MODELLING THE BACKWASH PROCESS IN MICROFILTRATION

Microfiltration separates fine solids from a liquid substrate, but requires a process to remove the accumulated solids from the filtering device. In this study, a backwash process was examined, in which a flow of high-pressure air in the direction opposite to the usual fluid flow, is used to dislodge and remove the particles.

The dynamics of the original air penetration of the filtering fibres, the flux balances of the displaced liquid, and the distribution of pressures through a network of modules were all studied. They fall naturally into a classification by increasing spatial scales.

1. Introduction

The 1986 Study Group considered mathematical aspects of hollow fibre ultrafiltration. This is a process in which a liquid is forced from the outside into the centre of a hollow fibre, typically 350 μ inner diameter and 650 μ outer diameter. The fibres are gathered in bundles (see Figure 1), and the bundles contained in cartridges of 5cm diameter and 90 cm long. The filtration process forces the liquid through the pores in the fibre walls, typically 0.1-0.2 μ in diameter. The 1986 Study Group was concerned, among other things, with various modes of failure of the tube under this inward pressure.

Figure 1: A filtration cartridge unit

The resulting separation can remove very fine particles, and even achieve liquid-liquid separation, but to be useful requires that the filtered material can be removed. This is done by forcing air flow in the opposite direction at high pressure (500-700 kPa), called *backwash.* The lumina are drained, and the air pressure is applied, with exit of liquid blocked. The opening of a backwash valve suddenly decreases the liquid pressure, creating a big pressure drop across the fibre wall. The makers have gone to some trouble to decrease the response time of this valve, which can be as low as 40 milliseconds, and one of the points of interest of this investigation is to see whether such effort is justified. This question is addressed by the development of a suitable microscale model in Section 3. The answer, according to this model, is that the very fast valves are now faster than the timescale of the process.

As preparation for the Study Group, a search of the literature was done, and some useful references are given at the end. The chapter by Porter is a modern general introduction. Other chapters in the book include a series of three on membranes, some fundamental papers on membrane transport, an introduction to ultrafiltration, and a chapter on reverse osmosis. A paper by Doshi, gives some useful theory on dynamics in the lumen, and the book of which it is part is still a useful general reference. The Lorne Workshop Proceedings is non-technical, but is a very interesting exercise in industry-government communication, to which Memtec has contributed. The paper of Balmann *et al.* is a very recent study involving modelling of fouling, whilst the paper of Klein *et al.* is one of a number of articles in a major 1980 symposium.

For a complete study of backwash, three scales of operation need to be considered:

1. *Microscale:* The gas-liquid interface is followed as it passes through the fibre wall. A model for this process is developed in Section 3.
2. *Mesoscale:* The flow of air and liquid through a bundle of 3000 fibres, within a tubular envelope (module), is considered. A model for this process is presented in Section 4.
3. *Macroscale:* The array consists of a number of modules connected by manifolds into a series and/or parallel grid. The flow dynamics following opening of the backwash valve has been modelled by Memtec, and some suggestions were made during the Study Group to improve the performance of this model (Section 5).

2. Backwash phenomena and general considerations

The events in a backwash sequence are:

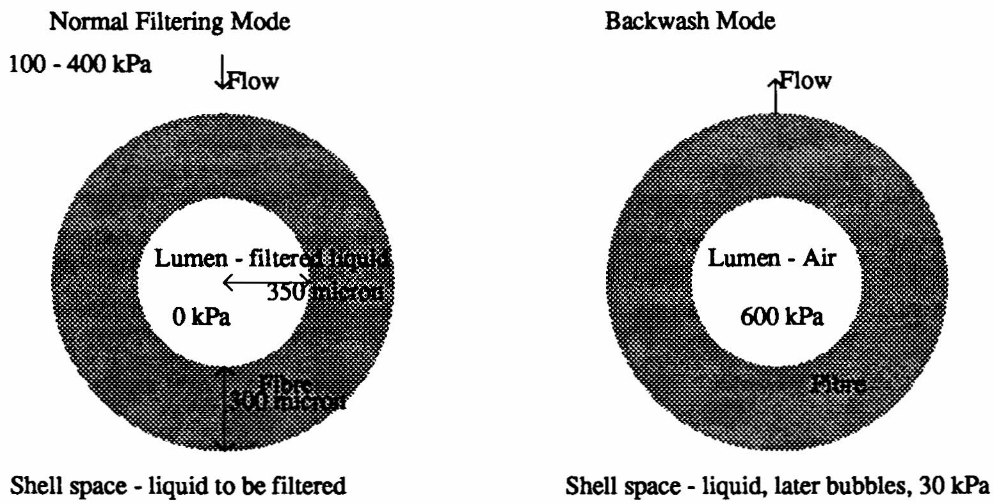

Figure 2: A fibre in cross-section in the two main modes of operation

1. Note the loss of effectiveness of the filter
2. Drain the lumina of the fibres (2 seconds)
3. Shut all shell side valves
4. Pressurise filtrate side with air to 600 kPa
5. Open backwash valve to let liquid escape (0.04-1 second)
6. Air flows for 3 seconds
7. Water is pumped through shell space while air flow continues
8. Pump and air stops
9. Rewet stage
   - Rewet with clean fluid in lumen
   - Open filtrate valve
   - Reapply 600 kPa air to pressurise lumen

- Open backwash valve 1 second and air valve

10. Resume filtering

When the backwash valve is opened (stage 6), the shell pressure drops to about 30 kPa, and an air front starts to advance through the fibre walls. This is relatively slow, although calculations presented later show it still occurs on a sub-second scale. As the air advances, water is displaced and forced from the bounding cylinders. The air must do work to impart kinetic energy to this water.

The fibre material is hydrophobic, and therefore presumably surface tension does not impede the movement of the front. Some discussion occurred in the group about the experimental observation of "bubble point pressure". This is a pressure of about 100 kPa which must be applied before any air will pass through the fibre, even after a very long time. Its value is used as a check on the quality and integrity of the fibres. It was thought to indicate hydrophilic behaviour, but since it now seems to be established that the fibre material is hydrophobic, the effect is attributed to the pressure needed to overcome the surface tension in the very small bubbles formed initially on the outer walls.

3. Backwash on the microscale

The basic equation for movement through the fibre of both phases is Darcy's law:

$$\nabla p = -\mu \mathbf{v} \epsilon / k \tag{1}$$

where p is the pressure, $\mathbf{v}$ the fluid velocity, μ the viscosity of the fluid, ϵ is the porosity and k the hydraulic permeability. Note that the 'superficial velocity', which is the volume flux per unit area, is $\epsilon \mathbf{v}$. The following complications were not taken into account:

- The material is deformed by the applied pressure (but only by 2-5%)
- The properties may be changed by the deformation
- The front separating air and liquid may be irregular (fingering)
- The filtrate material may have to be considered as a separate phase

A commonly used figure for permeability is 2×10^{-16} m^2 though it varies during the life of the fibre. In our calculations, the value used for the viscosity of water was $\mu_w = 1.0 \times 10^{-3}$ Pa s and the porosity = 0.7.

The suffices 0, i, and 1 will denote the inner surface, air/water interface and outer fibre surface respectively. Since the viscosity of air is much less than that of water, the pressure at the air-water interface p_i may be taken as the backwash air pressure, say $p_0 = 600$ kPa.

The condition at the outer surface of the fibre is more difficult. At backwash time, the fibre is surrounded by dirt etc., which creates very great resistance to radially inward flow. Whether it creates resistance to outward flow is not so clear; the material would seem unlikely to have much mechanical strength. The Study Group discussed this question, and decided to neglect the external resistance to flow; it was partly influenced by the unavailability of measurements. It would seem quite incorrect to assume that the inward flow resistance applies. A subsidiary question is whether the material's inertia is significant; again this is very hard to quantify, because the material does not have to accelerate to the flow velocity, but merely has to get out of the way.

We ignored the outer resistance, which would have the effect of increasing the time taken for air to penetrate the fibre wall. We did allow for a slight pressure at the outer surface of the fibre, say $p_1 = 30$ kPa.

With cylindrical symmetry, (1) becomes

$$\frac{\partial p}{\partial r} = -\mu_w v \epsilon / k \tag{2}$$

and with incompressibility, assuming k and μ constant:

$$\frac{\partial(rv)}{r\partial r} = 0$$

so

$$rv = r_i v_i$$

This gives

$$\frac{\partial p}{\partial r} = -\frac{\mu_w \epsilon r_i v_i}{rk}$$

so

$$(p_0 - p_i) = \mu_w r_i v_i \log(r_i - r_0)$$

and setting $v_i = dr_i/dt$ leads to the solution for the time t_1 for air to reach the outer surface

$$(p_0 - p_1)t_1 = -\frac{\mu_w \epsilon}{2k}[r_0^2 \log(r_1/r_0) + \frac{r_0^2 - r_1^2}{2}] \tag{3}$$

with

$$v_i = \frac{dr_i}{dt} = \frac{(p_0 - p_1)k}{\mu_w \epsilon r_i \log(r_1/r_i)} \tag{4}$$

Evaluation gives

$$t_1 = -\frac{(0.35/2)^2 \times 10^{-6} \times 0.7 \times 10^{-3} \times (2\ \log(0.65/0.35) + 1 - (\frac{0.65}{0.35})^2)}{4 \times 2 \times 10^{-16} \times 570 \times 10^3}$$

or

$$t_1 \approx 0.057 \text{ secs}$$

The initial velocity is

$$v = \frac{570 \times 10^3 \times 2 \times 10^{-16}}{10^{-3} \times 1.75 \times 10^{-4} \times \log(6.5/3.5) \times 0.7} = 1.5 \times 10^{-3} \text{m/s}$$

This may be used to check the assumption that the air pressure drop along the lumen of the fibre is negligible. Initially, the total flux F at the entry to the lumen is

$$F = 0.9 \times 2\pi \times 1.75 \times 10^{-4} \times 1.5 \times 10^{-3} \approx 1.4 \times 10^{-6} \text{m}^3/\text{s}$$

Use of the formula for Poiseuille flow in a pipe gives

$$\frac{dp_0}{dx} = \frac{8\mu_a \times F}{\pi r^4} \tag{5}$$

$$= \frac{8 \times 2 \times 10^{-5} \times 1.5 \times 10^{-6}}{\pi \times (1.75 \times 10^{-4})^4} \tag{6}$$

$$\approx 100 \text{ kPa/m} \tag{7}$$

A check on the validity of this model is that most of the work done by the air goes in overcoming viscous forces in pushing the water through the fibre. Some also goes into kinetic energy in the water movement in the cylinder. The power expended by the air on a module with 1 m^2 internal lumen area total is

$$\text{pressure} \times \text{area} \times \text{air velocity} = 6 \times 10^5 \times 1 \times 1 \times 10^{-3} = 600 \text{ watts}$$

A very rough comparison with the external liquid kinetic energy is given by noting that about 0.5 l of water is displaced in 0.3 seconds. If this has to pass through an orifice of 5 cm^2, the average speed would be 3 m/s, and the kinetic energy about 4 Joules, or an average power of about 13 watts. This figure is less than estimated above, but the energy increases rapidly as the size of the orifice diminishes.

The conclusion of this part of the analysis is that it takes about 0.06 seconds for the air to first reach the outside of the fibre, once the backwash valve is opened. This is somewhat longer than the opening time of the faster valves.

4. Mesoscale considerations

Here motion along the length of the fibres, in the lumen, and in the shell space is analysed. Q is the influx of air into each lumen, in m^3s^{-1}, and W is the return flux of water, in m s^{-1}. Generally, the suffix a indicates reference to the air phase in the lumen, whilst w refers to the water in the shell space. The equation for Poiseuille flow in the lumen is

$$Q = \frac{\pi r_0^4}{8\mu_a} \frac{P_R}{L} \frac{dp}{dz} \tag{8}$$

Here,

L is half the fibre length
P_R is the entry air pressure in Pa
z is the axial coordinate, non-dimensionalized against L
p is the air pressure, non-dimensionalised against P_R
μ_a is the viscosity of air

The equation relating movement of water through the fibre wall is, as in the previous section,

$$p - p_w = \frac{\mu_w}{k} \frac{\log(r_1/r_i)}{2\pi L P_R} \frac{dQ}{dz} \tag{9}$$

and the equation representing volume balance for the return flow in the shell space is

$$W = \frac{nQ}{1-\phi} \tag{10}$$

where ϕ is the volume fraction occupied by fibres and n is the number of fibres per m^2. The Navier-Stokes equation for the return flow is

$$\frac{W}{L}\frac{dW}{dz} + \frac{P_R}{\rho_w L}\frac{dp_w}{dz} + \frac{8\mu_w}{\rho r_1^2} W = 0 \tag{11}$$

The first term expresses the convection of momentum, and the second the force provided by the pressure gradient. The third term allows for the viscous drag of the fibres on the liquid by treating it as a body force. The force is obtained using the expression for Poiseuille pipe flow; the "effective radius" is taken to be the outer radius of the fibres. This is related to their packing density rather to an actual role played by that radius.

Gravity has been ignored, as the body force it produces is on this scale equivalent to a pressure differential of only a few kilopascals. More seriously, the pressure drop in the exit ducts has also been ignored; some information on this became available after the Study Group. Dr Ian Howells, in a separate report, has made a quantitative analysis taking some of these things into account[1]

Solving the equations for p gives

$$A\frac{d^2p}{dz^2} - B(\frac{dp}{dz})^2 - Cp = H = \text{constant} \tag{12}$$

where

$$A = 6.9 f_a \log_e(\frac{r_1}{r_i})$$

$$B = (\frac{n}{1-\phi})^2(\frac{\pi r_0^4}{8\mu_a})^2\frac{p_w P_R}{2L^2} \approx 1.86$$

$$C = 1 + (\frac{r_1}{r_0})^2\frac{\mu_w}{\mu_a}\frac{\pi n a^2}{1-\phi} \approx 3.57$$

Here f_a is a factor for describing fibre permeability, $f_a = 1.7 \times 10^{-15}/k$; f_a is about 1 for new fibres and 8.5 for old ones.

For r_i constant, this can be integrated

$$\frac{ds}{dp} = 2(\beta s + \gamma p + H)$$

where $s = (\frac{dp}{dz})^2$, $\beta = B/A$ and $\gamma = C/A$. This gives

$$(\frac{dp}{dz})^2 = s = De^{2\beta p} - \frac{\gamma p}{\beta} - \frac{H}{\beta} - \frac{\gamma}{2\beta^2} \tag{13}$$

and on further integration

$$1 - z = \int_{p_a}^{1} \frac{dp}{\sqrt{De^{2\beta p} - \frac{\gamma p}{\beta} - \frac{H}{\beta} - \frac{\gamma}{2\beta^2}}} \tag{14}$$

By applying $\frac{dp}{dz} = 0$ at z = 0 and using (9) and (14) at z = 1, the extra constants introduced can be determined.

The conclusion of this section is that flow in the cartridge can be treated mathematically, provided the assumption of an "effective resistance" of a fibre in (11) is acceptable.

[1] This valuable report could not be included here; it has however been communicated to Memtec. Ed.

5. Macroscale

For fluid motion through an array of modules it is useful to be able to model the pressure losses in the manifolds and their connections external to the cartridges. The dynamic response of the cartridges is also important. Mathematical modelling should, in principle, be able to resolve important design questions such as how many cartridges can be treated by one backwash valve (synchronizing several may not be easy), and what is their optimal arrangement.

The Study Group did not have time, nor really enough information, to treat this problem in detail. Nevertheless, there was active discussion. The existing method treats the system as analogous to a network of electrical resistances. It was pointed out that this is a way of representing the solution of the ordinary differential equations for the pressures in the various components by Laplace transformation, and if this is borne in mind, it is possible to incorporate more advanced dynamics and better solution. The following suggestions were made:

- One should be wary of the effect of highly transient events, for example, the transition to turbulence of accelerating flows. Even if the model correctly accounts for them, they will cause numerical difficulties.
- Taking the differential equation point of view, the existing solution method is simple Euler; a higher order method such as fourth order Runge-Kutta would allow longer time steps with only a little extra effort.
- In the electrical analogy, it is simple and desirable to allow reactive elements (capacitance and inductance). These represent stored energy, which is certainly present in the system. For example, the work done in creating kinetic energy is presently regarded as having been dissipated but, if it is regarded as having been done against an inductance (stored energy = $\frac{1}{2}L \times I^2$, L = inductance, I = current), this is formally analogous to the kinetic energy ($\frac{1}{2}mv^2$).
- Attention was drawn to the usefulness of Milman's Theorem in network analysis
$$E = \frac{\sum_i(E_i/R_i)}{\sum_i(1/R_i)}$$
for the combined effect of a number of voltage sources with internal resistance connected in parallel.

6. Conclusion

1. On the *microscale*, the model developed in Section 3 shows that the time taken for air to penetrate the fibre walls during backwash is at the smaller end of the range of backwash valve opening times, suggesting that it may be justified to use the existing fast valves but not to expend resources on seeking further speed. This conclusion is affected by uncertainty about the effect of the fouling material.

2. A *mesoscale* model was developed and used to calculate solutions. These solutions have qualitatively correct features, although detailed interpretation requires data which was not available at the Study Group.

3. The *macroscale* networks of cartridges was discussed in terms of models developed by Memtec. The Memtec models involved systems of ODEs, and suggestions were made for improvement of the solution procedure.

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