined by minimizing a least squares expression in which parameters occur non-linearly. Minimization therefore required the use of the powerful package LMM. In the present Section, an alternative version of the model is developed in which parameters enter linearly in the least squares problem, thereby permitting the use of simpler packages.

If we define $P^i_{mn} = \log p^i_{mn}$ and take logarithms in equation (2), we obtain

$$P^{i+4}_{mn} = P^i_{mn} + \log\{1 + \sum_{j=1}^{8} \alpha_j e^i_{mj}\} - \log\{1 + \sum_{j=1}^{8} \alpha_j e^i_{nj}\}$$

Now use the approximation

$$\log(1 + r) = r - \frac{r^2}{2} + \frac{r^3}{3} - \ldots$$

and retain only the first terms in $\log\{1 + \sum_{j=1}^{8} \alpha_j e^i_{mj}\}$. This gives

$$P^{i+4}_{mn} = P^i_{mn} + \sum_{j=1}^{8} \alpha_j (e^i_{mj} - e^i_{nj}), \tag{8}$$

which is an equivalent linear model to that in Section 3, but far simpler to implement since the parameters α_j $(j = 1, \ldots, 8)$ and the initial values P^0_{mn} enter linearly into P^i_{mn} in the least squares expression

$$E = \sum_{i=0,\ldots,I} \sum_{m,n \epsilon M} w^i_{mn} (Q^i_{mn} - P^i_{mn})^2 \tag{9}$$

in which $Q^i_{mn} = \log q^i_{mn}$. (Note that minor modifications as in equation (5) are required to predict $P^1_{mn}, P^2_{mn}, P^3_{mn}$.) Determination of the parameters α_j and the initial values P^0_{mn} requires the solution of a set of simultaneous linear equations for which simple standard packages are available.

The improved linear model could be used as follows:

- Use results available at time t_I to predict values at time t_{I+4} $(I = 6, \ldots, 13)$
- Compare the predictions with those of World Values Ltd and those of the preliminary model (Section 3)
- Investigate goodness of fit
- Investigate the behaviour of the parameters α_j and P^0_{mn} with respect to sign, statistical significance (through the t test), and variability as I is increased
- Investigate the sensitivity of the model to variations in the forecast period

It would be easier to undertake these investigations with the improved linear model than with the preliminary model (Section 3). The improved model will still lead to biased estimates for the reasons given in Section 3.

5. Kalman filtering approach

If the results of the investigation foreshadowed in the previous section showed that the parameters in the model appeared to change with time, then this indicates that a model which incorporates stochastic variability should be used. Such a model is provided by a Kalman filtering approach (Anderson & Moore, 1979).

We start by rewriting equation (8) in the form

$$P^{i+4}_{mn} = P^i_{mn} + \sum_{j=1}^{8} \alpha^i_j (e^i_{mj} - e^i_{nj}) + X^i_{mn} \qquad (10)$$

where the parameters α^i_j now vary with time and X^i_{mn} is assumed to be Gaussian white noise with zero mean and variance σ^2_x.

In order to use the standard Kalman filtering algorithm, we write for the state vector,

$$V^{iT}_{mn} = [P^i_{mn}\ P^{i-1}_{mn}\ P^{i-2}_{mn}\ P^{i-3}_{mn}\ \alpha^i_1\ \alpha^{i-1}_1\ \alpha^{i-2}_1\ \alpha^{i-3}_1\ \ldots$$

$$\ldots\ \alpha_8^i\ \alpha_8^{i-1}\ \alpha_8^{i-2}\ \alpha_8^{i-3}]$$

and introduce the transition matrix

$$F = \begin{bmatrix} A & B_1 & B_2 & \ldots & B_8 \\ O & A & O & \ldots & O \\ O & O & A & \ldots & O \\ . & . & . & \ldots & . \\ O & O & O & \ldots & A \end{bmatrix}$$

where A is the 4×4 matrix

$$A = \begin{bmatrix} O & O & O & 1 \\ 1 & O & O & O \\ O & 1 & O & O \\ O & O & 1 & O \end{bmatrix}$$

and B_j is the 4×4 matrix

$$B_j = \begin{bmatrix} O & O & O & e_{mj}^{i-4} - e_{nj}^{i-4} \\ O & O & O & O \\ O & O & O & O \\ O & O & O & O \end{bmatrix}$$

Standard results (Anderson & Moore, 1979) give the system equation

$$V_{mn}^{i+1} = FV_{mn}^i + \underline{X}_{mn}^i \tag{11}$$

where

$$\begin{aligned} \underline{X}_{mn}^{iT} &= [X_{mn}^{i-3} O\ O\ O\ \xi_1^{i-3} O\ O\ O\ \xi_2^{i-3} O\ O\ O\ \ldots \\ &\quad \ldots \xi_8^{i-3} O\ O\ O\ ] \end{aligned}$$

and the ξ_j^{i-3} are Gaussian white noise with mean zero and variance $\sigma_{\xi_j}^2$, so the covariance matrix B^i of $\underline{X}_{mn}^i$ is

$$\begin{bmatrix} \sigma_x^2 & O & O & \ldots & O \\ O & \sigma_{\xi_1}^2 & O & \ldots & O \\ O & O & \sigma_{\xi_2}^2 & \ldots & O \\ . & . & . & \ldots & . \\ O & O & O & \ldots & \sigma_{\xi_8}^2 \end{bmatrix}$$

The observation or measurement equation is

$$Q_{mn}^i = HV_{mn}^i + Z_{mn}^i \tag{12}$$

where

$$H = [1\ O\ O\ \ldots\ O]$$

The Kalman filtering algorithm proceeds as follows: start with the initial estimate

$$\begin{aligned} V_{mn}^{3T} &= [P_{mn}^{3}\ P_{mn}^{2}\ P_{mn}^{1}\ P_{mn}^{O}\ \alpha_1^3\ \dots\ \alpha_1^O\ \dots \\ &\quad \dots\ \alpha_8^3\ \dots\ \alpha_8^O] \end{aligned}$$

and the properties σ_x^2, $\sigma_{\xi_j}^2, j = 1, \dots, 8$, $\sigma_{Z_{mn}}^2$ and the covariance matrix R_{mn}^3 (of V_{mn}^3). The prediction stage is

$$V_{mn}^{i+1} = F^i\ V_{mn}^i \qquad i = 3, 4, \dots$$

which yields $\hat{V}_{mn}^{i+1|i}$. The prediction of the observation vector is $\hat{Q}_{mn}^{i+1|i}$

$$\hat{Q}_{mn}^{i+1|i} = H\ \hat{V}_{mn}^{i+1|i} + E(Z_{mn}^{i+1|i}).$$

Then calculate the covariance matrix $R_{mn}^{i+1|i}$ (of $V_{mn}^{i+1|i}$)

$$R_{mn}^{i+1|i} = F^i\ R_{mn}^{i|i}\ F^{iT} + B^i$$

where B^i is the covariance matrix of $\underline{X}_{mn}^i$. The correction stage is

$$\hat{V}_{mn}^{i+1|i+1} = \hat{V}_{mn}^{i+1|i} + K_{mn}^{i+1}(Q_{mn}^{i+1} - \hat{Q}_{mn}^{i+1|i})$$

in which K is the Kalman gain

$$K_{mn}^{i+1} = R_{mn}^{i+1|i}\ H^T(H\ R_{mn}^{i+1|i}\ H^T + \sigma_{Z_{mn}}^2)^{-1}$$

The update of the covariance matrix is

$$R_{mn}^{i+1|i+1} = R_{mn}^{i+1|i} - K_{mn}^{i+1}\ H\ R_{mn}^{i+1|i}$$

This prediction-correction procedure is to be repeated for all m, n in the index set M.

This model could be systematically investigated:

- This model is in a standard framework, and convergence would be guaranteed if F had constant coefficients which is, in fact, not the case here
- If the behaviour of the parameters α_j^i varies with time, then the Kalman filtering approach might give superior performance to the improved linear model of Section 4
- Comparisons of forecasts could be made, as before, through the mean square error $\mathcal{E}_{mn}$

- The performance of the Kalman filtering algorithm depends on σ_x^2, $\sigma_{\xi_j}^2, j = 1, \ldots, 8$, $\sigma_{Z_{mn}}^2$. These are "tuning" parameters and need to be chosen carefully. Literature to guide in the selection of these parameters is available (Anderson & Moore, 1979, Harvey & Peters, 1984) although expert assistance would be required by World Values Ltd if this model were to be implemented

- This model could also be used to investigate sensitivity to variations in the forecast period

- The model developed in this Section does not, however, take account of the constraints (1,2). To do this, the following straightforward modification is made. Rewrite equation (12) as

$$Q^i = HV^i + Z^i$$

where

$$Q^i = (Q_{12}^i, \ldots, Q_{18}^i, Q_{23}^i, \ldots, Q_{28}^i, \ldots, Q_{78}^i)^T$$

$H = h_{mn}$ is a 28 × 64 matrix with non-zero entries as follows:

$$1 \leq m \leq 7: \quad h_{mn} = 1 \ (n = 1), \ h_{mn} = -1 \ (n = 4m + 1)$$

$$8 \leq m \leq 13: \quad h_{mn} = 1 \ (n = 5), \ h_{mn} = -1 \ (n = 4(m - 7) + 5)$$

$$14 \leq m \leq 18: \quad h_{mn} = 1 \ (n = 9), \ h_{mn} = -1 \ (n = 4(m - 13) + 9)$$

$$19 \leq m \leq 22: \quad h_{mn} = 1 \ (n = 13), \ h_{mn} = -1 \ (n = 4(m - 18) + 13)$$

$$23 \leq m \leq 25: \quad h_{mn} = 1 \ (n = 17), \ h_{mn} = -1 \ (n = 4(m - 22) + 17)$$

$$26 \leq m \leq 27: \quad h_{mn} = 1 \ (n = 21), \ h_{mn} = -1 \ (n = 4(m - 25) + 21)$$

$$m = 28: \quad h_{mn} = 1 \ (n = 25), \ h_{mn} = -1 \ (n = 29)$$

$$V^i = [v_1^i \ v_1^{i-1} \ v_1^{i-2} \ v_1^{i-3} \ldots v_8^i \ v_8^{i-1} \ v_8^{i-2} \ v_8^{i-3} \\ \alpha_1^i \ \alpha_1^{i-1} \ \alpha_1^{i-2} \ \alpha_1^{i-3} \ldots \alpha_8^i \ \alpha_8^{i-1} \ \alpha_8^{i-2} \ \alpha_8^{i-3}]^T$$

$$v_m^i = \text{'value' of log (currency) } m \text{ at time } t_i$$

$$Z^i = [Z_{12}^i, \ldots, Z_{18}^i, Z_{23}^i, \ldots, Z_{28}^i, \ldots, Z_{78}^i)^T$$

The covariance matrix of the measurement error vector Z^i can be written as $\sigma^2 HH^T$ if we assume that all the Z_{mn}^i are Gaussian distributed with zero mean and same variance σ^2. The modification of equation (11) is again very straightforward.

- A special case is that α^i_m does not depend on t. If so, the model requires less computational effort.

6. Interest rates - short term and long term effects

Foreign currency exchange rates are clearly affected in the short term by interest rate movements. Such short term movements are additional to the long term effects of interest rates. One approach was to try to identify the short term interest rate component f in the expression

$$q^i_{mn} = p^i_{mn} \ \{1 + f^i_{mn}(\ldots)\}$$

where p^i_{mn} is a predicted exchange rate, q^i_{mn} is the market exchange rate, and the argument of the factor f includes variables that have to be identified. In an attempt to specify the factor f_{mn}, we examined the 'gain' g^i_m on investments over a period Δ:

$$g^i_m = (1 + e^{i*}_{m9})\ v^{i+\Delta}_m - v^i_m$$

Here v^i_m denotes the value of currency m and e^{i*}_{m9} is the short term interest rate at time t_i for country m.

The potential gains in various currencies could be compared by examining

$$g^i_m - g^i_n = v^i_n[\{(1 + e^{i*}_{m9})\frac{v^{i+\Delta}_m}{v^i_n} - \frac{v^i_m}{v^i_n}\} - \{(1 + e^{i*}_{n9})\frac{v^{i+\Delta}_n}{v^i_n} - 1\}]$$

In this expression, $(v^{i+\Delta}_m/v^i_n)$ and $(v^{i+\Delta}_n/v^i_n)$ would be predicted by an appropriate forecasting model, whilst $(v^i_m/v^i_n) = q^i_{mn}$ is the market realization of the exchange rate.

We looked for a general "principle of equilibrium" in which q^i_{mn} adjusts immediately to make $g^i_m - g^i_n = 0$. If such a principle held, this would provide a means for determining the factor f_{mn} described above.

This approach was not pursued further because of the following difficulties:

- what forecasting model to choose?
- what method to choose to give e^{i*}_{m9}? *e.g.* actual rates at t_i, average rates, expected rates, *etc.*
- what time period Δ to choose?
- how to relate this general principle of equilibrium to definite accounting and economic identities? *e.g.* the current account must balance

Accordingly, we looked for a simpler approach and now recommend that the dimensionless short term interest rate e^i_{m9} should be included in the list of economic properties that affect long term foreign currency exchange rates. Minor modifications would be required to incorporate e^i_{m9} in the models presented in Sections 4 and 5. Standard statistical tests could then be used to decide whether or not to include e^i_{m9}. In particular, the Kalman filtering approach would lend itself well to incorporation of e^i_{m9} in the correction (and not prediction) stage.

7. Conclusions

The preliminary model described in Section 3 yields promising results. Further development of this preliminary model is possible, but our opinion is that further development would be expedited using the improved linear model described in Section 4. This follows since the improved linear model has a simpler theoretical framework and would involve less computational work. The Kalman filtering model would be more difficult to use than the model of Section 4, but it would be necessary if the parameters α_j exhibited variability with time. In cases where such variability occurs, the Kalman filtering model is optimal.

The models described in this report should have the capability to improve the foreign currency exchange rate forecasts made by World Values Ltd.

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