

Synchronizing inventory and transport within supply chain management

Report on the problem presented by the

Infora Research Group

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1 Introduction

We study a problem presented by the Infora Research Group about distribution of tobacco in Serbia. The problem is to find the most cost-efficient way to distribute products from the production center in Senta to the customers. After leaving the production center, products can be stored in warehouses before being redistributed to cross-docking points. Cross-docking points are used to change the means of delivery (e.g. truck to van), but they cannot store large amount of goods. The following is the description of factors related to the supply costs.

1. *Locations of facilities*: There is one production center in Senta. Currently, there are 5 warehouses and 16 cross-docking points (see Figure 1), which can be relocated for optimality. There are 16141 customers.
2. *Renting cost for facilities*: Monthly renting cost for warehouses in Belgrade, Niš and Novi Sad is 3.5 €/m², and 2.75 €/m² for warehouses in other cities. Renting cross-docking points costs 100 € for a month.
3. *Capacity of warehouses*: Currently the capacity of the warehouse is 1500 m² in Belgrade, and 1000 m² in Niš, Novi Sad, Kragujevac, and Požarevac.
4. *Means of transportation*: The delivery from the production center to the distribution points is outsourced. Trucks are used from the distribution centers to the cross-docking points, and vans are used for the delivery to the costumers. There are two types of trucks with 3.5t and 5t capacity, and one type of van with 1.5t capacity.
5. *Costs for transportation*: From a warehouse to a cross-docking point and a cross-docking point to customers, costs should be paid for round trips. From the production center to the warehouses, cost is paid for one-way only. The cost is 35 ¢/km for 5t trucks, 30 ¢/km for 3.5t trucks and 20 ¢/km for 1.5t vans.
6. *Customer needs*: One van with 70–80% load can serve about 100 customers in a city in an eight-hour shift. All customers get the same amount of goods for each delivery, and the actual amount of delivered goods is determined by the frequency a customer is visited with.

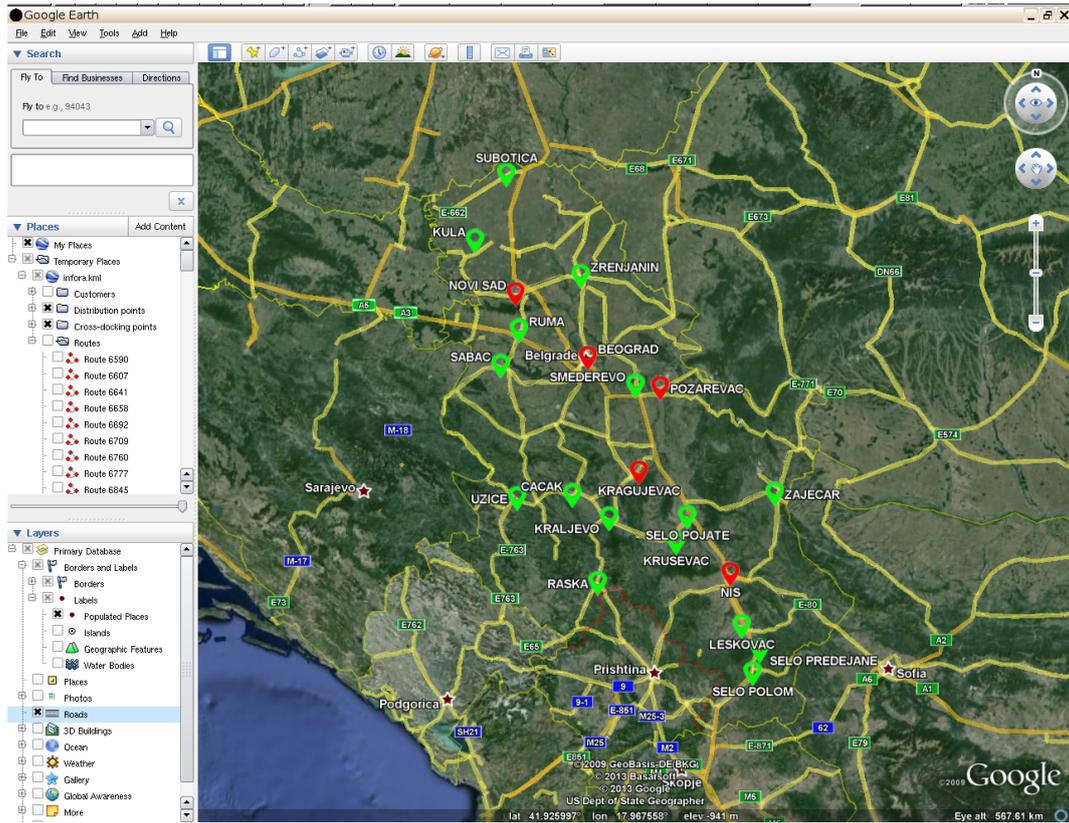


Figure 1: Current location of warehouses and cross-docking points.

Based on the factors listed above, we optimize the supply link, such as the number of warehouses/cross-docking points, their locations, and the optimal route of the delivery.

We have received a table from the Infora Research Group containing the data of all routes: the shops' addresses with geographical coordinates, the warehouses they are served from and the route(s) on which they are served. We converted the table to a database, and created a Google Earth ([1]) presentation from the given data. We found some problems:

- missing coordinates for some of the customers,
- some of the coordinates are incorrect (e.g. a customer in Bulgaria),
- some of the customers are assigned to routes which are not suitable for them as they lie far from other customers on the same route (see Figure 2),
- sometimes the order of the customers visited on the same route is illogical, as part of the route consist of driving back and forth between two regions instead of visiting customers within one region and then moving to the next region (see Figure 3),
- the database is incomplete in the sense that the cross-docking points are not included in the routes and the corresponding cross-docking points are not assigned to the routes; routes between the production place and the warehouses, resp. between the warehouses and the cross-docking points are not given.

We give an overview of a genetic algorithm in section 2. In section 3 and 4, genes and mutation for our problem is described in detail. The advantages and disadvantages of using genetic algorithm is addressed in section 5. In section 6, we present ideas and improvements for the implementation.

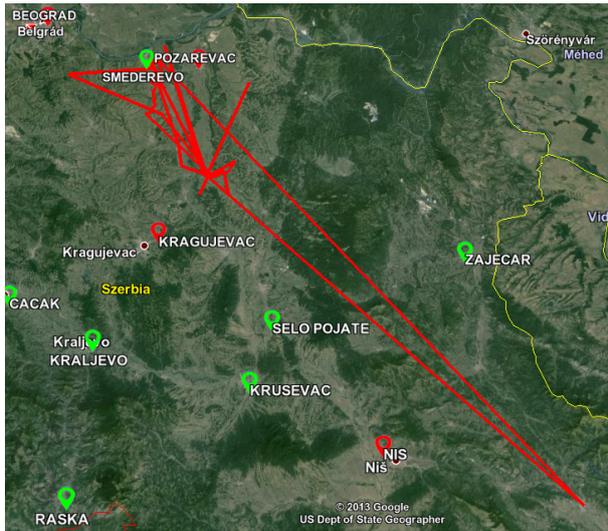


Figure 2: Incorrect route assignment: one customer is too far from the rest

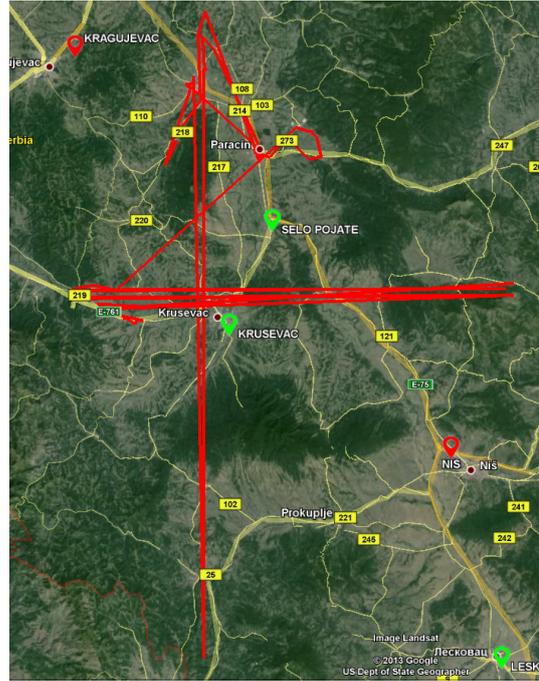


Figure 3: Incorrect route assignment: going back and forth between two regions

2 Overview of the genetic algorithm

The genetic algorithm ([2]) is a nondeterministic algorithm designed to find optimal or near-optimal solutions for optimization problems by imitating the evolution. For an optimization problem, we can think of the possible solutions as a species, the variables of the problem as a predetermined set of genes, and the cost function as a fitness score of the individual solutions. Given a set of possible solutions, the algorithm executes three consecutive steps: crossing of the solutions, mutation and selection.

2.1 Crossing

When two individuals reproduce, they share their genes, therefore providing the opportunity for the offspring to be better than both of its parents. During the crossing stage, we create offsprings by randomly mixing the genes of the selected individuals, and form the next generation from them. Without crossing, the average fitness score of the population would increase very slowly.

2.2 Mutation

Mutation is basically changing each gene of an individual with a small probability. During reproduction, this gives an opportunity to increase the genetic variation of the population. Without mutation, the set of possible genes would be very small, since crossing introduces no variation, only a mixture of genotypes.

2.3 Selection

In each generation, we shall select the fittest individuals to reproduce, and form the next generation from their offsprings. Without selection, it can happen that “bad genes” persist for a long time, therefore disrupting the convergence to the optimal solution. After the new generation

is created, the old one is discarded, and the process is repeated for a predetermined number of generations, when an appropriate fitness is reached, or until the fitness score does not change significantly for a few generations. The last generation of the solutions shall be near-optimal.

3 Genes for this problem

3.1 Genes for warehouses (WH) and cross-docking points (CDP)

There are 39 cities in Serbia with population higher than 20000. We assume that cross-docking points and warehouses are located in these cities – if needed, we can add any other place, or even exact locations. The gene containing the information about the location of WHs and CDPs will be an array of 39 integers. The possible states are:

- 0: no WH or CDP in the city,
- 1: WH,
- 2: CDP.

Along the genes, we need to store additional information about costs. The cost of a WH depends on its square footage, which will be computed from the demand, but the CDPs have fixed costs.

3.2 Supplying routes from the production center to warehouses

There are two pieces of information we have to store:

- the frequency of visits: a bitvector of seven bits, one for each weekday,
- the WHs visited: a bitvector of 39 bits.

The order in which the WHs are visited are determined by solving a one way travelling salesman problem.

3.3 Supplying routes from warehouses to cross-docking points

Every CDP must be supplied from a warehouse with a 3.5t or a 5t truck, and multiple CDPs can be supplied in one route. We need to store the following information for each route:

- Starting warehouse
- The frequency of visits: a bitvector of 7 bits, one for each weekday
- The type of truck used: with the 5t or 3.5t loading capacity
- The CDPs visited: a vector of 39 bits

The order in which the CDPs are visited are determined by solving a travelling salesman problem (TSP, [3]).

3.4 Supplying routes from cross-docking points to customers (CU)

There are 16141 customers which must be supplied with given frequency. Every customer is supplied by a van. For each supplying route, we store the following information:

- The frequency of visits: a bitvector of 7 bits, one for each weekday
- The CUs visited: a bitvector of 16141 bits

The order of the visited customers is once again determined by solving a TSP.

3.5 Conditions on the genes

For a genotype to provide a valid solution, some conditions have to be met.

- All WHs and CDPs must be visited. Although this condition is not really necessary since an unvisited CDP is extra cost and therefore it will be eliminated eventually by the genetic algorithm, enforcing this condition makes the algorithm more efficient.
- Every CU must be visited with at least the frequency they require.
- The demands of the CDPs are computed from the routes and the number of served customers. These demands must be fulfilled by the routes from WHs to CDPs and the demands of the WHs can be calculated from this. These demands must be fulfilled by the routes from the production center to WHs.

4 Mutations and crossovers

4.1 Mutations

Mutations in the genes storing warehouse and cross-docking point locations can be simply changing the status of every city with some probability. This means that in each city, we can

- set up a new warehouse or cross-docking point,
- promote cross-docking points to warehouses,
- demote warehouse to cross-docking point,
- eliminate an existing warehouse or cross-docking point.

The mutations on the routes are more than flipping bits. The possible mutations are

- change frequency of the route,
- add or remove waypoints on the route,
- change the truck used on the route from a WH to CDPs,
- add a new route,
- remove a route.

4.2 Crossovers

For genes containing the location of warehouses and cross-docking stations, we can do the crossover by slicing up the two genes to parts of the same size, and then swap them. This means that for the offsprings, we choose some warehouses and cross-docking points from the mother, and some others from the father. This method also works for genes containing information about the supplying routes: we choose some routes from the mother, and we choose some from the father. In this case, we also have to make sure that the offspring genes satisfy our conditions regarding the routes.

5 The advantages and disadvantages of the genetic algorithm

5.1 Advantages

- (i) The genetic algorithm can solve complex optimization problems, such as this one. We shall optimize the location of the cross-docking points and the supplying routes the same time, because optimizing one and then the other may result in highly sub-optimal solutions.
- (ii) It can handle problems with a large number of variables, and it does not need to know the connection between them.
- (iii) Due to the selection process and the large variation of genotypes granted by mutation, the convergence of the algorithm is fast.
- (iv) It can be easily implemented to run in parallel using multicore processors, GPUs, application accelerators or multiple computers.

5.2 Disadvantages

- (i) The algorithm does not find an exact solution, but instead a bundle of near-optimal ones.
- (ii) It may also find sub-optimal solutions, because it can converge to local extrema. A solution to this problem is to run the algorithm multiple times, starting from a different set of initial solutions.
- (iii) For this particular problem, there are many genes and it may prove difficult to code them efficiently. For example, it is possible that after crossing and mutating genes, we do not obtain a valid solution (some customers may be left unsupplied, we can have unused cross-docking points, etc). To avoid this, we have to check the validity of every genotype obtained by crossing and mutating existing solutions.

6 Ideas for implementation

To decrease the number of genes in our problem it is feasible to handle small towns with few (e.g. at most 25) customers in one route, as in a route connecting smaller towns, moving between the customers inside a town is negligible in comparison with the distance between the towns. 92% of the towns fall in this category, which means a significant decrease in the number of customers to be handled separately.

It seems logical to handle customers in cities where a warehouse or a cross-docking point is located, from that warehouse or cross-docking point. This means solving a multipath travelling salesman problem in the candidate cities before we start the genetic algorithm. This method decreases the number of customers to be handled individually even more: after applying this and the previous idea together only 17% of the customers remain.

We can also decrease the number of genes by assuming that all customers are covered only from a cross-docking point which is at most twice as far as the nearest cross-docking point, and all cross-docking points are covered from a warehouse which is at most twice as far as the nearest warehouse.

We can also fix the location of the warehouses (if moving them is too costly) and the cross-docking points, and even (part of) the routes if necessary, the genetic algorithm easily allows that.

For speeding up the computation, we can cache the shortest paths and TSP results which have already been computed as well as keep track of the costs and recompute them only if they are changing.

The algorithm does not change if we change anything here, e.g.: the frequency from weekly to biweekly or monthly, the candidate cities for warehouses and cross-docking points, the number of customers, the actual costs, the number and type of trucks and vans, etc. The genetic algorithm can also handle more complex conditions if necessary, e.g. delivery times, actual orders of the customers, etc.

7 Conclusions

The problem presented by the Infora Research Group is quite complicated and challenging. It contains location and route optimization tasks within a supply chain to achieve the lowest cost possible. In order to start with the implementation we would need the correct actual routes, so that we know the exact problem, and we can compare our results with the current costs. We hope to cope with this problem using the genetic algorithm, which seems suitable for handling the several variables used here.

8 Acknowledgements

We would like to thank the companies for the interesting problems, especially the Infora Research Group and its representative, Nebojša Gvozdenović, who was very supportive during the conference. This was our first, but certainly not the last participation on the conference series *European Study Group with Industry*, we enjoyed it very much. We would also like to thank the members of the Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, for organizing this conference.

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Synchronizing inventory and transport within supply chain management (territorial approach)

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1 Introduction

Infora Research Group presented a problem of optimizing the distribution costs of cigarettes from a single production facility placed in Senta to approximately 15000 shops in Serbia. Current solution includes 5 warehouses (located at Novi Sad, Beograd, Požarevac, Kragujevac, Niš). The cigarettes are transported from Senta to warehouses with big rented trucks paid on the bases of distance from Senta.

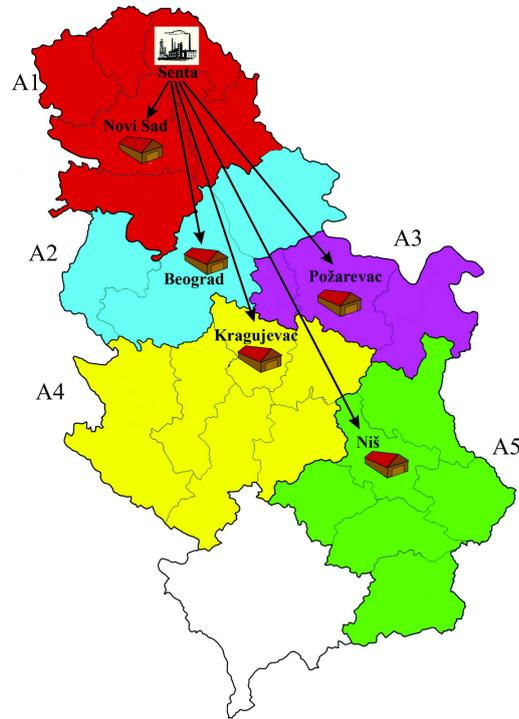


Figure 1: Placement of warehouses and division of territories into areas.

Cigarettes are stored in warehouses wherefrom smaller trucks, owned by distribution company, distribute them to cross-docking points where they are reloaded to several smaller vans which distribute them further to shops. Current number of cross-docking points covered by one warehouse is 5 in average. Vans can also be loaded at the warehouse with no extra renting cost and they supply shops near the warehouse.

The industry representative expected to get a mathematical model which can optimize the overall distribution costs by appropriate placement of warehouses and cross-docking points.

Major drawback was absence of reliable real data for van routes and shop demands. From the communication with the industry representative it turned out that there are big chances that reliable data on shop level could not be obtained in practise. Nevertheless, demand data for territories of the size of municipalities or similar could be obtained from historical data. Based on constraints mentioned above, our study group decided to develop the mathematical model which needs only demand data from a number of territories connected with the road network. The model is proposed with the current territorial division of Serbia into regions,

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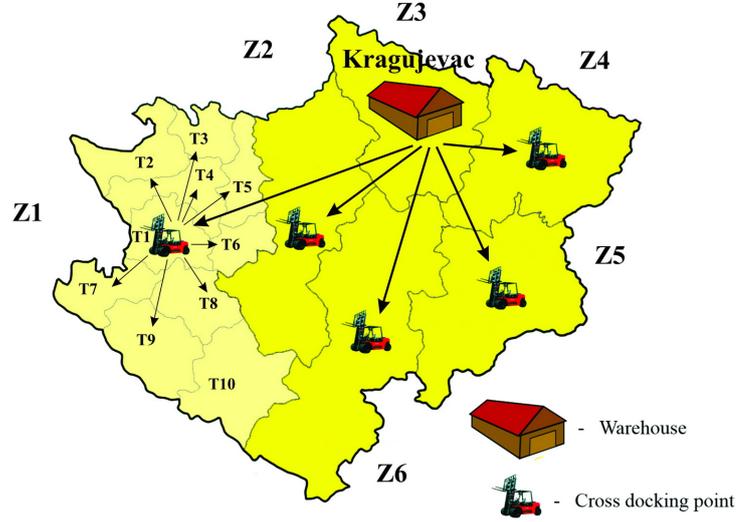


Figure 2: Placement of cross-docking points and division of territories into zones.

districts and municipalities in mind. Although municipalities are natural candidates for territories in the proposed model, smaller territories could sometimes be more appropriate.

2 Notation

- T - the set of territories. The whole market is divided into smaller territories T_i , for instance municipalities or smaller, such that demand data are known for that territory. Territories are organized as a graph with known distances between each two territories $d(T_i, T_j)$.
- W - the set of all possible places for warehouses W_k . For each possible warehouse place W_i , $T(W_i)$ denotes the territory on which it resides, R_{W_i} is overall renting cost for it over a unit time period. Renting cost may be a function of total demand covered by the warehouse.
- P - the set of all possible places for cross-docking points P_j . For each possible point P_j , $T(P_j)$ denotes the territory on which it resides, R_{P_j} is overall renting cost for it over a unit time period.
- D_{T_i} - known demand for each territory T_i over a unit period of time.

- L_i - overall distribution costs to individual shops on territory T_i under assumption that there is a cross-docking point on that territory.
- c_b, c_s, c_v - cost of transport per unit distance and unit quantity of goods for big trucks (from production facility to warehouses), small trucks (from warehouses to cross-docking points) and vans (from points to shops). Assumption is that all vehicles are reasonably well loaded so that this cost can be used as a constant in practise.

3 Modelling the total cost of distribution

For a fixed combination of warehouses W_1, \dots, W_{kmax} the set of all territories can be divided into areas A_1, \dots, A_{kmax} with respect to closeness to a warehouse based on known distances $d(T_i, T(W_k))$ (Fig. 1). It is assumed that all territories in area A_k are supplied through the warehouse W_k .

Within one area A_k there is a set of cross-docking points P_1, \dots, P_{jmax} , each point P_j placed on territory $T(P_j)$. All territories from the area are divided into zones Z_1, \dots, Z_{jmax} with respect to closeness to points P_j based on known distances $d(T_i, T(P_j))$ (Fig. 2).

For fixed positions of warehouses and cross-docking points, and corresponding partition into areas and zones, total cost of distribution could be calculated in the following way. First, knowing the demands D_i for each territory T_i , by summation we obtain the total demands D_{P_j} for each zone Z_j with cross-docking point P_j , and total demands D_{W_k} for each area A_k with warehouse W_k .

On single territory T_i total cost of distribution is L_i in case there is a cross-docking point on T_i . In the other case, when the nearest point P_j is on another territory, cost can be approximated as $L_i + d(T_i, T(P_j)) \cdot D_{T_i} \cdot c_v$.

Total cost of distribution from one cross-docking point P_j on its zone Z_j is

$$C_{P_j} = R_{P_j} + \sum_{T_i \in Z_j} (L_i + d(T_i, T(P_j)) \cdot D_{T_i} \cdot c_v) = \sum_{T_i \in Z_j} L_i + C_{P_j}^*$$

where the renting cost $R_{P_j} = R_{P_j}(D_{P_j})$ can depend on the demand, and

$$C_{P_j}^* = R_{P_j} + \sum_{T_i \in Z_j} d(T_i, T(P_j)) \cdot D_{T_i} \cdot c_v$$

does not depend on local costs L_i .

Total cost of distribution from one warehouse W_k on its area A_k is

$$C_{W_k} = R_{W_k} + \sum_{P_j \in A_k} (C_{P_j} + d(T(P_j), T(W_k)) \cdot D_{P_j} \cdot c_s) = \sum_{T_i \in A_k} L_i + C_{W_k}^*$$

where the renting cost $R_{W_k} = R_{W_k}(D_{W_k})$ can depend on the demand, and

$$C_{W_k}^* = R_{W_k} + \sum_{P_j \in A_k} (C_{P_j}^* + d(T(P_j), T(W_k)) \cdot D_{P_j} \cdot c_s)$$

does not depend on local costs L_i .

Total cost of distribution is

$$C = \sum_{W_k} (C_{W_k} + d(T(Production), T(W_k)) \cdot D_{W_k} \cdot c_b) = \sum_i L_i + C^*$$

where

$$C^* = \sum_{W_k} (C_{W_k}^* + d(T(Production), T(W_k)) \cdot D_{W_k} \cdot c_b).$$

Since the sum of local distribution costs $\sum_i L_i$ does not depend on number and positions of warehouses and cross-docking points, optimization of total cost function C is equivalent to optimization of C^* . The function C^* does not contain local costs L_i , so using this model these data are not needed.

Since the renting costs C_{W_k} for the warehouse W_k is allowed to depend on the total demand D_{W_k} it covers, costs of transport from central production facility in Senta to the warehouse, since it also depends on this demand, can be incorporated into the renting costs. This leads to the very similar model containing only warehouses and cross docking points, without the central production facility.

4 Optimizing the total cost of distribution

For optimization of the number and positions of warehouses W_k and cross-docking points P_j using the model described in previous section the data for local distribution costs L_i are not needed, only the data for demands D_i and territorial data of distances between territories. The territories could be organized as a weighted graph, each node representing one territory T_i with edge weights representing the distances. This graph could be assembled using the road map.

Optimization of the total cost function C^* can be organized in two levels. The first level optimizes the positions of warehouses on the map. For each proposed combination of warehouse positions, all territories are divided into areas A_k with respect to the nearest warehouse criteria. For each area A_k the second level of optimization provides the optimal number and positions of cross-docking points P_j on that area providing the minimal total cost $C_{W_k}^*$. These second level optimizations could be done in parallel. At the territory $T(W_k)$ where the warehouse is placed, the rental cost for cross-docking point should be put to zero, or this point should be fixed at warehouse position with other points being subject to optimization.

For this combinatorial optimization problem exact methods could be very time consuming, so heuristic approach could be more appropriate. Genetic algorithms, variable neighborhood search, simulated annealing, swarm optimization and other metaheuristics could provide good solutions. Constrains such as minimal or maximal demands and similar could be incorporated through penalty functions in the model.

The second level optimization problem is recognized in literature as a hub location problem ([1]). Similar class of problems ([2]) could be very efficiently solved using modern metaheuristics. Variable neighborhood search algorithm can solve problems with 100 nodes and 10 hubs for less than a second. The same problem can also be considered as the hub location problem with star network structure ([3]). Memetic algorithms (hybrid metaheuristic methods combining genetic algorithm and a local search procedure) are also very efficient for this type of problems ([4],[5]).

The model developed in previous section can be also considered as a hierarchical facility location problem ([6]). A mixed integer linear programming model for this problem is recently proposed in [7].

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Synchronizing inventory and transport within supply chain management

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PROBLEM DEFINITION

The practical problem considers synchronized optimization of inventory and transport, and focuses on producer-distributor relations. A producer has signed a contract with a distributor that guarantees exclusivity for sales in a certain region/country.

Given a region/country with its road network, historical data about sales, and all relevant costs (facilities, vehicles, workers, and haulage), it should be provided a way for a distributor to organize his sales/distribution network.

Since goods are owned by the distributor as soon as they leave production facility, and since stock out results in lost sales, the distributor's cash flow highly depends on inventory control. Given sales forecasts, should be found methods how the producer can manage its inventories. Finally, some suggestions about contracting models that lead to more profitable supply chains should be given.

Introduction

The problem of synchronizing inventory and transport within supply chain management attracts attention and has been studied for many years [1, 3, 5, 7]. The supply chains of large corporations involve hundreds of facilities (retailers, distributors, plants and suppliers) that are globally distributed and involve thousands of parts and products. The goals of corporate supply chains are to provide customers with the products they want in a timely way and as efficiently and profitably as possible. Fueled in part by the information revolution and the rise of e-commerce, the development of models of supply chains and their optimization has emerged as an important way of coping with this complexity. Indeed, this is one of the most active application areas of operations research and management science today. This reflects the realization that the success of a company generally depends on the efficiency with which it can design, manufacture and distribute its products in an increasingly competitive global economy [1].

Supply chain management is a dynamic operation research problem where one has to quickly adapt according to the changes perceived in environment in order to maximize the benefit or minimize the loss. Supply chain management can be defined as "a goal-oriented network of processes and stock points used to deliver goods and services to customers" [6]. Science of supply chains deals with an array of suppliers, plants, warehouses (WHs), customers, transportation networks and information systems that make up actual supply chains.

Many companies search for efficient distribution alternatives, as the lead times for customer order fulfillment need to be shortened while the costs and risks of warehousing need to be minimized. Cross-docking is an operation strategy that moves items through consolidation centers or cross docks without putting them into storage [3]. In our case we have one supplier that should provide some products through a network of shops. This scheme includes wide variety of problems, such as transportation scheduling problems and warehouse location problems. These problems are independently defined as optimization problems, and algorithms have been proposed for each problem. There are different approaches that have already been proposed for solving similar kind of problems. Some of them are using genetic algorithm [4], gravitational search algorithm [2], heuristic methods and also simulation based algorithms [7].

The main purpose in our work is to optimize the distribution network of MPC Holding Mercata that has a pioneer role in domestic wholesale development for Serbia and is committed to the wholesale and distribution of tobacco products and consumer goods.

The Warehouse Location Problem (WLP)

The (uncapacitated) WLP is a problem to minimize the sum of the transportation cost and the fixed cost of warehouses. Let S be the set of shops, W be the set of candidate locations for warehouses, f_j be the fixed cost for opening a warehouse $j \in W$, and c_{ij} be the cost to supply shop i from warehouse j . The WLP is defined as follows:

$$WLP(x, y) = \sum_{i \in S} \sum_{j \in W} c_{ij} x_{ij} + \sum_{j \in W} f_j y_j \rightarrow \min_{x, y} \quad (1)$$

subject to $\sum_{j \in W} x_{ij} = 1$ for each $i \in S$

$$x_{ij}, y_j \in \{0, 1\} \text{ for each } i \in S, j \in W$$

$$0 \leq x_{ij} \leq y_j \leq 1 \text{ for each } i \in S, j \in W$$

where x and y are decision variables ($x_{ij} = 1$ decides if store i is supplied from

warehouse j , $y_j = 1$ decides if warehouse j is open).

The first term of $WLP(x, y)$ represents the transportation cost between warehouses and shops and the second one represents the fixed cost of the warehouses. The first constraint means that each shop must be supplied by only one warehouse. The second constraint means that the shops must be supplied by open warehouses. The last constraint means that the variables are zero-one. For this problem, Beasley proposed a Lagrangian relaxation algorithm which can find optimal or near optimal solutions quickly [5].

Our approach

At the moment, the distribution network of MPC Holding Mercata in Serbia is:

- One factory (in Senta);
- Currently 5 WHs (Novi Sad, Beograd, Pozharevac, Kragujevac, Nish) for whole Serbia;
- Different numbers of cross-docking points (CDPs) supplied from a given WH;
- Different numbers of shops supplied from a given CDP (by smaller transport vehicles) or directly from the corresponding WH

To optimize the network it is possible to open new WHs/CDPs and change the position of any WH/CDP. Also, some WHs/CDPs can be closed.

We have to consider the following constraints (based on the received data by the contact person from the company):

- The transport vehicles from given WH are trucks and vans;
- The transport prices are: for 5 ton (capacity) truck 0.35 Eur/km; for 3.5 ton truck 0.30 Eur/km; for a van 0.20 Eur/km.
- One truck supplies up to 4 vans;
- The rental cost of CDP is up to 100 Eur per month (that is negligible).

For these reasons we do not need to consider the Traveling Salesman Problem. Instead of it we can apply the Dijkstra's algorithm to compute the lengths of the shortest paths and the tree of these paths (the root of the tree is a given WH).

In order to get an optimized solution we make the following:

- Redefine the terms of WLP depending on the possibility to add the intermediate level of CDPs (one possible way to do that is to define 2-stage WLP: 1'st stage "factory-WHs" and 2'nd stage "WH-CDPs");
- Decide WLP for WHs by Simplex Algorithm for 0-1 Integer Linear Programming (or by the algorithm suggested in [5]). Generally, it is NP-complete problem but for practical goals it is possible to get a solution in polynomial time. Also, the current positions of the WHs can be taken as initial;
- Split the region supplied from a given WH to the smallest possible parts. In our case, these are the territories supplied by the smallest used transport vehicle (vans), as in Figure 1 (a);
- Estimate the values of distances from any territory to the corresponding WH (this estimation is based on the road network of Serbia).

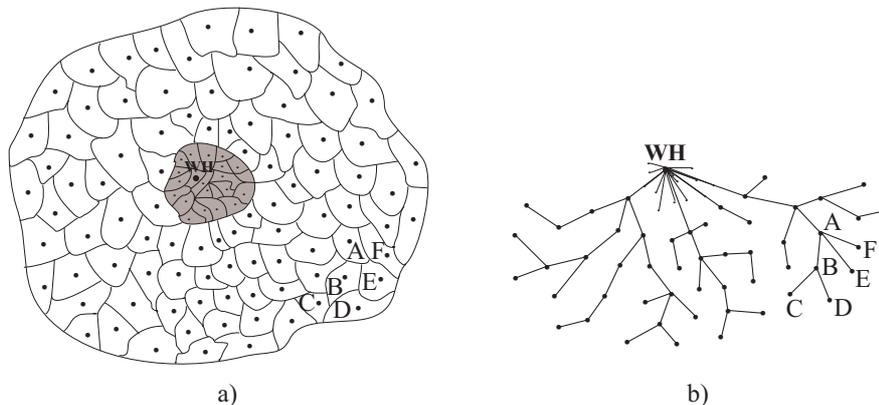


Figure 1: a) The WH area; b) A part of the computed tree of the shortest paths

Figure 2 represents the two possibilities for a given smallest part that we consider. First, it can be a set of small towns and villages (a). In that case we choose one town (or village) to be a 'center' of this territory and we calculate the distance from this point to the WH. In the second case this is a big town or city and we can assume that the distances from WH to all van's territories are the same (b). So, all these vans are associated with the same 'center' – this big town or city. The borders and the dots inside (in Figure 2) denote the shops supplied by the corresponding van, i.e. its territory. We assume that each van supplies its territory in a way near to the optimal one.

Let V be the set of all centers, including WH. We define a weighted graph $G = (V, R)$ where V is the set of vertices of G , R is the set of all edges between two adjacent

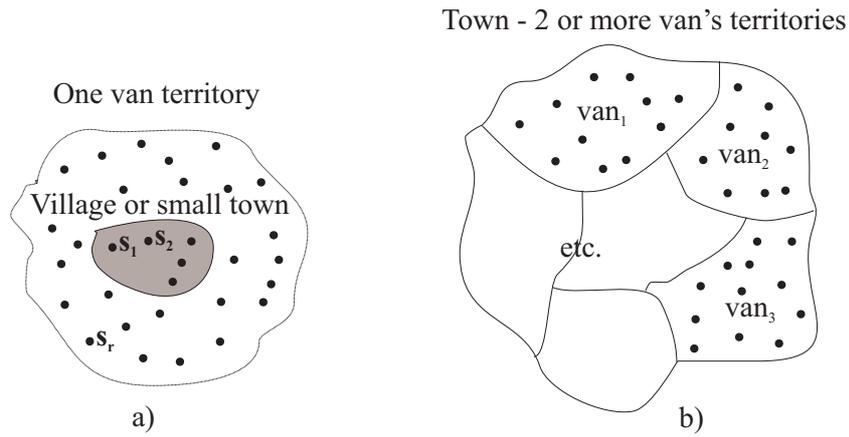


Figure 2: The smallest possible part

vertices (when there is a direct route between them), and L is the set of all weights of the edges in R .

By the defined graph we can apply the following method:

1. Use Dijkstra's algorithm to obtain the tree of the shortest paths (the root is that WH);
2. Choose the outermost leaves of the obtained tree and compute where to place CDP more efficiently;
3. Place CDP that supply some centers that are leaves of the tree;
4. Cut (exclude) already supplied centers (by the last placed CDP);
5. If there are nonsupplied centers go to Step 2;
6. End.

It can be seen that the optimization depends on:

- The distances from the WH to each center (so at any step we choose the longest path).
- The type of the territories A, B, \dots, F and the number of vans that supply them.
 - Figure 3 illustrates Step 2 in more details if the center is of type shown in Figure 2 (a). The optimization in this case can be computed as follows:

$$p_1 = (2(l_1 + \dots + l_5) + 2(l_1 + \dots + l_4 + l_6) + 2(l_1 + \dots + l_4))p_v,$$

where p_v is the price of van per km. Here p_1 is the transport price for supplying the centers B, C, D by vans.

$$p_2 = 2(l_1 + \dots + l_4)p_{bt} + 2(l_5 + l_6)p_v,$$

where p_{bt} is the price of big truck per km. Here p_2 is the transport price if we place a CDP in center B .

$$p_{opt} = p_1 - p_2 = 2(l_1 + \dots + l_4)(3p_v - p_{bt})$$

Here p_{opt} is the value of the obtained optimization. This value is positive because $3p_v > p_{bt}$ (as it was shown above).

- If the center is of type shown in Figure 2 (b) then this center becomes a CDP. Obviously, in this case the optimization is maximal.

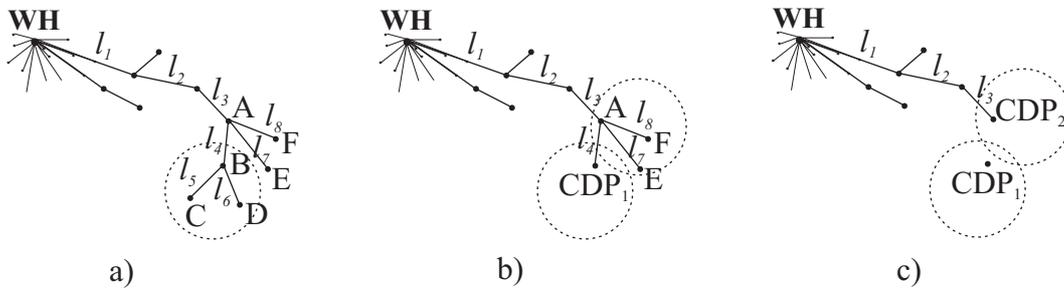


Figure 3: a) No placed CDP; b) One placed CDP; c) Two placed CDPs

Conclusions

By using the proposed model the distribution network can be optimized in general. The main advantage of this model is that (once computed) the tree of the shortest paths can be used many times until some change of the road network happens. Also, this model can be used in case of occasional supplies (after excluding the centers without demands).

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