ANALYSIS OF THE POTENTIAL MECHANISMS OF ROCKBURSTS

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Abstract

Sudden slip on geological faults or other discontinuities in rock may be preceded by an initial phase of "slow" fault creep. A simple plane strain model of a suddenly loaded fault is analysed to illustrate the possible transition from stable slip behaviour to accelerated, unstable slip. The model assumes that a peaked shear load is applied suddenly to the fault region. The rate of slip movement is assumed to be proportional to the difference between the applied shear stress and the cohesive and frictional slip resistance. It is found that the evolutionary fault movement can be described succinctly by a non-linear ordinary differential equation describing the activated length of the sliding fault as a function of time. The differential equation is found to depend on a single, dimensionless parameter whose value determines whether the fault slip decays monotonically or accelerates in an unstable manner.

1 Introduction

The occurrence of sudden rock failure near excavations (so-called "rockbursts") in deep level mining operations is of continuing concern to the South African mining industry both as a potential cause of serious injuries to miners and as an inhibiting factor for the exploitation of valuable mineral resources. Surface tremors may, in extreme cases, even cause damage to buildings or other infrastructure. Understanding some of the underlying deformation mechanisms of rockbursts can clearly have considerable benefits in devising engineering strategies to ameliorate the potential damage and safety risks of deep level mining. Three main areas of interest are:

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1. Prediction: Precursory signals of seismic activity or other changes may be used to infer the impending onset of large seismic events.

2. Excavation damage: An improved understanding of wave propagation effects, initiated by sudden rock failure, can be used to design improved support systems in underground excavations.

3. Localisation theory: Further advances in understanding the intrinsic structure of rock failure mechanisms can assist in the assessment of the likelihood of rock failure in the vicinity of existing excavations.

During the MISG study week, in January 2006, efforts were concentrated on formulating a simple model to address the first area of interest, relating to the prediction of a seismic event. In this model, fault creep is studied to investigate the transition from stable to unstable slip behaviour.

2 Fault Creep Model

In order to fix ideas, consider the case of a simple fault (plane of weakness) represented as a discontinuity surface in the plane $z = 0$. The fault is assumed to be located in the region $-b \leq y \leq b$ and to extend indefinitely in the x direction implying a state of plane strain with respect to the x-axis. The fault is loaded by a shear stress σ_{yz} and a normal "clamping" stress σ_{zz} as shown in Figure 1.

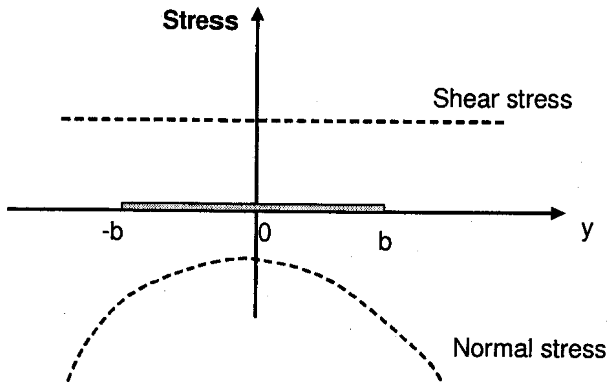

Figure 1: Stress components acting on a fault discontinuity.

The applied shear stress, σ^0_{yz} , is assumed to be constant and the normal stress, $\sigma^0_{zz}(y)$, is assumed to be compressive (negative) and to have a peaked, symmetrical shape with the absolute stress value increasing away from the centre of the fault at $y = 0$. It is postulated that slip on the fault is controlled by a "creep" law of the form

$$\frac{\partial D_s(y,t)}{\partial t} = \kappa\Big[\tau(y,t) - \rho(y,t)\Big] , \tag{1}$$

where $D_s(y,t)$ is the slip extent at position y on the fault at time t. $D_s(y,t)$ is equal to the absolute value of the difference between the displacement components u^+_y and u^-_y on opposite sides of the fault according to

$$D_s(y,t) = \Big|\Delta u_y(y,t)\Big| , \tag{2}$$

and the slip displacement discontinuity component, $\Delta u_y(y,t)$ is defined to be

$$\Delta u_y(y,t) = u^+_y(y,t) - u^-_y(y,t) . \tag{3}$$

The absolute value of the total applied shear stress, τ, is given by

$$\tau(y,t) = \Big|\sigma_{yz}(y,t) + \sigma^0_{yz}\Big| , \tag{4}$$

where $\sigma_{yz}(y,t)$ is the shear stress induced at position y along the fault by the slip displacement discontinuity component, $\Delta u_y(y,t)$. The slip resistance $\rho(y,t)$ is determined by the prevailing cohesion and friction on the fault at position y and time t. κ is a proportionality constant. The slip resistance ρ is assumed to be defined by the following relationship.

$$\rho(y,t) = S_0 - \beta D_s(y,t) + \mu\sigma_n(y) , \tag{5}$$

where
$S_0 =$ initial cohesion (MPa),
$\beta =$ cohesion slip weakening rate (MPa / m),
$\mu =$ coefficient of friction,
$\sigma_n(y) =$ absolute value of the normal "clamping" stress (MPa) applied to the fault at position y as illustrated in Figure 1. (i.e. $\sigma_n(y) = -\sigma^0_{zz}(y) > 0$).

Equation (5) is operative when the slip extent, D_s , is less than the maximum limit D^*_s implied by

$$D^*_s = \frac{S_0}{\beta} . \tag{6}$$

When $D_s = \frac{S_0}{\beta}$, the slip resistance is presumed to be equal to $\mu\sigma_n(y)$.

The induced shear stress, σ_{yz}, in an isotropic elastic material can be determined from the displacement discontinuity value Δu_y using an integral equation relationship for the stress components, σ_{yy} , σ_{zz} and σ_{yz}. This relationship can be expressed compactly using the following complex variable representation. (See, for example, Linkov and Mogilevskaya, 1994). At a general field point $Z = y + iz$ in the complex $(y,\, z)$ plane

$$\sigma_{zz} - \sigma_{yy} + 2i\sigma_{yz} = \frac{iG}{2\pi(1-\nu)} \int_{-b}^{b} \left\{ \frac{\bar{\Delta} d\zeta - \Delta d\bar{\zeta}}{(Z-\zeta)^2} + \frac{2(\bar{Z}-\bar{\zeta})\Delta d\zeta}{(Z-\zeta)^3} \right\} \quad (7)$$

$$\sigma_{yy} - \sigma_{zz} = \frac{iG}{2\pi(1-\nu)} \int_{-b}^{b} \left\{ \frac{\bar{\Delta} d\bar{\zeta}}{(\bar{Z}-\bar{\zeta})^2} - \frac{\Delta d\zeta}{(Z-\zeta)^2} \right\} , \quad (8)$$

where $i = \sqrt{-1}$, G is the material shear modulus and ν is Poisson's ratio. $\Delta = \Delta u_y + i\Delta u_z$ is the complex displacement discontinuity vector at position ζ of the fault segment. In the present case, ζ assumes values on the real line $-b \le \zeta \le b$ and the displacement discontinuity opening component $\Delta u_z = u_z^+ - u_z^- = 0$. Hence, σ_{yz} can be deduced from the imaginary part of the limiting form of equation (7):

$$\sigma_{zz} - \sigma_{yy} + 2i\sigma_{yz} = \lim_{z \to 0} \left\{ -\frac{iG}{\pi(1-\nu)} \frac{\partial}{\partial Z} \left[\int_{-b}^{b} \frac{\Delta u_y(\zeta) d\zeta}{Z-\zeta} \right] \right\} . \quad (9)$$

In order to simplify the analysis, assume that the slip profile $\Delta u_y(\zeta)$ has the following specific shape

$$\Delta u_y(\zeta) = a(b^2 - \zeta^2)^{3/2} , \quad (10)$$

where a and b are parameters that depend on the time, t. The characteristic shape of this slip profile is illustrated in Figure 2. It should be noted that when $\zeta = \pm b$ the slope of Δu_y is zero and the induced shear stress σ_{yz} remains finite near the edges of the slip region. This behaviour may be contrasted to the conventional fracture mechanics assumption of a uniformly loaded crack of fixed length where the crack slip or opening displacement is proportional to $x^{1/2}$ at a distance x from the crack tip, within the crack and where the stress values become infinite immediately ahead of the crack tip. In the present case, the far field crack-normal load is assumed to be peaked (Figure 1) and the fault slip region limits $\pm b(t)$ at time t are adjusted to ensure that the total shear stress is equal to the shear resistance at the edge positions $\pm b(t)$. Under these assumptions, equation (10) provides the

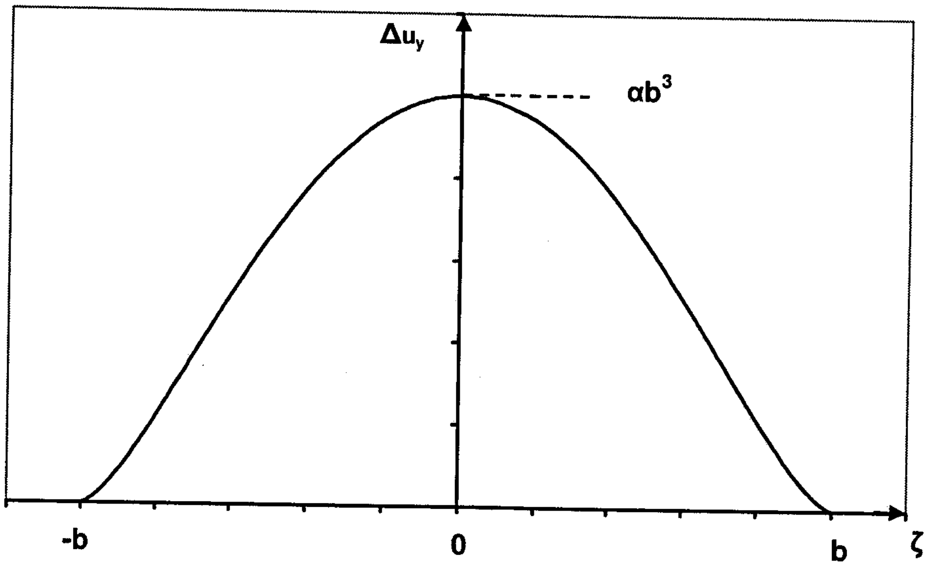

Figure 2: Assumed discontinuity slip profile shape.

appropriate asymptotic edge behaviour.

Substituting equation (10) into equation (9), it can be shown that

$$\int_{-b}^{b} \frac{a(b^2-\zeta^2)^{3/2}d\zeta}{Z-\zeta} = a\pi\left[(b^2-Z^2)\left(Z-\sqrt{Z^2-b^2}\right)+b^2\frac{Z}{2}\right] . \quad (11)$$

Differentiating the right hand side of equation (11) with respect to Z and employing the limiting relationship

$$\lim_{z\to\pm 0}\sqrt{Z^2-b^2} = \pm i\sqrt{b^2-y^2} \quad \text{for} \quad |y|\leq b\,, \quad (12)$$

gives the expression for the induced shear stress component σ_{yz} in the slip region as

$$\sigma_{yz} = -\frac{3aG}{4(1-\nu)}\left[b^2-2y^2\right] \quad \text{for} \quad |y|\leq b\,. \quad (13)$$

In equation (13) it is understood implicitly that the active slip region $-b(t)\leq y\leq b(t)$ is a function of time, t, following the imposition of the far field shear stress σ^0_{yz} at $t=0$. Substituting equation (13) into equation (4) and assuming that $\sigma^0_{yz}>0$, the total (positive) shear stress magnitude is given by

$$\tau(y,t) = \sigma_{yz}^0 = -\frac{3a(t)G}{4(1-\nu)}\left[b^2(t) - 2y^2\right] \quad \text{for} \quad |y| \leq b(t)\ , \tag{14}$$

where the slip amplitude $a(t)$ and the half-length parameter $b(t)$ are written as explicit functions of time.

3 Determination of the active slip length

The evolution of the slip patch half-length $b(t)$ requires the general coupled solution of equation (1) and equation (9) to determine the slip function $\Delta u_y(y,t)$. However, using the postulated representation of the slip function, given by equation (10), the simplifying assumption is made that the slip behaviour is governed essentially by the stress and slip state at the particular point $y = 0$. From equations (14), (10) and (5)

$$\tau(0,t) = \sigma_{yz}^0 - \frac{3a(t)Gb^2(t)}{4(1-\nu)}\ , \tag{15}$$

$$\rho(0,t) = S_0 - \beta a(t)b^3(t) + \mu\sigma_n^0\ , \tag{16}$$

where σ_n^0 is the absolute value of the compressive normal stress component σ_{zz} at $y = 0$. The further explicit assumption is made that the peaked shape of the normal stress component, illustrated in Figure 1, is given by the function

$$\sigma_n(y) = \sigma_n^0 + \gamma y^2\ , \tag{17}$$

where γ is a fixed parameter. The half-length $b(t)$ is defined by the condition that $\tau(b,t) = \rho(b,t)$ and that at this point the slip value $D_s(b,t) = 0$. Using equations (5), (14) and (17) this implies the equality constraint

$$\sigma_{yz}^0 + \frac{3a(t)Gb^2(t)}{4(1-\nu)} = S_0 + \mu\sigma_n^0 + \mu\gamma b^2(t)\ . \tag{18}$$

Hence,

$$a(t)b^2(t) = \frac{4(1-\nu)}{3G}\left[S_0 + \mu\sigma_n^0 - \sigma_{yz}^0 + \mu\gamma b^2(t)\right]\ . \tag{19}$$

Finally, noting that $D_s(0,t) = a(t)b^3(t)$ and substituting equations (15) and (16) into equation (1) yields the ordinary differential equation

$$\frac{d}{dt}\left(ab^3\right) = \kappa\left[\sigma_{yz}^0 - \frac{3Gab^2}{4(1-\nu)} - S_0 + \beta ab^3 - \mu\sigma_n^0\right]\ . \tag{20}$$

At time $t = 0$, the shear slip on the fault is zero ($D_s(0,0) = 0$) and the initial activated half-length, b_0 , is deduced from equation (18) to be

$$b_0^2 = \frac{(\sigma_{yz}^0 - S_0 - \mu\sigma_n^0)}{\mu\gamma} , \tag{21}$$

with the implicit understanding that $\sigma_{yz}^0 - S_0 - \mu\sigma_n^0 > 0$ and $\mu\gamma > 0$.

4 Non-dimensional form of the slip equation

It is apparent that the unknown slip profile parameters $a(t)$ and $b(t)$ in equation (20) depend on the following nine material and loading parameters: S_0, β, μ, κ, G, ν, σ_{yz}^0, σ_n^0 and γ. However, this complex model can be simplified significantly by defining the non-dimensional slip length, B, and time, T, according to the relationships

$$B = \frac{\beta b}{G} , \tag{22}$$

$$T = \beta\kappa t . \tag{23}$$

From equations (21) and (22), the non-dimensional expression for the initial activated fault half-length is given by

$$B_0^2 = \frac{\beta^2 b_0^2}{G^2} = \frac{\beta^2}{G^2}\left[\frac{\sigma_{yz}^0 - S_0 - \mu\sigma_n^0}{\mu\gamma}\right] . \tag{24}$$

Employing equations (22), (23) and (24) and eliminating $a(t)$ between equations (19) and (20), yields the following differential equation for the slip length, B, that includes only the two non-dimensional parameters B_0 and ν

$$(3B^2 - B_0^2)\frac{dB}{dT} = B^3 - B_0^2 B + \frac{3}{4(1-\nu)}(2B_0^2 - B^2) . \tag{25}$$

The initial condition is that when $T = 0$, $B = B_0$.

The cohesion falls to zero at the centre of the slip region, $y = 0$, when $D_s(0,t^*) = D_s^* = S_0/\beta$. At this time $a(t^*)b^3(t^*) = S_0/\beta$ and, using equations (19) and (21), the critical slip length, B^* , can be deduced to satisfy

$$(B^*)^3 - B_0^2 B^* = \frac{3S_0\beta^2}{4(1-\nu)\mu\gamma G^2} . \tag{26}$$

Also, employing the residual slip resistance

$$\mu\sigma_n(y) = \mu\sigma_n^0 + \mu\gamma y^2 , \tag{27}$$

yields the appropriate differential equation when $B > B^*$

$$(3B^2 - B_0^2)\frac{dB}{dT} = (B^*)^3 - B_0^2(B^*) + \frac{3}{4(1-\nu)}(2B_0^2 - B^2)\,. \tag{28}$$

Both equations (25) and (28) have a separable structure allowing direct quadrature. This can be carried out in terms of simple analytic functions for equation (28) once the transition time T^* is determined. In order to expose the structure of equation (25), it is useful to define the non-dimensional slip length variable, Y , and the parameter λ by

$$Y = \frac{B}{B_0}\,, \tag{29}$$

$$\lambda = \frac{3}{4(1-\nu)B_0} = \frac{3}{4(1-\nu)}\left(\frac{G}{\beta}\right)\left[\frac{\mu\gamma}{\sigma_{yz}^0 - S_0 - \mu\sigma_n^0}\right]^{1/2} > 0\,. \tag{30}$$

It is of particular interest to note that λ depends on all the basic model parameters except the slip rate proportionality constant κ which is subsumed into the non-dimensional independent "time" variable, T. The analogous equations to (25) and (28) may then be written, respectively, as

$$(3Y^2 - 1)\frac{dY}{dT} = Y^3 - Y + \lambda(2 - Y^2) \quad \text{for} \quad 1 \le Y \le Y^*\,, \tag{31}$$

$$(3Y^2 - 1)\frac{dY}{dT} = (Y^*)^3 - Y^* + \lambda(2 - Y^2) \quad \text{for} \quad Y > Y^*\,, \tag{32}$$

where Y^* is inferred from equations (24), (26) and (30) to be determined by

$$(Y^*)^3 - Y^* = \frac{3S_0\beta^2}{4(1-\nu)B_0^3\mu\gamma G^2} = \frac{\lambda S_0}{\sigma_{yz}^0 - S_0 - \mu\sigma_n^0}\,. \tag{33}$$

It can be seen that the right hand side of equation (31) is always positive as long as $Y^2 \le 2$. When $Y^2 > 2$ the right hand side of equation (31) can be written as

$$f(Y) = (Y^2 - 2)\left[\frac{Y^3 - Y}{Y^2 - 2} - \lambda\right] = (Y^2 - 2)\Big[g(Y) - \lambda\Big]\,, \tag{34}$$

where

$$g(Y) = \frac{Y^3 - Y}{Y^2 - 2}\,. \tag{35}$$

It may be shown that $g(Y)$ has a local minimum value,$\lambda_{\min} \approx 2.9696$, at $Y^2 = (5+\sqrt{17})/2$ in the region $Y^2 > 2$. Consequently, if $\lambda < \lambda_{\min}$ then $f(Y)$ will never be zero when $Y^2 > 2$ and since

$$\frac{f(Y)}{(3Y^2-1)} \sim O(Y) \qquad \text{as } y \to \infty, \tag{36}$$

the slip length may be expected to grow exponentially until the cohesion loss is complete. From equation (30), the condition $\lambda < \lambda_{\min}$ corresponds to the requirement that the initially activated length B_0 is sufficiently large – an intuitively plausible condition. Alternatively, if $\lambda > \lambda_{\min}$ (and the initially activated fault length is sufficiently small) then when $f(Y)$ falls to zero, fault creep slip will no longer proceed. This is, again, a plausible conclusion. Clearly, the parameter λ defines the basic condition for stable or unstable slip evolution following the imposition of the initial shear load on the fault. Notably, parameter λ does not depend on the slip rate proportionality constant κ which only plays a role in determining the time scaling of the slip length evolution. The constant κ does not influence the fault slip stability behaviour.

The influence of the parameter λ on the slip length evolution trajectory, determined by equation (31), is illustrated in Figure 3 for the specific values $\lambda = 1$, $\lambda = 2$ and $\lambda = 4$. This demonstrates the unstable and stable evolution of the slip activation length, Y, when $\lambda < \lambda_{\min}$ and $\lambda > \lambda_{\min}$ respectively. It may also be noted that λ plays an analogous role to the critical slip nucleation length parameter G/β highlighted in the analysis of fault slip stability by Uenishi and Rice, 2003.

5 Conclusions

A simple fault creep model has been analysed. It is found that the fault movement can be described in terms of a single non-linear ordinary differential equation. This differential equation can be expressed in terms of non-dimensional variables, Y and T, representing the active "length" of the fault and the "time" respectively and, remarkably, a single dimensionless parameter λ. Parameter λ is inversely related to the length of the initially activated region of the fault plane. It is demonstrated that a critical value,$\lambda_{\min} \sim 2.9696\ldots$, of the parameter λ determines the stable or unstable evolution of the fault length trajectory. Specifically, if $\lambda < \lambda_{\min}$ the slip extent will extend in an unstable manner and if $\lambda > \lambda_{\min}$ the slip extent will

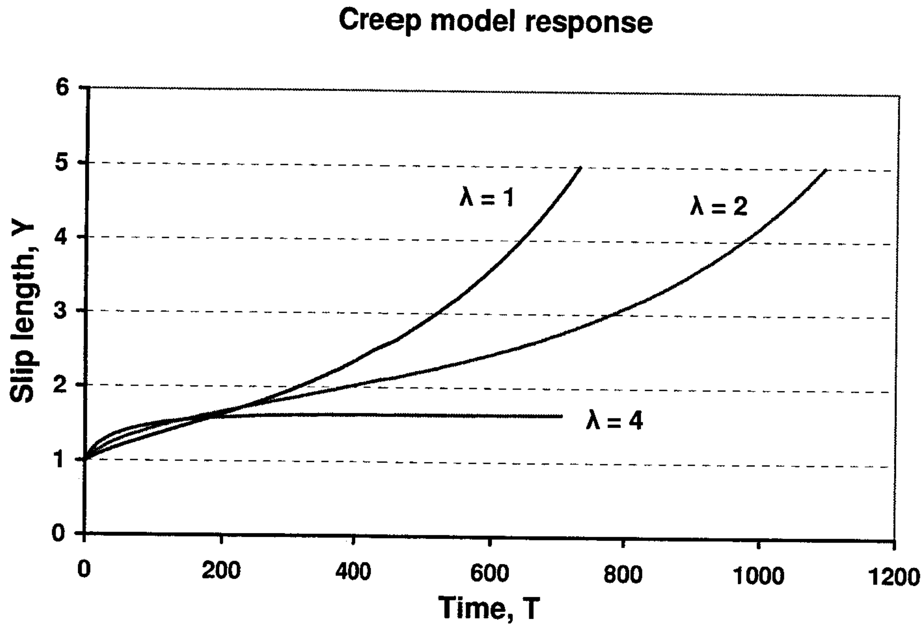

Figure 3: Explicit evolution of the non-dimensional fault activation length, Y, for three values of the critical stability parameter λ.

be limited. It may be of interest to explore the slip evolution model further in three dimensions and to investigate multiple interacting fault structures as well as elastodynamic behaviour.

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