Evaluation of Customer Lifetime Value

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Suggested Model (Berger & Nasr)

$$CLV = C\sum_{i=0}^{n} \frac{r^i}{(1+d)^i} - M\sum_{i=1}^{n} \frac{r^{i-1}}{(1+d)^{i-0.5}}$$

* Applicability?
  –Not in current form!
* Modified Model?
  Yes! For 1 customer in 1 service:

$$CLV = \sum_{i=1}^{n} \frac{C_i \prod_{j=1}^{i} r_j}{(1+d)^i} - \sum_{i=1}^{n} \frac{M_i \prod_{j=1}^{i-1} r_j}{(1+d)^{i-0.5}}$$

$$set[\prod_{j=1}^{m} r_j] = 1 \quad if \quad m < 1$$

C_i=i^{th} **month's gross contribution**
$M_i = i^{th}$ **month's marketing spend**
$r_i = i^{th}$ **month's retention rate**
$n =$ **number of months,** d **=discount rate.**

Determining the retention rate, r_i**.**
Proposed method:
use the recursion formula

$$r_{i+1} = 1 - (1 - r_i)e^{-kM_i}$$

Assumption: Marketing in the current month effects retention this month!

Now we need an initial value, r_1**.**
We have 2 suggestions.
Both base the value for any given customer on a group of "similar" customers.
The value is calculated for the group and is applied to each individual with those characteristics.

One is simple but under-estimates retention.

Suppose we have historical data of account closures for up to N months. Let n_i be the number of accounts that closed after i months.
eg:

months	*customers*
1	n_1
2	n_2
3	n_3
$\vdots$	$\vdots$
N	n_N

$$\begin{aligned} P(j) &= \frac{n_j}{(\sum_{i=1}^{N} n_i) - (\sum_{i=1}^{j-1} n_i)} \\ &= \text{prob (leave in month k | survived k-1 months)} \end{aligned}$$

$$r_i = 1 - \frac{\sum_{j=1}^{N} P(j) n_j}{\sum_{j=1}^{N} n_j}$$

weighted average on number of customers.

This should take account of any lock-in period.

The other takes currently active customers into account.

We wish to establish the relationship between the data & the model.

Model:
Prob(Account lasts through the 1^{st} month)$=r$
Prob(Account lasts through 2 months)$=r^2$
$\vdots$

Hence:
Prob(Account lasts exactly 1 month)$=1-r$
Prob(Account lasts exactly 2 months)$=r(1-r) = r - r^2$
Prob(Account lasts exactly 3 months)$=r^2(1-r) = r^2 - r^3$
$\vdots$
This relates to closed accounts.
Whence: Prob (account closes BEFORE reaching p months)

$$\begin{aligned} &= \sum_{i=1}^{p-1} r^{i-1}(1-r) \\ &= (1-r) + (r - r^2) + (r^2 - r^3) + \cdots \\ &= 1 - r^{p-1} \end{aligned}$$

$\Rightarrow$ **Prob(Account is still active in p^{th} month.)**$=r^{p-1}$
This relates to active accounts.

What is the overall probability that we see the data we have if it obeys the model we impose?

$$p = (1-r)^{n_1}[r(1-r)]^{n_2}[r^2(1-r)]^{n_3}\cdots r^{m_1}(r^2)^{m_2}(r^3)^{m_3}\cdots$$

where
m_1 = no of accounts still active & only established for 1 month
m_2 = no of accounts still active & only established for 2 months
⋮

We now choose r to maximize P.
This is where the relationship between the data & the model is strongest.
PCCW's next question was about combining CLV's to get the overall CLV.

Assume there are K services:

$$CLV_{TOTAL} = \sum_{k=1}^{K} CLV_k, \qquad \text{where}\cdots$$

$$CLV_k = \sum_{i=0}^{n_k} \frac{C_{ik}\prod_{j=1}^{i} r_{jk}}{(1+d)^i} - \sum_{i=1}^{n_k} \frac{M_{ik}\prod_{j=1}^{i} r_{jk}}{(1+d)^{i-0.5}}$$

Where M_{ik} is a function of r_{i-1k} but this is company Marketing policy.
and C_{ik} = Revenue$_{ik}$ − Coot of Sales$_{ik}$
Now Revenue$_{ik}$ = Usage$_{ik}$ ∗ unit price$_{ik}$
Let UP = unit price & US = usage.
Unit price is determined by company strategy and should be kept separate from other Marketing costs, M_{ik}.
We suggest that usage$_{ik}$ is a function of unit price$_{ik}$
M_{ik} and cross-effects of advertising $M_{ij} * M_{ik}, j \neq k, j = 1, \cdots, k$.

$$US_{ik} = f_1(M_{ik}) + f_{2j}(M_{ij} * M_{ik}) + f_3(UP_{ik})$$

again $j \neq k, j = 1, \cdots, k$. Now

$$f_1 = \frac{L}{1 - c_1 e^{-rM_{ik}}}$$

$$L = m_k US_{max}$$

where m_k needs to be estimated by PCCW & depends on expandability of service

$$US_{i-1k} = \frac{L}{1-c_1}$$

r is estimated from historical data. f_3 is the price sensitivity.
PCCW may already know this!
If not, we have a couple of suggestions ⋯.
eg $f_3 = aUP_{ik} + b$
so b = maximum usage$_k$ − US_{i-1k}
$0 = a(UP_{i-1k}) + b$

Alternatively ⋯
price sensitivity is proportional to the slope.
Finally f_{2j} could be given the form: $f_{2j} = c_2(M_{ij} * M_{ik})^2$ & c_2 is determined from historical data.
The fourth PCCW question was about optimizing the Marketing strategy to improve profits.
Now CLV_{TOTAL} can be optimized with respect to M_{ik} subject to $\sum_{i=1}^{n} \sum_{k=1}^{K} M_{ik} = \text{Budget}$.

SUMMARY.

- **Total CLV can be optimized against the promotional costs associated with all PCCW services.**
- Price sensitivity is build into the model.
- **Retention rates are now a function of the promotional costs.**
- Cross-effects of advertising are taken into account through the usage function.
- **unit price of each PCCW service is now in the model and is separate from the other promotional costs.**
- The impact of unit price on usage is also built into the model.
- **The new model can revert to the original model if M_{ik}, C_{ik} and r_{ik} are all set to be constants $M, C\&\ r$.**