火焰温度分布的重构问题

4 引言

这一问题是由浙江大学王飞博士提出的。工业锅炉的测视图和俯视图分别由图 1 和图 2 所示：

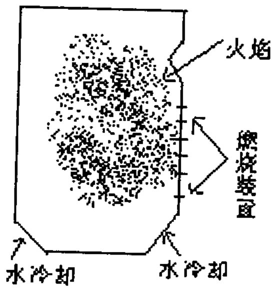


图 1

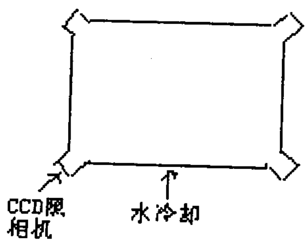


图 2

问题是：能否通过 CCD 照相机的测量数据快速地决定火焰地温度分布？

CCD 照相机测量的数据情形如图 3 和图 4 所示 (其中 $m \approx 20$):

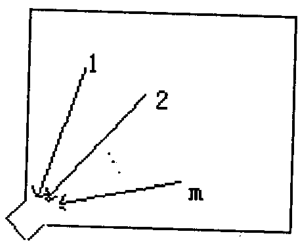


图 3

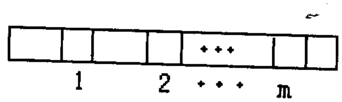


图 4

我们拟采用的步骤为：

1) 对问题的数学模型进行考察；
2) 考虑采用适当的数值方法。

5 建立模型

1. 建立辐射模型 (参见【1】)

在任何一点 $\underline{x}$ 有

a) 物质 (粒子) 的温度 $T(\underline{x})$

b) 有大小为 Q ，沿方向 $\underline{y}$ 的电磁辐射 $Q(\underline{x},\underline{y})$ ，其中 $|\underline{y}|=1$

射线 I_λ, r, j 其中 r 为位置， j 为方向。

作为黑体，物质向所有方向均匀地辐射

$$\varepsilon\sigma T^4, 0 \le \varepsilon \le 1 (T^{4.03})$$

考察一个粒子上静态能量守恒：

$$S + \alpha \int_{|\underline{y}|=1} Q dy = 4\pi\beta\sigma T^4$$

其中： $\alpha \approx \beta \approx K_\lambda$

沿着一条射线的能量

$$c\underline{y} \cdot \underline{\nabla} Q = \beta\sigma T^4 - \alpha Q$$

最后一项表示能量的吸收。

再考察边界条件，进入的射线具有一样的辐射强度 (图 5)

$$Q = \varepsilon\sigma T_\omega^4$$

射出的射线（图 6 ）的辐射强度可以由 CCD 照相机测得：

$$Q(\underline{y}^-, 0) = I_\lambda, r, s_e$$

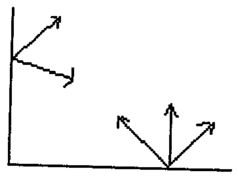


图5

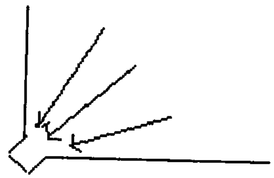


图6

我们的目的是给定 I_λ, r, s_e 决定合理的 $T(\underline{x})$ 。
问题是：

0) $S(\underline{x})$ 是未知的；
1) $\alpha \approx \beta \approx K(\underline{x})$ 是未知的，它依赖于粒子大小，反射率等；
2) 问题可能是超定或欠定的；
3) 如何选择好的数值方法。

6 解法

进行如图 (7)$N \times N$ 的剖分，将着眼于求每个区域中的 $T(x)$ 和 $K(x)$ 。

a) 建立粒子的热平衡，沿着从炉壁开始的若干射线对射线方程进行积分 (图 8)

$$Q = \varepsilon\sigma T_\omega^4 .$$

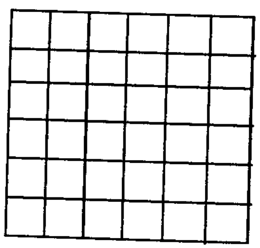


图7

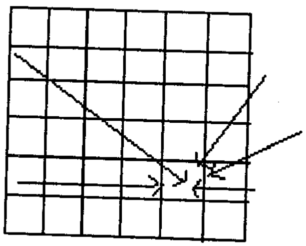


图8

b) 建立照相机的射线守恒 (图 9) 得到方程组，其中未知量 $T(x), S(x), K(x)$ 均有 N^2 个。方程数：关于粒子的方程有 N^2 个，关于照相机的方程 $4m$ 个。

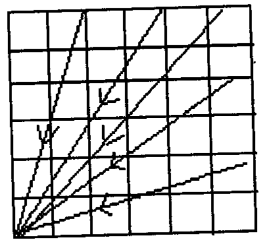


图9

7 近似解

1) 不计算 $S(x)$

未知量 $T(x)$ 和 $K(x)$ 各为 N^2 个，而方程数为照相机方程共 $4m$ 个。

2) 不计算 $S(x)$，取 $K(x)$ 为均匀的。此时，未知量为 $T(x)$ 共 N^2 个，方程数为 $4m$ 个。

若 $N^2 \leq 4m$ 是超定的或方程与未知量的个数相等。

若 $N^2 > 4m$ 是欠定方程组。

8 关于方程组求解的注记

若在某种情形下可以选择超定或欠定方程组，那么最好是方程个数与未知量个数一致，较好的是超定，有许多经典的方法如最小二乘法等，较差的是轻微欠定，最差的是严重欠定。

若欠定是无法避免时，可以做许多复杂的事情，包括模拟退火方法等。

若我们有线性方程组

$$A\underline{x} = \underline{b}$$

那么已有完整的理论求解，较差的是

$$\underline{x} = (A^T A)^{-1} A^T \underline{b}$$

它是严重病态的，或者

$$\underline{x} = (A^T A + \lambda B)^{-1} A^T \underline{b}$$

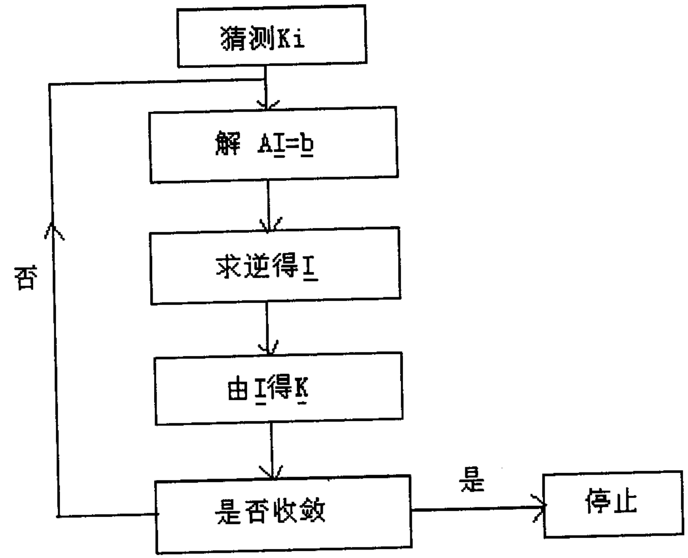
适当选取 λ 和矩阵 B ，此方法较好，计算量仅为 $O(n^3)$ ，比非线性节省得多。

具体的想法是，解

$$\begin{cases} known = a_{11}K_1I_1 + a_{12}K_2I_2 + \cdots + a_{1m^2}K_{m^2}I_{m^2} \\ known = a_{21}K_1I_1 + a_{22}K_2I_2 + \cdots + a_{2m^2}K_{m^2}I_{m^2} \\ \vdots \\ known = a_{N1}K_1I_1 + a_{N2}K_2I_2 + \cdots + a_{Nm^2}K_{m^2}I_{m^2} \end{cases}$$

其中 $known$ 表示已知量。

我们可以用以下步骤解 K_i 和 I_i 。



这样就无须求解非线性方程组。

(本报告由 C.Please 和 A.Fitt 撰写)

Reconstruction of the temperatures inside the furnace from the projection temperature

Abstract
We propose a method for solving the problem which was proposed by the researchers from Zhe Jiang University

9 Introduction

The "project temperature" is an average value along the project beam which is not only related to temperature distribution but also related to radiance distribution of the flame. It is known that the "project temperature" can reflect the combustion situation inside the furnace.

10 Formulation of the problem

We use ordinary array CCD, whose response range is the same as the wavelength range of visible light, as the detectors. Suppose that we can ignore the influence of gas element at infrared region and the scatter effect of pulverized coal for size of coal granule is quite small. In this case, for one ray-l(line), the radiantion transfer equation (YuKun Qing 1981) can be written as:

$$\frac{dI_{\lambda,r}(s)}{ds} = -K_{\lambda}(s)I_{\lambda,r}(s) + K_{\lambda}(s)I_{\lambda,br}(s) \qquad (1)$$

where $K_{\lambda}(s)$ is the radiation absorption coefficient at position s, $I_{\lambda,r}(s)$ is the radiation intensity at position s and $I_{\lambda,br}(s)$ is the radiation intensity of blackbody which is corresponding to the temperature at position s and can be calculated by Planck radiation theory.

The boundary conditions are:

- At $s = 0$,
$$I_{\lambda,r}(0) = I_{r,w} \tag{2}$$
where $I_{r,w}$ is the initial radiation intensity.

- At $s = s_e$,
$$I_{\lambda,r}(s_e) = I_{r,e} \tag{3}$$
where s_e is the length between the starting point and the end point.

Problem: Suppose that, for several line l_j, $j = 1, 2, \cdots, N$, $I_{r,w}$ and $I_{r,e}$ are given, find the radiation absorption coefficients $K_\lambda(s)$ and the radiation intensity $I_{\lambda,r}(s)$.

11 Previous Methods

The method, which is used by the engineers, is as follows:

1. Diving the domain into $G = M * N$ small rectangles and assuming that, at each rectangle, K and I are constants K_i and T_i.

2. Solving the differential equation (1) with the initial condition (2).

3. By the condition (3), getting a nonlinear equations and solving these equations by optimization method.

12 The method we suggest

We can solve this problem by two steps:

Step 1: We denote
$$p(s) = -K_\lambda(s) I_{\lambda,r}(s) + K_\lambda(s) I_{\lambda,br}(s). \tag{4}$$

Integrating the equation (1) and applying the boundary conditions (2), (3), we have

$$\int_{l_j} pds = I_{r,e,l_j} - I_{r,w,l_j} = M_j, j = 1, 2, \cdots, N \tag{5}$$

where we add l_j in the subscript to declare the it's relation with the line l_j.

Then we can solve the following computerized tomography problem first:

Reconstruct $p(s)$ from M_j.

There are references on this topic. We just refer to [1].

Step 2: By the equation (1) and initial condition (2), we have

$$I_{\lambda,r}(s) = I_{r,w} + \int_0^s p(s)ds \tag{6}$$

and

$$K_\lambda(s) = \frac{p(s)}{I_{\lambda,br} - I_{r,w} - \int_0^s p(s)ds}. \tag{7}$$

The advantages of our way are that

1. We divide the problem into two problems. One is a computerized tomography problem which is ill-posed and another is a very simple problem for which we only need to do some elementary calculus.

2. The ill-posedness of the original problem comes from the first problem –CT problem. Since there are so many works on this topic. We can know that the ill-posedness of the original problem is the same as CT problem which is not so strong. This means that it is possible to get the stable numerical solution.

3. Since the second problem is a simple, I think, from the analysis of the first problem, we can suggest the engineers whether we need to add more CDD cameras or not.

13 Conclusion

We hope that the way we suggested can work well and we are waiting for the numerical simulation results.

References

[1] F. Natterer, Numerical methods in tomography. Acta numerica 8 (1999), 107–141,