Estimating the Spread of Fire in Buildings

Problem presented by

A.W.E.

Aldermaston

Executive Summary

AWE is developing a general fire modelling capability at a city scale and want to upgrade their current capability by incorporating suitable building-scale models into the simulation. The main question that needs answering is: given an initial state in which fires are known to be burning in a particular location or region of a building of given dimensions, is it possible to determine estimates for the rate at which fire spreads throughout this building? In particular, estimates for the temperature at the outer walls as a function of time.

This report addresses the problem in three ways. First, the state of the art in the literature on fire modelling is reviewed and a mid-complexity model (referred to as "the Kyoto model") which treats rooms as individual compartments is recommended for further study. This is then simplified into an ODE compartmentalised model only those processes that are believed to be essential. A fully dimensional version of the model is developed including estimates for all the physical paramaters. After nondimensionlisation, the model is simulated for a building comprising a chain of rooms separated by fire doors. Finally, a homogenised reaction-diffusion PDE model is developed in one and two space dimensions. In this model walls are treated as areas of greater porosity. Simulation results reveal the sensitivity of the fire spread to the internal layout of the building. These final two models have both been implemented as a Matlab and C-code deliverable respectively.

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Contents

1 Introduction

(1.1) This report gives details of the work done during the 91st European Study Group with Industry held at Bristol from 15-19th April 2013. The problem, as presented by AWE is set out in section 2 below. The key part of the remit was not to produce a high complexity computational model, but to brainstorm how to produce a simplified model that nevertheless captures the true physics of fires at the building scale.

(1.2) The rest of the report is outlined as follows. First, in section 2 a precise statement of the problem is given. Section 3 then conducts a partial literature review and makes some initial assumptions. Section 4 then presents three different methods of solution. First, in 4.1 we present the bare bone details of a compartmentalised ODE-based building-scale model from the literature. Next, Section 4.2 derives a simplified version of this model and presents results simplified, but fully parametrised version of this model for the spread of the fire through a 1D chain of rooms. Appendix A.1 presents more details of the derivation of this model, including reasoning for approximate values for all the key parameters in the model. The final solution, in Section 4.3 involves modelling fire spread through the building via an inhomogeneous PDE, with the details given in Appendix A.2. Finally, Section 5 draws conclusions and suggests avenues for future work.

2 Problem statement

(2.1) How fire spreads is of great importance when considering the safety of critical assets. As such, AWE plays an important role in supporting UK Government departments in assessing risks associated with the spread of fire.

(2.2) Currently AWE has a large scale, grid-based model for urban areas that ignores the effects of individual buildings. However, it is known that different buildings types will change the spread of fire significantly. Thus suitable building-scale models must be incorporated into the simulation to assess the risk at specific locations.

(2.3) In principle it is possible to incorporate a great deal of information about the layout and composition of buildings. But it is extremely expensive in terms of money and time to acquire and incorporate this information into a model. Therefore, it is desirable to know what can be determined with only a minimal description of the building.

(2.4) Thus our main objective is to determine estimates for the rate of fire development, spread and decay in a building with minimal description of the building. That is, given an initial state in which fires are known to be burning in a particular location or region of a building of given dimensions, we

want to determine estimates for the rate at which fire spreads throughout this building.

3 Initial assumptions and literature review

(3.1) We require a computationally inexpensive time-dependent solutions at a building scale for two and potentially three-dimensional spread of fire through a building. In particular we want to determine the time evolution of the pattern of fire spread (and amount of damage), the radiative heat flux/temperature at the boundaries, and the amount of available fuel represented by the building fabric and contents.

(3.2) We deemed it out of our remit to model the initial ignition mechanism, or the how fire spreads from building to building.

(3.3) We conducted a partial literature review on fire modelling. There are several classes of model. For region scale models there is a large literature on wildland surface fires. A review can be found in [26], and an older one in [25]. These generally lead to PDE models with potentially non-homogeneous terms representing different vegetation. These models can potentially be adapted to deal with urban environment.

(3.4) At the building scale, there are several highly comprehensive codes there have been some recent highly accurate studies involving comprehensive models that incorporate detailed descriptions of reaction-chemistry, fluid dynamics, thermodynamics, gas flow, and specific information on materials and building structure [19, 2, 3, 7]. In particular the work [2] describes a highly detailed model of the fire spread in the World Trade Centre.

(3.5) However, these approaches involve computational intensive numerical simulations that would be prohibitively slow and expensive to use for in larger-scale application at the city scale. Hence this report shall mostly consider the use of simplified models that are more computationally tractable but can still reproduce good estimates of the spreading of the fire.

(3.6) There is also a comprehensive fire modelling package available at NIST [21] which falls into a similar category, but might be considered as a detailed industry standard software that can be used to benchmark simple mathematical models against.

(3.7) We began by developing a simple cellular automata model for fire spread in a 2D building, see e.g [24] for the concept of such a model. Each cell represents a room on a single floor. (Or alternatively, a floor in a tall building). Simple scalar quantities like percentage of fuel left in each cell are represented and updated in discrete time steps according to values of the same quantities in the neighbouring rooms. Extension to 3D are possible where the effect of

buoyancy can be modelled by giving different update rules for vertical or horizontal fire spread. This idea was not further pursued as it did not seem obvious how to include the true physics into the update rules.

(3.8) However, we did find another kind of model in the literature, that could be described as a compartmental model. This is like a cellular automata but there are continuous variables within each cell or compartment that represent physical quantities and are updated using ordinary differential equations. A particular example of such a model shall be described further in section 4.1 below, which we refer to as the Kyoto model, see [15, 14]. The most promising thing about the Kyoto model is that the publications appear to contain results at a city scale, by applying the model first at the building scale and then scaling up to model a complete city region.

(3.9) A good non-technical report on the spread of fire through buildings can be found at the US CFBT website [13]. It highlights that fire goes through six different phases:

1. **Ignition** The phase in which the fire starts. This can happen in one room or in several rooms and it will grow at certain rate which will depend on the amount of fuel, the flux of oxygen and the layout of the building.
2. **Growth** This phase consists of the spreading of the fire through the total surface of fuel. Depending on the flammability of the material, the amount and the energy released in the combustion during this phase, a big blast, so-called *flashover* may occur.
3. **Flashover** A flashover is the near-simultaneous ignition of most of the directly exposed combustible material in an enclosed area. An example of flashover is when a piece of furniture is ignited in a domestic room. The fire involving the initial piece of furniture can produce a layer of hot smoke which spreads across the ceiling in the room. The hot buoyant smoke layer grows in depth, as it is bounded by the walls of the room. The radiated heat from this layer heats the surfaces of the directly exposed combustible materials in the room, causing them to give off flammable gases. When the temperatures of the evolved gases become high enough, these gases will ignite engulfing the entire room in flames. The flashover signals the end of the growth stage leaving the fire in the fully developed stage. Eventually a lack of fuel will lead to the decay stage.
4. **Full development** This phase consists of the spreading of the fire through the total surface of fuel and the surface of the room but the ability to communicate with the exterior is still weak.
5. **Structure fire** During this phase the fire communicates with other rooms, either through windows, doors, pipes or other ways.
6. **Collapse** The most flammable fuel of the room has already been consumed and the rate of heat released starts to decrease until it is ex-

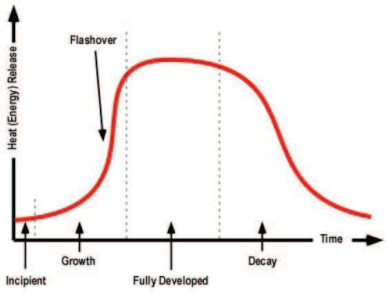

Figure 1: Figs/flashover.pdf

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(3.10) According to [13] conditions for flashover are defined in a variety of different ways. In general, ceiling temperature in the compartment must reach at least $500 - 600°C$ or the heat flux (a measure of heat transfer) to the floor of the compartment must reach $15 - 20kW/m$. When flashover occurs, this causes compartments (such as a door leading to another room) to blow out and for gasses to escape at a substantial velocity.

(3.11) Heat spread mechanisms for fires include:

1. **Convection** The transfer of heat from one place to another by the movement of fluids this is the dominant form of heat transfer in liquids and gases. Although often discussed as a distinct method of heat transfer, convective heat transfer involves the combined processes of conduction (heat diffusion) and advection (heat transfer by bulk fluid flow). Convection can be "forced" by movement of a fluid by means other than buoyancy forces. In some cases, natural buoyancy forces alone are entirely responsible for fluid motion when the fluid is heated, and this process is called "natural convection." This means that when there is a fire hot gasses will move upwards, and so the fire will spread very quickly up stairwells and lift shafts. Obviously anything object that comes into contact with the fire will receive a very large heat transfer.
2. **Conduction** Conduction is the transfer of heat energy by microscopic diffusion and collisions of particles or quasi-particles within a body due to a temperature gradient and is the dominant form of heat transfer in solids. This means that if one room in a building is on fire, heat can be passed to neighbouring rooms through walls, even if there are no gaps for the fire itself to travel through.
3. **Thermal radiation** Radiation is electromagnetic radiation generated by the thermal motion of charged particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation. This means that the surrounding area will receive heat energy from the fire without having come into contact with either the smoke

or the flames. Radiation is the dominant form of heat transfer in developed large fires, although it can also be a cause of ignition due to concentrated sun rays. Radiation from the fire will cause the ambient temperature to increase, when the majority of the exposed surfaces in a space are heated to their autoignition temperature and emit flammable gases flashover will occur.

(3.12) The literature suggests that fire spreads through the building in different ways:

1. **Direct** Flames communicate directly with other rooms.
2. **Radiative** Ignited rooms emit radiation to other rooms which get heated. After certain time of being exposed to this radiation, rooms achieve a critical temperature in which it gets ignited as well. Furthermore, flames spread easier through pre-heated rooms.
3. **Thermal plume** Fire spreads via mass transport of the smoke down doors and windows.

(3.13) Initial brainstorming and consultation with AWE led us to conclude that the spreading of fires within a building are likely to be dependent on many factors:

1. Building:
   - Dimensions
   - Shape (N-sided, e.g. L-shape, U-shape)
   - Usage (e.g. office, warehouse, apartment block)
   - Windows (for ventilation)
   - Internal layout
   - Internal firebreaks
2. Room
   - Dimensions
   - Fuel loading
3. Initial ignitions
   - Number
   - Location

(3.14) We did not have time to investigate varied geometry. But we were led to conclude that the layout of rooms is a key property, as is the assumed availability of fuel within each room. Essentially, we tried to investigate two kinds of building; a warehouse (one room) and an office (layout of many rooms). We also did not have time to investigate genuinely 3D effects.

(3.15) Our working assumption was that fire started on one edge of the building, that all windows were open, that there was sufficient fuel in each room for the fire to spread rapidly via convection (thermal plumes) and to remain

alight throughout the fire spread process. However, we assumed that walls and doorways would remain intact until their individual flashover point was reached.

4 The solution

4.1 The Kyoto model

(4.1) The Kyoto model is a computational model where each room or significant open area within a building is treated as a separate compartment within which all physical quantities are spatially lumped. The model has been developed by Himoto and Tanaka at the University of Kyoto (CHECK) across a number of publications, culminating in [15, 14].

(4.2) A schematic of what is assumed within each room, and between rooms is given in figure 2, which also includes the space between two buildings.

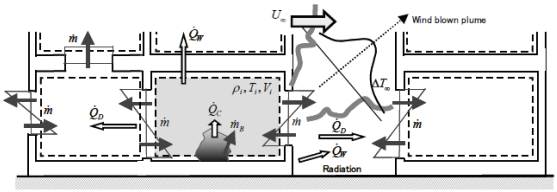

Figure 2: A schematic of the Kyoto model, taken from [15, 14]

(4.3) The basic physics in each room (with index i) can be summarised by the following conservation laws where summations are taken across all neighbouring rooms.

$$\frac{dm_i}{dt} = \dot{m}_{F,i} - \sum_j (\dot{m}_{i,j} - \dot{m}_{j,i})$$

$$\frac{dQ_i}{dt} = (\dot{Q}_{B,i} + c_p \dot{m}_{F,i} T_p) - \sum_i (\dot{Q}_{L,i} + \sum_j (c_p \dot{m}_{i,p} T_i - c_p \dot{m}_{j,i} T_j))$$

$$\frac{d}{dt}(m_i Y_{X,i}) = \dot{\Gamma}_{X,i} - \sum_j (\dot{m}_{i,p} Y_{X,i} - \dot{m}_{p,i} Y_{X,i})$$

These represent respectively conservation of mass, energy and momentum. See [15, 14] for the meaning of all the variables and parameters, and also for many of the transfer rules depending on the type of boundary assumed (wall, vent, ceiling etc.).

(4.4) It is impressive that Himoto and Tanaka are able to use the model to simulate a real fire in an entire city region. See figure 3

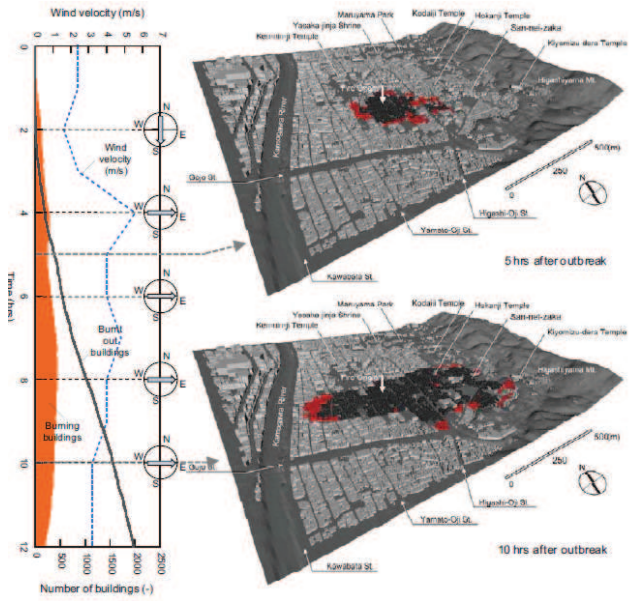

Figure 3: City district scale output from the Kyoto model, taken from [15, 14]

(4.5) A drawback of the kyoto model for the purposes of the study group is its large generality and the huge number of physical parameters that need to be taken into account in order to model any realistic scenario. It was also not entirely clear to us how to incorporate non-trivial combustion kinetics into the model, which might lead to the critical notion of flashover.

4.2 A simplified ODE model

(4.6) Inspired by the Kyoto model, we have set up a simplified compartmental model that considers a sequence of N rooms connected by doors, which are all initially closed. A fire is started at one end of the sequence of rooms (*e.g.* on one side of a building) and spreads through the rooms sequentially. An important aspect of the model is the presence of the closed doors, which block mass transfer (and the resulting convective heat transfer). The doors are assumed to maintain their integrity until they reach a critical temperature W_c, the *flashover point, at which they instantly disintegrate.*

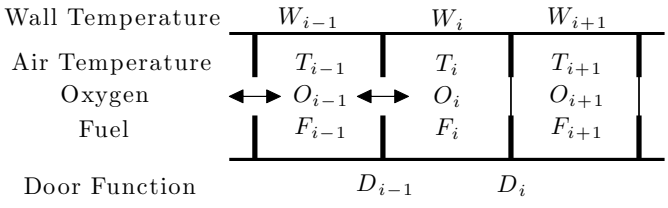

Figure 4: Setup for the ODE model.

(4.7) The basic combustion model is assumed to obey an an Arrhenius temperature dependence and a simple linear dependence on oxygen and fuel concentrations. Applying the law of mass action and results in four ordinary differential equations for the wall and air temperatures and the fuel and oxygen concentrations.

(4.8) The details of the model are given in Appendix A.1, where the 21 dimensional parameters that enter the model are listed, assuming for simplicity that each of the rooms is identical. Typical values for each of these constants is determined, in particular assume a room dimensions of size of $30m^3$, with a door size of $6m^2$ and surface of fuel (thought of as wooden and soft furniture, paper etc.) being comparable to the wall surface area.

(4.9) These constants are then reduced to eight dimensionless parameters. The key dimensionless parameter ε measures the thermal capacity of the room relative to that of the walls. It is found that this parameter is typically small, which indicates that the temperature, oxygen and fuel are mostly in a quasi steady state, slaved to the slowly evolving wall temperature W. It can be seen that this ε is the key timescale within a room for determining the rate of spread of fire and is independent of the physical dimensions; thus it suggests that a building with one large room might be overcome around 10 times faster than the same building with 10 rooms each one tenth of the size.

(4.10) Simulations of the model are presented in Appendix A.1, where the simple effects of varying the number of rooms and the thickness of the doors is demonstrated. Further possible simplifications of the model are also outlined, as are ways to extend it to more spatial dimensions.

4.3 A PDE model

(4.11) The PDE model begins from a different starting point, namely research from the literature in the 1980s and 90s that use continuum mechanics descriptions of flame propagation and combustion of solid fuels. These models take the form of partial differential equations (PDEs) for the spatial distribution of resources and a measure of the spread of the fire (either in terms of the temperature or an indicator of the completion of reactions).

(4.12) Although more sophisticated models have subsequently been derived (see Appendix A.2), the benefits of using these simple models are that they are generic, robust and require few specialised parameters.

(4.13) We therefore take a simple model for fire spreading through a single floor of a building, treated as a porous medium (with the building's walls as the solid matrix) with solid fuel and oxygen available in the void space. Combustion is then modelled by the same Arrhenius law as in the ODE model, which for

simplicity is approximated by a step function. The diffusion of oxygen and temperature follow the rules for a porous medium with the local porosity ϕ being 1 for open space and zero for a solid wall, with the porosity being depleted by a fire. This results in coupled system of two PDEs for the Oxygen and Temperature fields and ODEs for the local fuel concentration and porosity.

(4.14) Simulations are carried out in 1D, see A.2, in which it was found that, provided the initial concentration of fuel was high enough, a uniform flame front would spread from a fire started at one end of the domain. The speed of propagation is observed to vary like the square root of the initial Fuel concentration. Fire was observed to spread more slowly through walls.

(4.15) Simulations were also carried out in 2D, see A.2. Here different configurations of walls within the building were seen to significantly alter the spread of the fire. This suggests that the areas that will be burned and the rate of reaching the 'far' side of the building will depend strongly on the configuration of the building and the source of the fire.

5 Conclusions and recommendations

(5.1) We have produced two new mathematical models of fire spread within a building that can take different layouts of rooms and model the fire spread, a compartmentalised ODE model and a PDE model. These have been implemented in simple Matlab and C codes. These models are fully parametrised and can be used to investigate the sensitivity of the solutions to different physical assumptions.

(5.2) Both models have solutions where a flame front travels through the building as a form of travelling wave. The speed of travelling is in general non-uniform due to the presence of walls and doors. This property of the solutions suggest that yet further simplifications may be possible in the case of a large building with many rooms. In that limit, further analytical work might be able to derive approximate expressions for the effective speed of fire spread.

(5.3) The first model is inspired by the Kyoto model, but seems to ignore some of the key physics. This was predicated by the assumption that fire spread within rooms is much quicker than that through thermal barriers between rooms (walls). We also have not included modelled buoyancy, where transfer of heat vertically is argued to be an order of magnitude more efficient than laterally. The model has so far only been applied to a 1D chain of rooms. In principle though it could be extended to 2D and 3D layouts.

(5.4) It is worth investigating whether a reduced model based upon the asymptotic approximation $\varepsilon \ll 1$ could be carried out to obtain a simple and fast

algorithm with little data required to populate the model. Homgenisation in the case of many rooms may also yield a further reduction of parameters and AWE can do parameter testing to see which parameters matter.

(5.5) The major weakness of the PDE model is that it takes a reaction-diffusion approach, assuming the building to be a porous medium with walls, doors etc being modelled as an inhomogenity in the medium, and does not directly include convection. At first site this might appear simplistic given our conclusion that convection rather than condition is likely to be major mechanism of fire spread. Incorporation of simple convective terms should indeed be possible and should be considered in any further development of the model.

(5.6) Other effects not included that might be considered for incorporation into the PDE model include influences of radiation, different temperatures for gas and solids, and some effective heat-transfer coefficients to describe the boundary conditions at the exterior of the building (these would presumably be like Saffman, Beavers and Joseph conditions at the interface between porous media and other domains – see for example the paper [18]). We also need to potentially make the model 3D, to allow for buoyancy effects which are likely to allow the fire to preferentially spread vertically.

(5.7) We recommend that AWE try running simulations themselves of the two models.

(5.8) We also recommend that AWE investigate forming links with either the authors of the Kyoto model, or with one of the producers of the high-fidelity computational models, such as NIST. The ability to run these models may enable AWE to benchmark lower fidelity building models that they might wish to incorporate within their larger city-scale simulation model.

A Appendices

A.1 Details of simple ODE model

This section is written by Ian Hewitt

Setup

Consider the sequence of N rooms as depicted in Fig. 4. The model considers each room as a lumped element, characterised by its average air temperature T, average oxygen concentration O, average vaporised fuel concentration F, and average wall temperature W. The low thermal capacity of the air means that the air temperature rapidly increases during a fire; the walls, which have a much higher heat capacity, take longer to warm up. The separate treatment of wall temperature is therefore a key feature of this model. Since

W represents an average wall temperature, the door into the next room is assumed to be at this same temperature and the critical flashover point occurs when W reaches W_c. At that moment the door opens, and allows mass and heat exchange with the next room. This process is described by a variable D, representing the area of the door: D is initially zero, but once $W > W_c$ it switches to the full door area $\tilde{D}$ (this is an irreversible switch).

The fire is described by a reaction, which we take to occur at a rate $Ce^{-E/RT}OF$, *i.e.* an Arrhenius temperature dependence and a simple linear dependence on oxygen and fuel concentrations. Oxygen is present in each room initially, at a prescribed concentration O_0. F describes vaporised fuel, of which there is initially assumed to be none. The fuel is produced by volatilising of *solid* fuel that is present in the room; this fuel production is assumed to occur when the temperature of the solid fuel exceeds a volatilising temperature T_p; any excess heating of the solid fuel is used to vaporise the fuel according to its latent heat of vaporisation, L, and its available surface area, S. In principle, the solid fuel temperature could be tracked as an independent variable. To keep the model as simple as possible, however, we take it to have the same temperature W_i until it reaches the critical temperature T_p. Thus W_i could be interpreted as the average temperature of the various 'surfaces' within the room.

Heat transport between the air and the surface is taken to occur at a rate $\rho c K_w(T - W)$. K_w is simply treated as a constant here. It is likely that the air in the room is undergoing turbulent convection, and this heat transfer results from the forced convection. When the door to the next room is opened, heat transfer is taken to occur at a rate $\rho c K \Delta T$, where ΔT is the temperature difference with the next room. This transfer is primarily the result of buoyancy driven exchange of air. It is likely that K may in fact depend on ΔT, but for the simplest model we take it as constant.

Equations

The model consists of conservation equations for heat, oxygen, fuel and wall heat in each room. Subscript i is used to denote the ith room and the door to its right. There are exchange terms with the neighbouring room provided the door to that room is open. The equations are

$$V_i \frac{dT_i}{dt} = K\left[D_{i-1}(T_{i-1} - T_i) - D_i(T_i - T_{i+1})\right] + \frac{\Delta H}{\rho c} C e^{-E/RT_i} O_i F_i$$
$$- K_w A_i (T_i - W_i) - K_w S_i \left[(T_i - W_i)\mathcal{H}(T_p - W_i) + (T_i - T_p)\mathcal{H}(W_i - T_p)\right],$$

$$V_i \frac{dO_i}{dt} = K\left[D_{i-1}(O_{i-1} - O_i) - D_i(O_i - O_{i+1})\right] - \alpha C e^{-E/RT_i} O_i F_i,$$

$$V_i \frac{dF_i}{dt} = K\left[D_{i-1}(F_{i-1} - F_i) - D_i(F_i - F_{i+1})\right] - \beta C e^{-E/RT_i} O_i F_i + \frac{\rho c}{L} K_w S_i (T_i - T_p)\mathcal{H}(W_i - T$$

$$\frac{\rho_w c_w}{\rho c} d_w \left[A_i + rS_i \mathcal{H}(T_p - W_i)\right] \frac{dW_i}{dt} = K_w A_i (T_i - W_i) + K_w S_i (T_i - W_i)\mathcal{H}(T_p - W_i).$$

Here $\mathcal{H}(\cdot)$ denotes the Heaviside function, used to account for a switch that occurs at the critical temperature T_p: for $W < T_p$ the heat loss to the surfaces goes entirely into warming them up, for $W > T_p$, part of it goes into volatilising the solid fuel, and the rest continues the warm the walls. The proportion is set by the relative surface areas of fuel, S, and walls, A. The wall thickness d_w should probably be interpreted as a conductive length scale into the walls, since it is unlikely that the full thickness of the walls heats up on the timescales we are concerned with. Since the value of wall temperature is most important for triggering the flashover of the doors, it may be most sensible to take d_w to be the thickness of the doors.

Each door is described by

$$D_i = \begin{cases} \tilde{D}_i & \tilde{W}_i \geq W_c \\ 0 & \text{otherwise,} \end{cases}$$

where $\tilde{W}_i$ is the maximum wall temperature of the room over all time since the door was constructed, and $\tilde{D}_i$ is the area of the open door.

Dimensionless model

Typical values of room volume, wall area, solid fuel area, and door area, are taken to be V_0, A_0, S_0 and D_0, respectively. We scale the temperatures T and W with the volatilising temperature, $T_0 = T_p$, oxygen concentration with its initial value O_0, fuel concentration with F_0, chosen as

$$F_0 = \frac{\rho c T_0}{L} \frac{K_w S_0}{K D_0}, \tag{1}$$

(which balances its production rate with the transport rate). The time scale is chosen to be the longest timescale in the problem, *i.e.* the rate of heating the walls,

$$t_0 = \frac{\rho_w c_w d_w}{\rho c K_w}. \tag{2}$$

After scaling the variables with these typical values, the equations become

$$\begin{aligned} \varepsilon\lambda V_i \frac{dT_i}{dt} &= D_{i-1}(T_{i-1} - T_i) - D_i(T_i - T_{i+1}) + \mu_T e^{-T_m/T_i} O_i F_i \\ &\quad - \lambda A_i (T_i - W_i) - \lambda\nu S_i \left[(T_i - W_i)\mathcal{H}(1 - W_i) + (T_i - 1)\mathcal{H}(W_i - 1)\right], \\ \varepsilon\lambda V_i \frac{dO_i}{dt} &= D_{i-1}(O_{i-1} - O_i) - D_i(O_i - O_{i+1}) - \mu_O e^{-T_m/T_i} O_i F_i, \\ \varepsilon\lambda V_i \frac{dF_i}{dt} &= D_{i-1}(F_{i-1} - F_i) - D_i(F_i - F_{i+1}) - \mu_F e^{-T_m/T_i} O_i F_i + S_i (T_i - 1)\mathcal{H}(W_i - 1), \end{aligned}$$

ρ	0.5 kg m^{-3}	α	5	T_a	300 K
ρ_w	2000 kg m^{-3}	β	1	W_c	900 K
c	1000 J kg^{-1} K^{-1}	ΔH	8.4×10^5 J mol^{-1}	d_w	0.03 m
c_w	960 J kg^{-1} K^{-1}	L	3.6×10^5 J mol^{-1}	V_0	30 m^2
R	8.3 J mol^{-1} K^{-1}	K_w	0.1 m s^{-1}	A_0	50 m^2
E	1.1×10^4 J mol^{-1}	K	10 m s^{-1}	S_0	50 m^2
C	5 m^3 mol^{-1} s^{-1}	T_p	500 K	D_0	6 m^2

Table 1: Approximate values for physical parameters.

ε	λ	ν	μ_T	μ_O	μ_F	T_m	W_c
0.005	0.08	1	0.16	0.02	0.83	2.6	1.8

Table 2: Approximate values of dimensionless parameters.

$$[A_i + \nu S_i \mathcal{H}(1-W_i)]\,\frac{dW_i}{dt} = A_i(T_i - W_i) + \nu S_i (T_i - W_i)\mathcal{H}(1-W_i).$$

All the variables are now dimensionless (note that if all the rooms have the same geometry and solid fuel content, the V_i, A_i and S_i are all simply 1). There are several dimensionless parameters:

$$\varepsilon = \frac{\rho c V_0}{\rho_w c_w d_w A_0}, \qquad \lambda = \frac{K_w A_0}{K D_0}, \qquad \nu = \frac{S_0}{A_0},$$

represent respectively the thermal capacity of the room relative to the walls (expected to be small), the efficiency of heat transport to the walls relative to that through the doors, and the ratio of solid fuel area to wall area;

$$\mu_T = \frac{\Delta H}{L}\frac{CO_0}{KD_0}\lambda\nu, \qquad \mu_O = \frac{\alpha C}{KD_0}\frac{\rho c T_0}{L}\lambda\nu, \qquad \mu_F = \frac{\beta C O_0}{KD_0},$$

represent the strength of the reaction relative to transport, for heat, oxygen and fuel, respectively; and

$$T_m = \frac{E}{RT_0}$$

is the dimensionless reaction temperature. W_c becomes the dimensionless critical door temperature.

Some typical values for these parameters are shown in table 2. The fact that ε is small indicates that the temperature, oxygen and fuel are mostly in a quasi steady state, slaved to the slowly evolving wall temperature W. Physically, this is the timescale given by (2), which is evidently a key property for determining the rate of fire propagation. Note that it is a timescale for an individual room, and is independent of the physical dimensions; thus it suggests that a building with one large room might be overcome around 10 times faster than the same building with 10 rooms each one tenth of the size. For the values in table 1, the timescale is around 19 minutes.

Results

For these example solutions, we start with identical conditions ($T = W = T_a$, $O = O_0$, $F = 0$) in each room and open the door from the outside world to the first room. The outside world has temperature T_m (*i.e.* there is a raging fire outside), oxygen concentration O_0 and fuel concentration zero, (*i.e.* the air outside is well mixed, so that there is a constantly replenished supply of oxygen and any fuel is rapidly transported away). On breaking through the last door in the building, the outside conditions there are held at the initial air temperature T_a, oxygen concentration O_0 and fuel concentration zero (these conditions have no effect until the last door is breached).

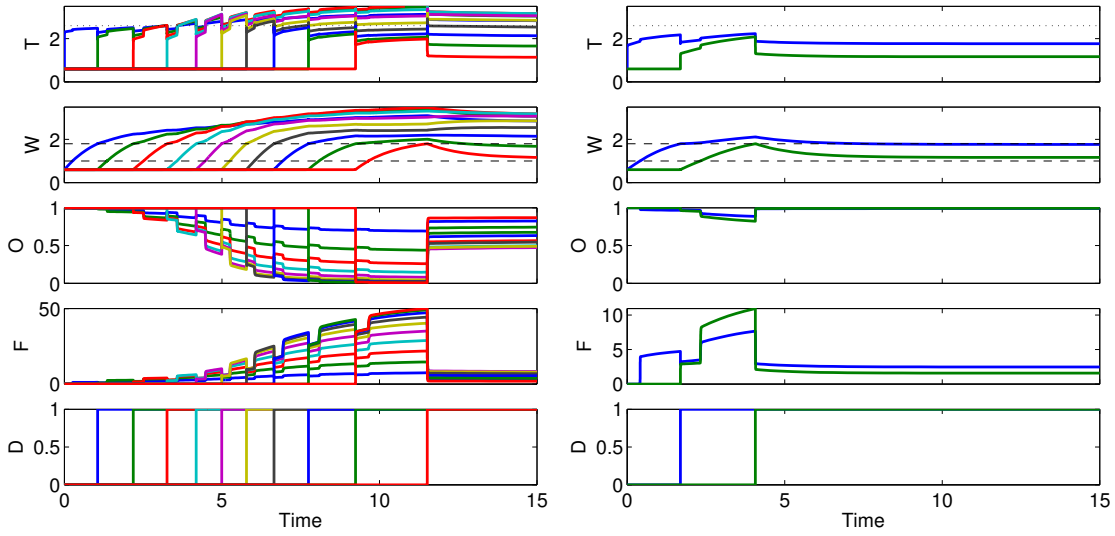

Figure 5: Example solutions driven by temperature T_m, oxygen concentration O_0, and fuel concentration 0 outside the first room. Different rooms are shown by different coloured lines; the first room is blue, the second green etc. The left solution shows a building with 10 identical rooms; the right solution shows the same building with just 2 rooms that are five times as long. Parameter values are as in table 2.

Figure 5 shows two solutions, with different coloured lines corresponding to the conditions in each room. The first is for a building with 10 rooms, and the second shows the same building with just 2 rooms, each five times as long. It is clear that the time taken to spread through the building (*i.e.* until the last door opens) is considerably shorter in the second case, despite the fact that it takes longer to break through each room. Notice also that the fire becomes much more vigorous in the first case. This is because it is held up for longer by the doors, allowing much more fuel to be vaporised and the combustion reaction to really take off. As a result the oxygen is used up, and the growth of the fire becomes oxygen limited. In the second case, the temperature never gets up to the reaction temperature T_m, and there is a plentiful supply of oxygen.

Figure 6 shows two other solutions with 10 rooms, but with slightly different conditions. In the first, the doors are twice as large, which allows more rapid

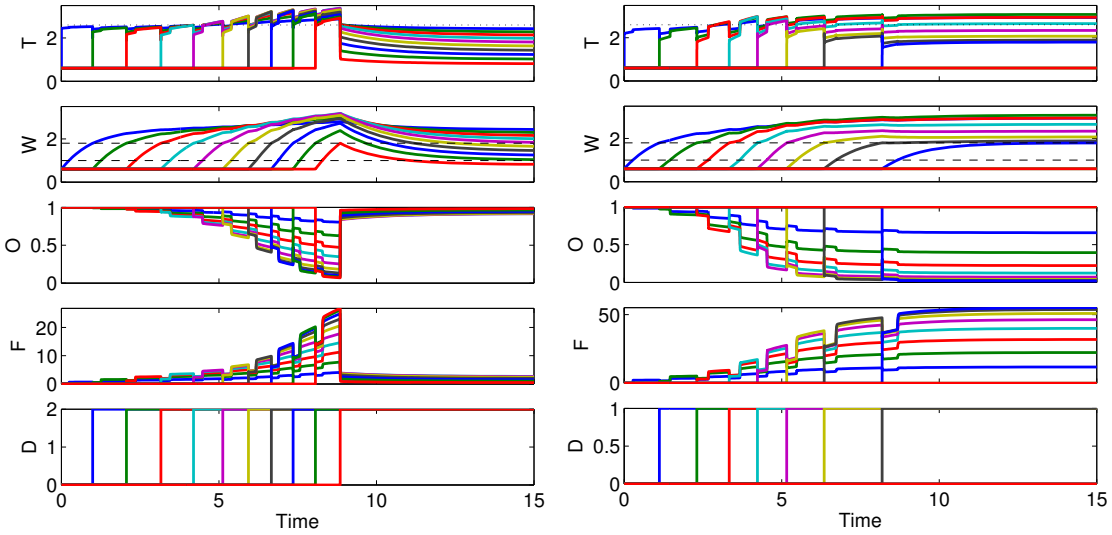

Figure 6: Similar solutions to the first case in figure 5 for a sequence of 10 rooms, but with doors that are twice the size (left), or with twice the solid fuel area (right).

transfer between adjacent rooms and, as might be expected, reduces the time for the fire to spread. In the second case, there is twice the amount of fuel. This has the rather non-intuitive effect of slowing the spread of the fire to the extent that it never spreads through the building. This is because the larger amount of fuel leads to a more vigorous reaction that consumes oxygen more quickly. Once the fire has spread into 8 rooms there is insufficient oxygen to allow it to progress further.

Extensions

The analysis of this model has not been exhausted by any means. Indeed it has barely been started. One could analyse the behaviour in more detail and work out which of the dimensionless parameters provide the greatest control. This would be of benefit for the original problem, because a minimalist description is what is required.

- A significant simplification of this model is to treat the mass of air in each room as fixed. Exchange with neighbouring rooms is what causes the convective transfer through the doors, but in order to be consistent with the constant mass assumption it must be assumed that there is always a compensating air flow in the opposite direction. It is possible that this might be justified under certain conditions, but also possible that this simplification is erroneous. Further work might establish this.
- The model could be easily extended to more dimensions, with several doors and connections per room. Such increasing generality would make it more and more like the Kyoto model.

- We assume that the initial supply of solid fuel in the room is not depleted. This is probably sufficient for understanding the initial spread of the fire, but if one wanted to know more about how the fire evolves and then dies down, one could include an additional conservation equation for the solid fuel in each room.

A.2 Details of reduced PDE models for fire spreading

This section is written by Tom Witelski

A lot of research was done in the 1980s and 1990s on formulating mathematical models for flame propagation and combustion of solid fuels [8, 6]. Based on continuum mechanics descriptions, these models are expressed in sets of partial differential equations for the spatial distribution of resources and a measure of the spread of the fire (either in terms of the temperature or an indicator of the completion of reactions). The studies focused on a few research directions: propagation of flame fronts [20, 5], wildfires in forest/brushland [1, 12], porous medium [9, 10, 22] and applications like smoking of cigarettes [27, 11]. While later studies have produced more sophisticated and specialised models, some of the virtues in applying these earlier models are:

Their generic structure is robust and captures the dominant physical mechanisms involved. This is in-line with the limited knowledge expected to be available about the variety of materials and designs to be encountered in fires in different types of buildings.

- In basic form, they need a minimal amount of data on fuel and geometry to specific the problem. This will be appropriate for the city-scale simulations, where only limited data can be provided on each building.
- They can be generalised to yield more specialised models, but were developed in times of more limited computing resources, so their forms are computationally tractable and efficient.

Taking elements from these papers (mostly from the work on flame fronts and propagation in porous medium), we describe a simple model for fire spreading through a single floor of a building. We treat the building as a porous medium (with the building's walls as the solid matrix) with solid fuel and oxygen available in the void space. Combustion will be modelled by the simple reaction

$$\text{Fuel} + O_2 \xrightarrow{k(T)} \text{burned product and released heat}, \tag{3}$$

where an Arrhenius law, $k(T) = k_0 \exp(-E/[RT])$, will be used to describe the dependence of the reaction rate on the temperature. The model will describe the evolution of the concentrations of the fuel and oxygen $(F(x,y,t), O(x,y,t))$ and the temperature field $T(x,y,t)$. The interior structure of the building will be given in terms of a porosity, $0 \leq \phi(x,y,t) \leq 1$, which may evolve as the building suffers structural damage from the fire. The porosity ϕ gives the local void fraction, so $\phi = 1$ is open space while

$\phi = 0$ represents a solid impermeable wall. Intermediate values of ϕ could describe doors, windows or other structures providing less resistance to the spread of fire.

The model is given by first order kinetic equations for the depletion of the fuel and oxygen in (3),

$$\frac{\partial F}{\partial t} = -k(T)FO, \tag{4a}$$

$$\frac{\partial O}{\partial t} = D_O \nabla \cdot (\phi \nabla O) - k(T)FO, \tag{4b}$$

where we note that the solid fuel is assumed immobile, but the oxygen spreads by diffusion through the void space. The temperature evolves according to a reaction-diffusion equation from the heat released by combustion, possibly with a different diffusion coefficient than oxygen,

$$\frac{\partial T}{\partial t} = D_T \nabla \cdot (\phi \nabla T) + \alpha k(T)FO, \tag{4c}$$

note that the temperature scale can be normalised between some ambient temperature ($T = 0$) and the maximum temperature at peak combustion ($T = 1$). Finally, combustion will also govern the rate of increase of porosity as structures degrade from the fire,

$$\frac{\partial \phi}{\partial t} = \beta k(T)(1 - \phi). \tag{4d}$$

To further simplify this model, a widely used approximation is to replace the Arrhenius exponential dependence on temperature with an equivalent step function that switches on above an ignition threshold (see Fig. 7)

$$k(T) = \begin{cases} 0 & T < T_{\text{ign}} \\ 1 & T > T_{\text{ign}} \end{cases} \tag{5}$$

Use of this approximation has been considered in more detail in [4].

Simulations of system (4abcd) require initial conditions on all four fields $(F, 0, T, \phi)$ and boundary conditions on $0, T$. In our preliminary trials, we found that explicit finite difference methods on uniform grids (second order accurate in space, first order with respect to time) were very easy to implement and performed with good speed once (5) was used. We present results from a few illustrative simulations. The typical scenario all of the simulations will share is an imposed temperature and source of oxygen on one side or at a point along the boundary of the building (from fire breaching the exterior walls).

One-dimension simulations

From simulations in one-dimension, it was seen that the model produced flame fronts that propagated with constant speed, see Fig. 8. This base-line

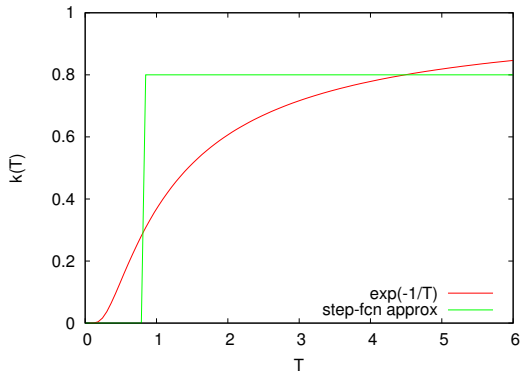

Figure 7: Plot of the Arrhenius reaction rate function $k(T) = \exp(-1/T)$ and an approximating step-function given by (5).

simulation could be imagined to represent fire spreading through a large warehouse. The simulation was carried out with uniform initial conditions on fuel, oxygen, and temperature, $F(x,0) = O(x,0) = 1$ and $T(x,0) = 0$. The porosity was uniformly set to $\phi(x) = 1$ in the interior to correspond to open space.[1] The diffusion coefficients were taken to be small compared to the reaction rate, $D_0 = D_T = 0.01$, $\alpha = 1$ with $T_{\text{ign}} = 0.2$. The exterior source of fire is represented by a boundary condition – at $x = 0$ the temperature is imposed as $T(0,t) = 1$. Travelling wave behaviour is also exhibited in single reaction-diffusion equations like Fisher's equation, e.g. [29], but such further-simplified models may be difficult to interpret since they lack degrees of freedom to independent specific conditions on the fuel and temperature.

To see the effect of changing the initial fuel level, simulations were run with $F(x,0) = F_0$ for $0 < F_0 \leq 1$, see Fig. 9. Below some critical minimum density of fuel (for these parameters, it was $F_0 > 0.24$), the fire decays and halts after moving a limited distance. For greater amounts of fuel, the fire spreads with constant speed as in Fig. 8, but with two modifications: (i) the average temperature behind the front scales linearly with F_0 and (ii) the speed of propagation also depends on F_0 similar to $v \approx \sqrt{F_0}$, see Fig. 9. Some preliminary testing was done on how nonuniform initial distributions of fuel with $\bar{F}_0 = \frac{1}{L}\int_0^L F(x,0)\,dx$ effects the spreading of fire. While the local speed of propagation becomes non-uniform, large variations in the average speed expected with F_0 were not observed, but this may need more careful study.

The presence of internal structure in the building can have a much stronger effect on the spread of the fire. Figure 10 shows simulations comparable to Fig. 8 except for the addition of interior walls that the fire must burn through if it is to spread further. Walls were represented by piecewise-constant changes in the porosity, $\phi(x,0) = 0$ on $x \in [x_* - \sigma, x_* + \sigma]$, with

[1] Permanent (non-degrading) exterior walls at $x = 0$ and $x = 1$ were set with $\phi = 1/2$.

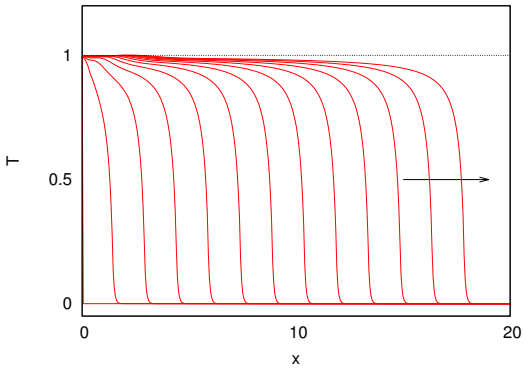

Figure 8: Evolution of the temperature $T(x,t)$ for fire spreading with constant speed through a uniform one-dimensional domain. Temperature profiles are shown at equally spaced times.

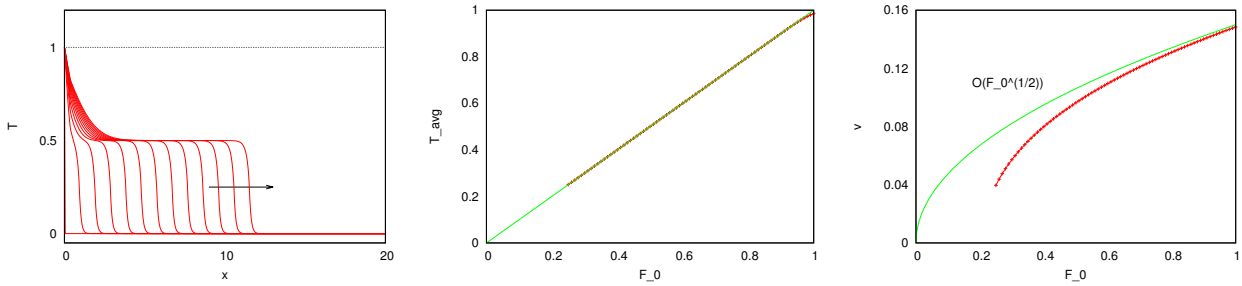

Figure 9: Influence of the initial level of available fuel F_0 on the spread of the fire: (left) Temperature profiles as in Fig. 8 but with $F_0 = 0.5$, (middle) The average temperature behind the front scales linearly with F_0, (right) The speed of propagation grows like $O(\sqrt{F_0})$.

one wall in Fig. 10(left) at $x_* = 10$ and three walls in the (middle) plot, at $x_* \in \{5, 10, 15\}$. The rate at which a wall burns down is set by the parameter β in (4d) and can be adjusted to appropriate values for different types of materials. In these simulations, $\beta = 0.001$ to indicate moderately fire-resistant material. The right plot in Fig. 10 shows the position of the advancing fire front from Fig. 8 and the two cases with walls. In each case, the speed moving through empty space is always the same. It is seen that the speed (the inverse slope in the plot) is much slower (on the order of β/α) through the thin domains occupied by the walls. Effectively the fire stops spreading at a wall, burns through it at the rate set by (4d) and then continues on. If a wall if very thick or β is very small, then the fire can run out of fuel and be stopped by the wall.

Two-dimensional simulations

In one-dimension, if blocked by a wall the fire has no alternative except to wait to burn through it. In two-dimensions, depending on the configuration

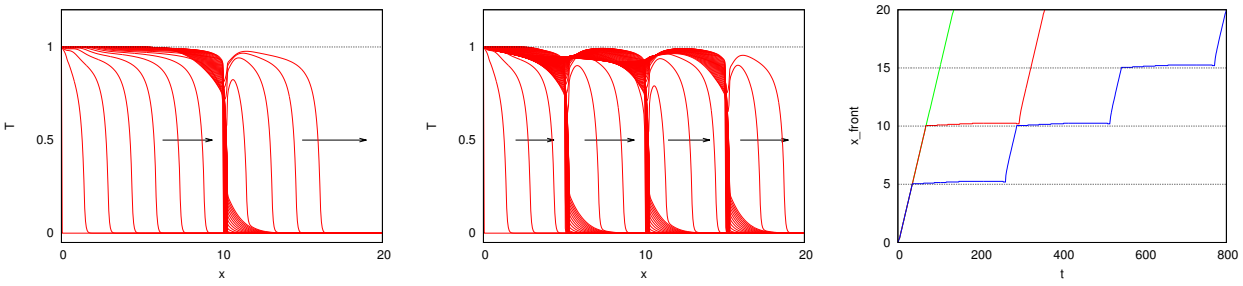

Figure 10: Spread of fire through buildings with interior walls: (left) Temperature profiles at equally spaced times with a wall at $x_* = 10$, (middle) $T(x,t)$ profiles with walls at $x_* \in \{5, 10, 15\}$, (right) position of the leading edge of the fire as a function of time with no-walls (green), one wall (red), three walls (blue).

of the walls and on how long it takes to burn through the walls, the fire may instead progress faster by spreading around the walls. Figure 11 shows a building with three interior walls being hit by a uniform front of fire coming from two different orientations: (left) in the y-direction, (right) in the x-direction, with the walls being aligned in the y-direction. The walls are simulated to be made of the same material as in the previous simulations ($\beta = 0.001$), but are thicker here. Some burning into the thickness of the walls is visible, but in general, the fire primarily moves around the walls rather than getting slowed down much by the walls. In the left case the fire is ideally oriented to propagate straight through the building as if the walls were entirely absent. In the right case, the progress of the fire is significantly slowed down by the need to travel along the hallways; it will take almost twice as long for the fire to reach the far-side of the building in this case.

Figure 12 shows a building with a different configuration of interior walls (perhaps representing rooms and a hallway in a hotel). The figure shows the advancing fire front starting from a point source in one room. The fire that enters the hallway spreads to the far-side of the building long before the thicker walls between the rooms can burn through. This illustrates that the areas that will be burned in a fire strongly depends on the configuration of the building and the source of the fire. In this example if the fire began on the $y = 20$ wall of the hallway, it will likely spread into all of the rooms at effectively the same time.

Suggestions for a homogenised, reduced model

The observations from the one- and two-dimensional simulations suggest that homogenisation theory [16, 23] might be applicable to replace the interior wall structure of the building with some effective averaged building-scale properties. This would analogous to the derivation of Darcy's law for flow in porous media eliminating the micro-scale properties of the soil [17, 28]. From the two-dimensional simulations we expect the model to

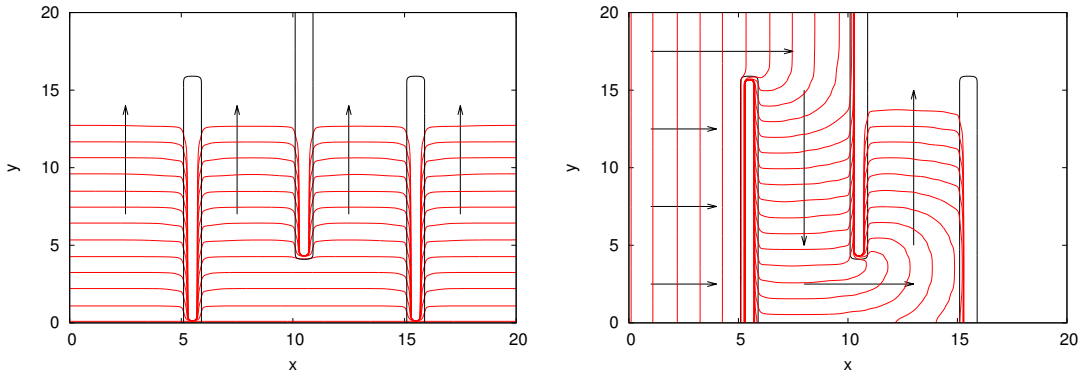

Figure 11: Influence of the initial level of available fuel F_0 on the spread of the fire: (left) Temperature profiles as in Fig. 8 but with $F_0 = 0.5$, (middle) The average temperature behind the front scales linearly with F_0, (right) The speed of propagation grows like $O(\sqrt{F_0})$.

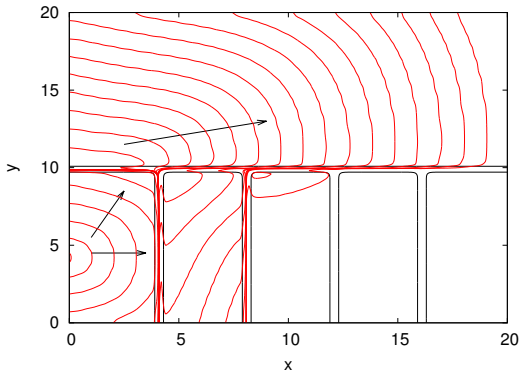

Figure 12: Influence of the initial level of available fuel F_0 on the spread of the fire: (left) Temperature profiles as in Fig. 8 but with $F_0 = 0.5$, (middle) The average temperature behind the front scales linearly with F_0, (right) The speed of propagation grows like $O(\sqrt{F_0})$.

include anisotropic effects, as in

$$\frac{\partial F}{\partial t} = -k(T)FO \tag{6a}$$

$$\frac{\partial O}{\partial t} = D_O \nabla \cdot (\mathbf{D}\nabla O) - k(T)FO \tag{6b}$$

$$\frac{\partial T}{\partial t} = D_T \nabla \cdot (\mathbf{D}\nabla T) + \alpha k(T)FO \tag{6c}$$

where

$$\mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{pmatrix} = \begin{pmatrix} D_{11}\cos^2\Theta + D_{22}\sin^2\Theta & (D_{11} - D_{22})\sin\Theta\cos\Theta \\ (D_{11} - D_{22})\sin\Theta\cos\Theta & D_{11}\sin^2\Theta + D_{22}\cos^2\Theta \end{pmatrix} \tag{6d}$$

where the interior structure is described in average form by two relative diffusion coefficients (D_{11}, D_{22}) and the angle Θ giving the orientation of the principle axes.

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