Ideal stresses produced in Welding

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1. Introduction

This is a brief summary of some calculations of the stress field induced by a moving point heat source on a relatively thin sheet of metal. First, the temperature field is calculated. Then the stress field according to linear thermoelasticity is calculated.

Some features of the stress field can help to explain the existence and the location of some cracks that occur in a welding process. A laser or electron beam can produce the heat source.

The basic assumptions are:

(i) plane temperature field $T'(x', y')$

(ii) steady field in frame fixed in source

(iii) source is at a point

(iv) plane stress $\frac{\partial \sigma'}{\partial z'} = 0$

2. Temperature Field

The physical set up appears in Figure 1.

In a coordinate system fixed in the uniformly moving heat source the heat equation for the temperature $T'(°K)$ is

$$\rho c U \frac{\partial T'}{\partial x'} = k \left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'} \right) + Q_0 \delta(x') \delta(y'^2)$$

U = speed of heat source in coordinates fixed in material at rest

ρ = density,

$$
\begin{aligned}
c &= \text{specific heat,} \\
k &= \text{thermal conductivity,} \\
Q_0 &= \text{rate of energy addition per depth } (z')
\end{aligned}
$$

A scaling, as follows, puts the problem in dimensionless form in terms of these variables

$$x = \frac{Ux'}{\kappa}, \qquad y = \frac{Uy'}{\kappa}, \qquad T(x,y) = \frac{T'(x',y')}{T^*}$$

where

$$
\begin{aligned}
\kappa &= \text{diffusivity } = \frac{k}{\rho c} \\
T^* &= \text{characteristic temperature } = \frac{Q_0}{\kappa}
\end{aligned}
$$

In general there is a melting isotherm $T = T_M(x,y)$ and a pool of melt around the point source. The thermal problem is solved neglecting the heat of fusion (usually relatively small) and assuming that the properties of the melt are the same as those of the solid.

The basic solution is well known

$$T(x,y) = e^{\frac{x}{2}} K_0\left(\frac{r}{2}\right), \qquad r = \sqrt{x^2+y^2}$$

The behavior of the modified Bessel function is

$$
\begin{aligned}
K_0\left(\frac{r}{2}\right) &= -\frac{1}{2\pi}\log r + \cdots \qquad r \to 0 \\
&\cong \sqrt{\frac{\pi}{r}} e^{-\frac{r}{2}} + \cdots \qquad r \to \infty
\end{aligned}
$$

Thus as $r \to \infty$

$$T(x,y) \cong \sqrt{\frac{\pi}{r}} e^{\frac{1}{2}(x-r)} + \cdots$$

The qualitative form of an (asymptotic) isotherm is shown in Figure 2.

3. Thermal Stresses

In linear thermoelasticity thermal expansion acts as a pressure. The thermal strain is proportional to T'

$$\text{strain} = \alpha T'$$

$\alpha =$ coeff. of thermal expansion. Thus the stress-strain law is modified to read (cf Timershenko and Goodier, Theory of Elasticity (TG))

$$\sigma'_x = \frac{E}{1-\nu^2}\left(\frac{\partial u'}{\partial x'} + \nu\frac{\partial v'}{\partial y'} - (1+\nu)\alpha T\right) \quad \text{normal stress in } x'$$

$$\sigma'_y = \frac{E}{1-\nu^2}\left(\frac{\partial v'}{\partial y'} + \nu\frac{\partial u'}{\partial x'} - (1+\nu)\alpha T\right) \quad \text{normal stress in } y'$$

$$\tau' = \frac{E}{2(1+\nu)}\left(\frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'}\right) \quad \text{shear stress}$$

where

$$\begin{aligned} E &= \text{elastic modulus} \\ u', v' &= \text{displacements in } x', y' \\ \nu &= \text{Poisson ratio} \end{aligned}$$

See Figure 3.

(TG) show that for this case a potential ϕ exists (no rotation introduced into displacement field) so that

$$u' = \frac{\partial \phi'}{\partial x'}, \qquad v' = \frac{\partial \phi'}{\partial y'}$$

This problem can also be put into dimensionless form with the following scaling

$$\phi'(x', y') = \frac{\kappa^2}{U^2}(1+\nu)\alpha T^* \phi(x, y)$$

$$\sigma' = \frac{E\alpha T^*}{1-\nu}\sigma$$

$$\tau' = \frac{E\alpha T^*}{1-\nu}\tau$$

Then the basic equations for the potential are

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = T \qquad \text{equilibrium}$$

and from the heat equation it follows that

$$\frac{\partial \phi}{\partial x} = T$$

modulo a harmonic function. The resulting stress field is

$$\begin{aligned}
\sigma_x &= \frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial^2 \phi}{\partial y^2} - T = (1-\nu)\left(\frac{\partial T}{\partial x} - T\right) \\
\sigma_y &= \frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial^2 \phi}{\partial y^2} - T = -(1-\nu)\frac{\partial T}{\partial x} \\
\tau &= (1-\nu)\frac{\partial T}{\partial y}
\end{aligned}$$

The temperature field is given by the basic solution for the heat source. Note that there are no dimensionless parameters left in the solution. The stresses are calculated neglecting the melt.

$$T = e^{\frac{x}{2}} K_0\left(\frac{r}{2}\right)$$

and

$$\begin{aligned}
\sigma_x &= -\frac{1}{2}e^{\frac{x}{2}}\left\{K_0\left(\frac{r}{2}\right) + K_1\left(\frac{r}{2}\right)\cos\theta\right\}(1-\nu) \\
\sigma_y &= -\frac{1}{2}e^{\frac{x}{2}}\left\{K_0\left(\frac{r}{2}\right) - K_1\left(\frac{r}{2}\right)\cos\theta\right\}(1-\nu) \\
\tau &= -\frac{1}{2}e^{\frac{x}{2}} K_1\left(\frac{r}{2}\right)\sin\theta
\end{aligned}$$

It turns out then that the principal stresses are

$$\sigma_{1,2} = -\frac{1}{2}(1-\nu)e^{\frac{x}{2}}\left(K_0\left(\frac{r}{2}\right) \pm K_1\left(\frac{r}{2}\right)\right)$$

The principal stress directions are constant on rays since the principal directions are

$$\beta = \frac{\theta}{2}, \qquad \frac{\theta}{2} + \frac{\pi}{2}$$

(cf Figure 4)

Since $K_1\left(\frac{r}{2}\right) > K_0\left(\frac{r}{2}\right) > 0$

$$\sigma_1 = \text{ compression}, \qquad \sigma_2 = \text{ tension}$$

When $\theta = \frac{\pi}{2}$, right angles to the direction of motion, the tension acts normal to a line at $\frac{\pi}{4}$ to the x-axis. In certain experiments cracks at some distance from the source were observed in this direction [1]. This would be possible if the tensile stress were sufficiently high and a tension failure occurred.

Some numerical values for this experiment in $Ti - Al$ are

$$\begin{aligned}
\epsilon_M &= \text{ thermal strain } = \alpha T_M \\
&= (10 \times 10^{-6})(1500^\circ) = .015 \\
\sigma_M &= E\epsilon_M = (1.6 \times 10^{-6})(.015) = 240,000\text{psi} \\
\sigma_{yield} &\cong 170,000 \text{ psi} \\
T^* &= \frac{Q_0}{k} = \frac{10^4 \frac{watt}{cm}}{10^{-1} \frac{watt}{cm^\circ K}} = 10^{5\circ} K \\
T &= 10^5 e^{\frac{Ux'}{2k}} K_0 \left(\frac{Ur'}{2\kappa}\right), \frac{U}{\kappa} = \frac{1 \frac{cm}{sec}}{.1 \frac{cm^2}{sec}} = \frac{10}{cm}
\end{aligned}$$

This gives roughly the dimensions of the melt as on Figure 4 and shows that the yield stress can be exceeded outside this region.

4. Comments

This calculation indicates general features of the stress field and is a first step in the understanding of the problem. The real problem involves much more physics. The melt should be incorporated in the solution. More detailed characterization of resolidified material is needed. The irreversibility of the problem needs to be considered. (O. Ryzhov) For example, if an energy pulse is put into a solid and then turned off the melt grows and shrinks. The temperature distribution is different when the melt is growing or shrinking but at the same radius. Certain periodic cracks [1] appearing along the bead and normal to it are not explained at all by the considerations so for.

There is a large literature on this subject which needs to be digested. Different explanation are offered to give the same results.

References

1. R.A. Patterson, P.L. Martin, B.K. Damkroǵer, and L. Christodovlov, "Titanium Aluminide: Electron beam weldability," *Welding Research Supplement*, Jan. 1990, pp. 395-445.

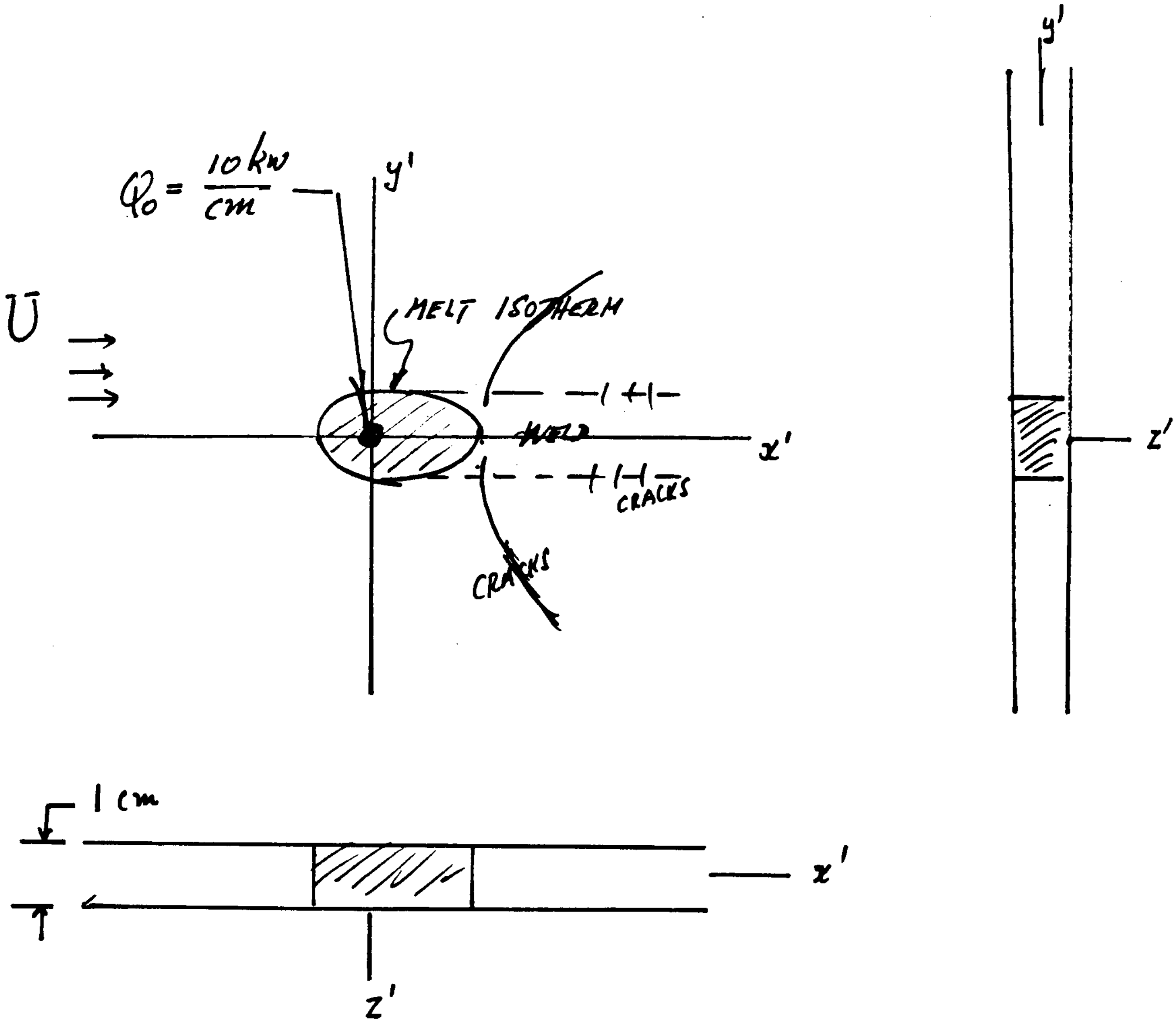
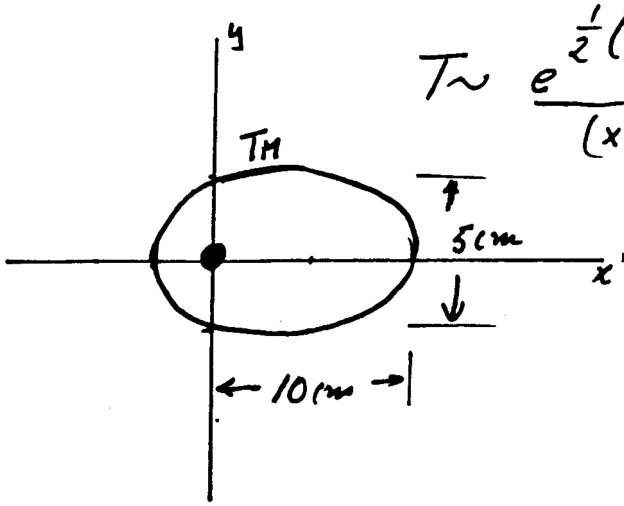


FIG 1. GENERAL SET-UP



$$T \sim \frac{e^{\frac{1}{2}\left(x-\sqrt{x^2+y^2}\right)}}{\left(x^2+y^2\right)^{\frac{1}{4}}}$$

FIG 2. MELT POOL

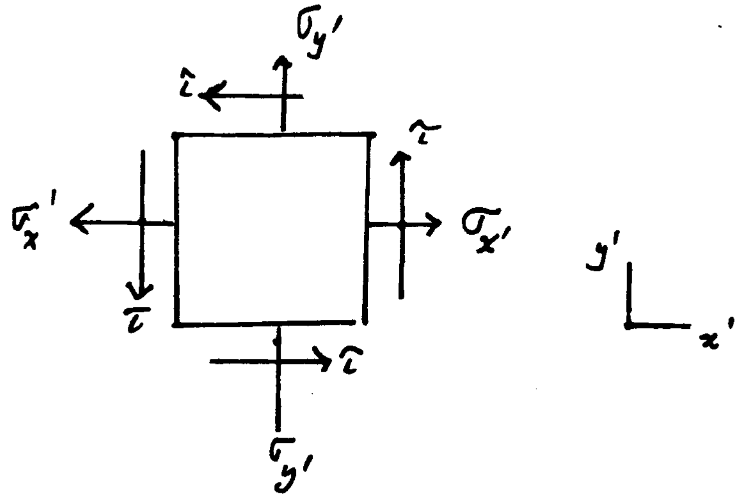


FIG 3. PLANE STRESSES

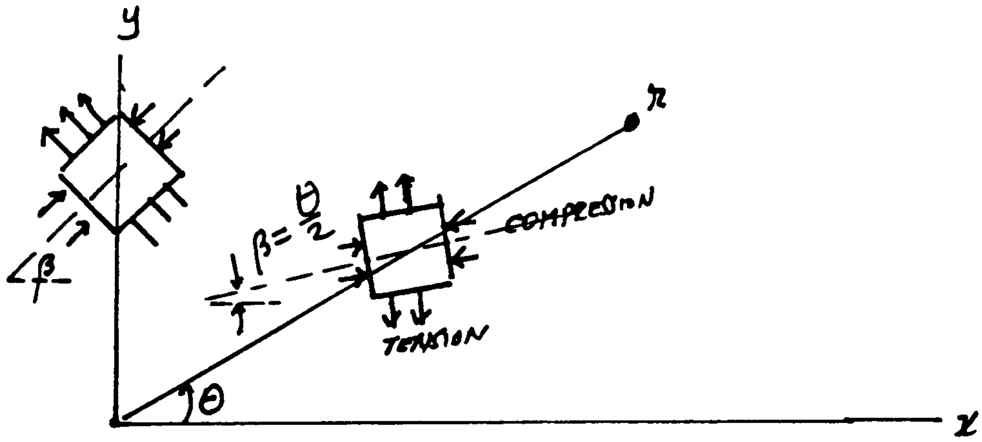


FIG 4. PRINCIPAL STRESSES